29 Quantum Electrodynamics of a Qubit

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A detailed understanding of the basic physical laws governing the exchange of quantum information, as well as the interaction between material qubits and the quantized electromagnetic field, is of central importance for realizing quantum information networks and for suppressing decoherence due to spontaneous emission of photons. In this section, some basic physical aspects of this interaction are explored in the special case of a single material qubit.

The energy exchange between a material qubit interacting with the electromagnetic field is dominated by the absorption and emission of photons [1]. Whereas absorption and stimulated emission of photons is conditioned on photons which are already present in the electromagnetic field, spontaneous emission of photons occurs randomly and even if the electromagnetic field is in its ground state (vacuum) [2]. It is this random and uncontrollable feature of this latter process which causes spontaneous decay and decoherence of qubits. Therefore, suppressing its undesired and uncontrollable features is one of the major challenges in the context of quantum information processing. For this purpose powerful error correction methods have been designed recently [3–5]. Alternatively, spontaneous decay of qubits can also be suppressed at least partially by an appropriate engineering of their coupling to the electromagnetic field.

The quantum dynamics of a material qubit interacting with the electromagnetic field depend significantly on the structure of the field modes. If a qubit is coupled to a single-field mode only, its quantum state can be transferred to the field mode and back again in a reversible way as described by the Jaynes–Cummings–Paul model [6,7]. This reversible energy exchange manifests itself in vacuum Rabi oscillations of the qubit between its excited state and its ground state, for example [7]. But with increasing number of interacting field modes this reversible character of the qubit–field dynamics is lost gradually [8–11]. In particular, in the limit of a continuum of accessible field modes the reversibility of the state exchange between qubit and field is lost completely. Typically, under such circumstances an initially excited qubit decays to its ground state spontaneously [12]. As a result, a controllable and reversible transfer of the quantum state of such a qubit to the electromagnetic field and back again becomes impossible. In general, the spontaneous decay rate of the qubit depends on the density of field modes it is coupled to. For purposes of processing quantum information, for example, this latter dependence can be exploited for suppressing spontaneous decay process by an appropriate engineering of the mode structure of the electromagnetic field [13]. Photonic crystals [14] are particularly well suited for this purpose.

In this section, we discuss basic physical aspects of the interaction between a single qubit and the electromagnetic field. In particular, we focus on the following main questions: How

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does the interaction of a qubit with the electromagnetic field depend on the structure and the density of states of field modes? How does the reversible dynamics of the coupling to a single mode of the radiation field change to an irreversible energy transfer to the electromagnetic field as the number of interacting field modes is increased and the continuum limit is approached? What is the characteristic time evolution of the spontaneous decay of a qubit embedded in a photonic crystal? How do bandgaps influence this decay? Is it possible to form bound qubit–field states within a photonic crystal?

In order to address these questions in Section 29.1 we first of all analyze the dynamics of the spontaneous decay of a single qubit which is assumed to be located at the center of a spherically symmetric metallic cavity. The spherical symmetry of the electromagnetic field modes the qubit is coupled to allows us to address many aspects of this problem even analytically. Within this model system we can explore the influence of the cavity size on the dynamics of the spontaneous decay process of the qubit. The continuum limit of this model is achieved in the limit of an infinitely large cavity. Basic aspects of the dynamics of spontaneous decay of a qubit in a photonic crystal, the influence of photonic bandgaps on this decay process, and the possibility of forming bound qubit-field states are explored in Section 29.2.

29.1 Quantum Electrodynamics of a Qubit in a Spherical Cavity

In this section we discuss the spontaneous emission of a photon by a single (infinitely heavy) qubit which is assumed to be located at the center of a spherically symmetric metallic cavity. With the help of a semiclassical path representation valid for highly excited field modes, the probability amplitude of observing the qubit in its initially excited state at any later time is expressed as a sum of probability amplitudes which are associated with repeated returns of the spontaneously emitted photon to the center of the cavity where it interacts again with the qubit [10]. In this way we obtain a unified description of the spontaneous emission process which, in the spirit of the Feynman path integral approach [15], sheds light onto the underlying elementary physical processes involved in the gradual transition from reversible to irreversible energy exchange between a single qubit and the electromagnetic field.

29.1.1 The model

We consider a single qubit, i.e. a quantum mechanical two-level system with (bare) energy eigenstates $|g\rangle$ and $|e\rangle$ of well-defined parity and corresponding energies $E_{\rm g} = 0$ and $E_{\rm e} > 0$, interacting with the electromagnetic radiation field in a spherical cavity. The field-modes are identified by the mode indices $l \in I$ and ω_l are their frequencies. This two-level system is assumed to be localized at the center of a spherical cavity, say at position $\mathbf{x} = \mathbf{0}$, and the spatial extension of its charge distribution is assumed to be much smaller than the wave lengths of the electromagnetic field modes it is coupled to significantly. Furthermore, this qubit is supposed to be infinitely heavy so that its center of mass motion is not affected by momentum transfer from the electromagnetic field to the qubit. Thus, in the dipole approximation [1] the

Hamiltonian of this quantum system is given by

$$\hat{H} = \hbar\overline{\omega}_0 \hat{b}_0^\dagger \hat{b}_0 + \sum_{l\in I} \hbar\omega_l \hat{a}_l^\dagger \hat{a}_l - \hat{\mathbf{d}} \cdot \hat{\mathbf{E}}(\mathbf{x}=0) \tag{29.1}$$

with the dipole operator of the two-level system

$$\hat{\mathbf{d}} = \hat{b}_0 \langle g|\hat{\mathbf{d}}|e\rangle + \hat{b}_0^\dagger \langle e|\hat{\mathbf{d}}|g\rangle \tag{29.2}$$

and with the atomic ladder operators $\hat{b}_0 = |g\rangle\langle e|$ and $\hat{b}_0^\dagger = |e\rangle\langle g|$. The position of the qubit is denoted by $\mathbf{x}$ while its (bare) excitation energy is given by $\hbar\overline{\omega}_0 \equiv (E_\mathrm{e} - E_\mathrm{g})$. The creation and annihilation operators of mode $l \in I$ of the electromagnetic field are denoted by $\hat{a}_l^\dagger$ and $\hat{a}_l$, respectively. Correspondingly, the expansion of the electric field operator $\hat{\mathbf{E}}(\mathbf{x})$ in terms of the orthonormal set of mode functions $\mathbf{u}_l(\mathbf{x})$ is given by

$$\hat{\mathbf{E}}(\mathbf{x}) = -\mathrm{i}\sum_l \sqrt{\frac{\hbar\omega_l}{2\epsilon_0}} \{\mathbf{u}_l^*(\mathbf{x})\hat{a}_l^\dagger - \mathbf{u}_l(\mathbf{x})\hat{a}_l\} \tag{29.3}$$

with ϵ_0 denoting the dielectric constant of the vacuum. In Eq. (29.1) only the interaction of the two-level system with the almost resonantly coupled modes ($l \in I$) of the electromagnetic field is taken into account. To a good degree of approximation these couplings can be described approximately by the rotating-wave approximation [1]. Thereby, the interaction operator $-\hat{\mathbf{d}} \cdot \hat{\mathbf{E}}(\mathbf{x}=0)$ is approximated by the expression

$$\mathrm{i}\langle g|\hat{\mathbf{d}}|e\rangle \hat{b}_0 \sum_l \sqrt{\frac{\hbar\omega_l}{2\epsilon_0}} \mathbf{u}_l^*(\mathbf{x}=0)\hat{a}_l^\dagger - \mathrm{i}\langle e|\hat{\mathbf{d}}|g\rangle \hat{b}_0^\dagger \sum_l \sqrt{\frac{\hbar\omega_l}{2\epsilon_0}} \mathbf{u}_l(\mathbf{x}=0)\hat{a}_l.$$

The couplings to all other modes ($l \notin I$) which are not taken into account by the dipole and rotating-wave approximation can be treated at a later stage perturbatively. These modes give rise to a radiative level shift, i.e. $\overline{\omega}_0 \to \omega_0$, where $\hbar\omega_0$ denotes the physically observed energy difference of the qubit-system considered [compare with the discussion following Eq. (29.13)]. In the case of a realistic atom these radiative energy shifts are the well-known Lamb shifts [16,17]. It is worth mentioning that for a proper treatment of the influence of these off-resonant modes the dipole approximation is no longer applicable [18].

Within this model we aim at describing the influence of the mode structure of the cavity onto the spontaneous emission of photons by the qubit. Thus, we want to restrict ourselves to an initial condition in which the two-level system is prepared in its excited state $|e\rangle$ and the electromagnetic field is in its vacuum state $|\{0\}\rangle$. Due to the coupling between the two systems the two-level system will exchange its excitation predominantly with the resonantly coupled modes of the electromagnetic field with $\omega_l \approx \omega_0$. As long as we restrict ourselves to this particular initial condition we can replace the two-level system also by a harmonic oscillator by interpreting the operators $\hat{b}_0^\dagger$ and $\hat{b}_0$ of Eqs. (29.1) and (29.2) as the creation- and destruction-operators of a harmonic oscillator. This is possible because by energy conservation in this case only the ground and first excited state of this harmonic oscillator participate in the dynamical evolution. Such a replacement offers advantages because the dynamical evolution of the qubit interacting with the electromagnetic field reduces effectively to the diagonalization of a system of coupled harmonic oscillators.

29.1.2 Mode structure of the free radiation field in a spherical cavity

Before addressing this diagonalization let us first of all determine the mode structure of the free electromagnetic radiation field in a spherical cavity with ideal metallic boundary conditions.

In the Coulomb gauge [1] the electromagnetic field can be decomposed into two parts, namely an instantaneous Coulomb interaction between charged particles and the transverse radiation field. Thus, in the Schrödinger picture the radiation field is described by a vector potential $\hat{\mathbf{A}}(\mathbf{x})$ which fulfills the transversality condition $(\nabla\cdot\hat{\mathbf{A}})(\mathbf{x}) = 0$. This vector potential can always be decomposed into complete orthonormal sets of mode functions $\mathbf{u}_l(\mathbf{x})$ according to

$$\hat{\mathbf{A}}(\mathbf{x}) = \sum_l \sqrt{\frac{\hbar}{2\epsilon_0\omega_l}}\{\mathbf{u}_l(\mathbf{x})\hat{a}_l + \mathbf{u}_l^*(\mathbf{x})\hat{a}_l^\dagger\}.$$

These mode functions are orthonormal solutions of the Helmholtz equation

$$(\nabla^2 + (\omega_l/c)^2)\mathbf{u}_l(\mathbf{x}) = 0 \tag{29.4}$$

fulfilling the appropriate boundary conditions.

In order to generate such a complete system of mode functions for a spherical cavity with ideal metallic boundary conditions the tangential component of $\mathbf{u}_l(\mathbf{x})$ and the normal component of $(\nabla \wedge \mathbf{u}_l)(\mathbf{x})$ have to vanish on the surface of the spherical boundary. The resulting solutions $\mathbf{u}_l(\mathbf{x})$ of Eq. (29.4) determine the discrete set of all possible eigenfrequencies ω_l. Thereby the mode index l identifies all possible mode functions. Thus, in the Schrödinger picture the electric and magnetic field operators are given by Eq. (29.3) and by

$$\hat{\mathbf{B}}(\mathbf{x}) = \sum_l \sqrt{\frac{\hbar}{2\epsilon_0\omega_l}}\{(\nabla \wedge \mathbf{u}_l)(\mathbf{x})\hat{a}_l + (\nabla \wedge \mathbf{u}_l)^*(\mathbf{x})\hat{a}_l^\dagger\}.$$

In particular, in the case of a spherical cavity of radius R with ideal metallic boundary conditions one may choose two different classes of mode functions, namely

$$\mathbf{U}_{nLM}(\mathbf{x}) = \mathcal{N}_{nL}\; j_L(k_{nL}r)\mathbf{X}_{LM}(\mathbf{x}/||\mathbf{x}||),$$
$$\mathbf{V}_{nLM}(\mathbf{x}) = \mathcal{N}_{nL}\; \frac{\mathrm{i}}{k_{nL}}\nabla \wedge j_L(k_{nL}r)\mathbf{X}_{LM}(\mathbf{x}/||\mathbf{x}||)$$

with the vector spherical harmonics [19]

$$\mathbf{X}_{LM}(\mathbf{x}/||\mathbf{x}||) = -\frac{\mathrm{i}}{\sqrt{L(L+1)}}\mathbf{x} \wedge \nabla\; Y_L^M(\mathbf{x}/||\mathbf{x}||),$$

and with the (ordinary) spherical harmonics $Y_L^M(\mathbf{x}/||\mathbf{x}||)$ ($L \in \mathbb{N}_0, -L \leq M \in \mathbb{Z} \leq L$). The wave numbers of the mode functions are denoted by $k_{nL} \equiv \omega_{nL}/c$. Furthermore, $j_L(kr)$ is the regular spherical Bessel function [20] with the asymptotic behavior

$$\frac{(kr)^L}{(2L+1)!!} \xleftarrow{kr \to 0} j_L(kr) \xrightarrow{kr \gg 1} \frac{\sin(kr - L\pi/2)}{kr},$$

and with $(2L+1)!! = (2L+1)(2L-1)(2L-3)\cdots 5\cdot 3\cdot 1$. The normalization constants $\mathcal{N}_{nL}$ are given by

$$\mathcal{N}_{nL} = \left\{ \int_0^R dr\; r^2 j_L^2(k_{nL} r) \right\}^{-1/2} \quad \overrightarrow{n\gg 1} \quad k_{nL}\sqrt{\frac{2}{R}}.$$

The eigenvalues ω_{nL} of the mode functions $\mathbf{U}_{nLM}$ and $\mathbf{V}_{nLM}$ are determined by the conditions $j_L(k_{nL}R) = 0$ and $d(xj_L(x))/dx \mid_{x=k_{nL}R} = 0$, respectively. In the case of highly excited modes, i.e. $k_{nL}R \gg 1$, we find

$$k_{nL}R \overrightarrow{{}_{k_{nL}R\gg 1}} \pi n + (L+1)\pi/2 \tag{29.5}$$

so that the density of states is constant, i.e. $dn/d(\hbar\omega_{nL}) = R/(\pi\hbar c)$ with c denoting the speed of light in vacuum. For these highly excited modes only the mode functions $\mathbf{V}_{nL=1M=0}(\mathbf{x})$ are nonvanishing at the center of the cavity where the qubit is located. Therefore, in the dipole approximation the coupling between the qubit and the electromagnetic radiation field is dominated by these particular modes.

29.1.3 Dynamics of spontaneous photon emission

From the considerations of the previous sections, it is apparent that in the dipole and rotating-wave approximations the spontaneous decay of a qubit located at the center of a spherical cavity with ideal metallic boundary conditions can be described by the Hamiltonian

$$\hat{H} = \hbar\overline{\omega}_0 \hat{b}_0^\dagger \hat{b}_0 + \sum_{l\in I} \hbar\omega_l \hat{a}_l^\dagger \hat{a}_l + \sum_{l\in I} \{\alpha_l \hat{b}_0 \hat{a}_l^\dagger + \alpha_l^* \hat{b}_0^\dagger \hat{a}_l\} \equiv \sum_{k,m\in\{0\}\cup I} \hat{B}_k^\dagger h_{km} \hat{B}_m \tag{29.6}$$

with $\hat{B}^T = (\hat{b}_0, \hat{a}_1, \hat{a}_2, \ldots)$ and with the Hermitian matrix

$$h_{km} = \begin{pmatrix} \hbar\overline{\omega}_0 & \alpha_1^* & \alpha_2^* & \cdots \\ \alpha_1 & \hbar\omega_1 & 0 & \cdots \\ \alpha_2 & 0 & \hbar\omega_2 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \tag{29.7}$$

In cases in which the approximately resonantly coupled modes $l \in I$ are highly excited, i.e. $k_l R \gg 1$, the coupling constants α_l are given by

$$\alpha_l = \mathrm{i}\langle g|\hat{\mathbf{d}}|e\rangle \cdot \mathbf{u}_l^*(\mathbf{x}=0)\sqrt{\frac{\hbar\omega_l}{2\epsilon_0}}. \tag{29.8}$$

The matrix (29.7) can be diagonalized by a unitary transformation $\mathcal{U}$, i.e.

$$\hat{H} = \sum_{k\in\{0\}\cup I} \hat{P}_k^\dagger \Lambda_k \hat{P}_k$$

with $\hat{B}_k = \sum_m \mathcal{U}_{km}\hat{P}_m$ and $\sum_{mn} \mathcal{U}_{km}^\dagger h_{mn} \mathcal{U}_{nr} = \Lambda_k \delta_{kr}$. The operators $\hat{P}_k^\dagger$ and $\hat{P}_k$ are the creation and destruction operators of the "quasi particles" which describe the dressing of

the qubit by the radiation field. The eigenvalues Λ_r (dressed energies) are determined by the condition [10]

$$f(\Lambda) \equiv \hbar\overline{\omega}_0 - \Lambda - \sum_{l\in I} \frac{|\ \alpha_l\ |^2}{\hbar\omega_l - \Lambda} = 0 \ \ \rightarrow \ \ \Lambda_r. \tag{29.9}$$

For the elements $\mathcal{U}_{kr}$ of this unitary transformation we obtain the relations

$$\mathcal{U}_{kr} \equiv \mathcal{U}_k(\Lambda_r) = \left\{ \begin{array}{ll} \left(1 + \sum_{l\in I} \frac{|\alpha_l|^2}{|\hbar\omega_l - \Lambda_r|^2}\right)^{-1/2} & , \quad k = 0 \\ -\alpha_k(\hbar\omega_k - \Lambda_r)^{-1}\mathcal{U}_{0r} & , \quad k \in I \end{array} \right. . \tag{29.10}$$

As a result of this diagonalization of the Hamiltonian (29.6), the time evolution of any coherent state $|\beta_0, \beta_1, \beta_2, \ldots\rangle \equiv |\{\beta_i\}\rangle$ with $\hat{b}_0|\{\beta_i\}\rangle = \beta_0|\{\beta_i\}\rangle$, $\hat{a}_l|\{\beta_i\}\rangle = \beta_l|\{\beta_i\}\rangle$ ($l \in I;\ \beta_0, \beta_l \in \mathbb{C}$) is given by

$$\hat{U}(t)|\{\beta_i\}\rangle = |\{\beta_i(t)\}\rangle \quad \text{with} \quad \beta_i(t) = \sum_{k,m\in\{0\}\cup I} \mathcal{U}_{ik} e^{-\mathrm{i}\Lambda_k t/\hbar} (\mathcal{U}^\dagger)_{km} \beta_m,$$

and $\hat{U}(t) \equiv e^{-\mathrm{i}\hat{H}t/\hbar}$. Thus, if initially, at $t = 0$, the qubit is prepared in its excited state $|e\rangle$ and no photons are present in the radiation field, the probability $P(t)$ of observing the qubit again in its excited state at a later time t is given by

$$P(t) = |\ f_0(t)\ |^2 \ \ \text{with} \ \ f_0(t) = \sum_{r\in\{0\}\cup I} |\ \mathcal{U}_{0r}\ |^2\ e^{-\mathrm{i}\Lambda_r t/\hbar}. \tag{29.11}$$

In the subsequent discussion we concentrate on cases in which the approximately resonant modes of the spherical cavity are highly excited so that the coupling constants are given by Eq. (29.8). If the radius R of the spherical cavity is very large, many cavity modes are significantly coupled to the qubit. In this case many dressed energies Λ_r contribute to the sum of Eq. (29.11) so that an analysis in terms of dressed states of the interacting system is not very practical. In such cases considerable physical insight can be obtained by a semiclassical path representation [10] of the probability amplitude $f_0(t)$ which applies to the cases in which the relevant cavity modes are highly excited, i.e. $k_l R \gg 1$ with $k_l \approx \omega_0/c$. In such a semiclassical path representation $f_0(t)$ is represented by a sum of probability amplitudes which are associated with repeated returns of the spontaneously emitted photon to the center of the cavity where it interacts repeatedly with the qubit. Assuming that all cavity modes which are significantly coupled to the qubit are highly excited [compare with Eq. (29.5)], Eq. (29.11) can be rewritten in the form

$$\begin{aligned} f_0(t) &= -\frac{1}{2\pi\mathrm{i}} \int_{-\infty+\mathrm{i}0}^{\infty+\mathrm{i}0} d\Lambda\ e^{-\mathrm{i}\Lambda t/\hbar} \sum_{r\in\{0\}\cup I} \frac{|\ \mathcal{U}_{0r}\ |^2}{\Lambda - \Lambda_r} \\ &= -\frac{1}{2\pi\mathrm{i}} \int_{-\infty+\mathrm{i}0}^{\infty+\mathrm{i}0} d\Lambda\ e^{-\mathrm{i}\Lambda t/\hbar} \frac{|\ \mathcal{U}_0(\Lambda)\ |^2}{f(\Lambda)} \frac{df}{d\Lambda}(\Lambda) \end{aligned} \tag{29.12}$$

with the characteristic function $f(\Lambda)$ of Eq. (29.9) being approximately given by

$$f(\Lambda) \rightarrow \hbar\omega_0 - \Lambda + \hbar\frac{\Gamma}{2} \cot\left(\frac{\Lambda R}{\hbar c}\right). \tag{29.13}$$

Thereby, the summation over all highly excited cavity modes has been performed with the help of contour integration. The parameter

$$\Gamma = \left[\frac{2}{\hbar} \mid \alpha_l \mid^2 \frac{R}{\hbar c}\right]_{\hbar\omega_l=\Lambda} \equiv \left[\frac{\omega_l^3}{3\pi\epsilon_0\hbar c^3} \mid \langle e|\hat{\mathbf{d}}|g\rangle \mid^2\right]_{\hbar\omega_l=\Lambda}$$

of Eq. (29.13) is a smooth function of Λ and in the spirit of a Mittag–Leffler expansion [21] all singularities of $f(\Lambda)$ are contained in the cotangent function. For $\Lambda = \hbar\omega_0$, the value of Γ is equal to the spontaneous decay rate which according to Fermi's Golden rule describes the spontaneous decay $|e\rangle \rightarrow |g\rangle$ of the qubit in the infinite cavity limit $R \rightarrow \infty$. In our subsequent treatment we shall assume that Γ is independent of Λ and that it is equal to this spontaneous decay rate. This corresponds to the flat-continuum approximation [22] in the infinite cavity limit. Furthermore, we have incorporated an approximately Λ-independent frequency shift into the renormalized physically observable transition frequency ω_0 of the qubit system. It is assumed that this renormalized transition frequency includes also the radiative corrections of the off-resonant modes ($l \notin I$).

With the help of Eqs. (29.12) and (29.13) $f_0(t)$ can be written in an equivalent form as

$$\begin{aligned} f_0(t) &= e^{-\mathrm{i}\omega_0 t-\Gamma t/2} \\ &\quad + \frac{\hbar\Gamma}{2\pi}\int_{-\infty+\mathrm{i}0}^{\infty+\mathrm{i}0} d\Lambda \frac{e^{-\mathrm{i}\Lambda t/\hbar}\, e^{\mathrm{i}W(\Lambda)}}{(\Lambda-\hbar\omega_0+\mathrm{i}\hbar\Gamma/2)^2} \sum_{M=1}^{\infty}\left[e^{\mathrm{i}W(\Lambda)}\chi(\Lambda)\right]^{M-1} \end{aligned} \tag{29.14}$$

or

$$\begin{aligned} f_0(t) &= e^{-\mathrm{i}\omega_0 t-\Gamma t/2} + \sum_{M=1}^{\infty}\sum_{r=0}^{M-1}\Theta(t-\frac{2R}{c}M)\binom{M-1}{r} \\ &\quad \times\ e^{-\mathrm{i}(\omega_0-\mathrm{i}\Gamma/2)(t-2RM/c)}\frac{[-\Gamma(t-2RM/c)]^{1+r}}{(1+r)!} \end{aligned} \tag{29.15}$$

with the Θ-function defined by $\Theta(x) = 1$ for $x \geq 0$ and $\Theta(x) = 0$ for $x < 0$. In the spirit of a Feynman path integral approach [15] $f_0(t)$ is represented as a sum of probability amplitudes which are associated with $M \geq 1$ returns of the spontaneously emitted photon to the center of the spherical cavity. Equations (29.14) and (29.15) correspond to a semiclassical limit of such a Feynman path integral representation as they apply for highly excited cavity modes only. According to the first terms of Eqs. (29.14) and (29.15) the spontaneous emission of a photon is characterized by an exponential decay of the qubit with the spontaneous decay rate Γ. Each time the photon returns to the center of the spherical cavity it interacts again with the qubit. These successive qubit–photon interactions are turned on at multiples of the classical photon return time $T = 2R/c$, and are described by the probability amplitudes of Eqs. (29.14) and (29.15) with $M \geq 1$. Due to the spherical symmetry of the cavity the probability amplitudes of all possible photon paths interfere constructively at the center of the cavity. In spite of this constructive interference the initial-state probability $P(t)$ does not rise again to its initial value of unity at times $t \approx 2R/c$. Physically speaking this is due to the fact that the re-excitation of the qubit takes a time of the order of $1/\Gamma$. However, during this time the qubit can also re-emit this photon again spontaneously. The resulting characteristic time evolution of this

competition between re-excitation and re-emission is described by the first term of the sum of Eq. (29.15) with $M = 1$.

At its subsequent returns to the center of the cavity the photon already contains information about its previous time evolution. In particular, according to Eq. (29.14), each one of the photonic returns contributes to $f_0(t)$ with an additional phase of magnitude

$$W(\Lambda) = \frac{2\Lambda R}{\hbar c}.$$

Moreover, the scattering matrix element

$$\chi(\Lambda) = 1 - \frac{\mathrm{i}\hbar\Gamma}{\Lambda - \hbar\omega_0 + \mathrm{i}\hbar\Gamma/2} \equiv \frac{\Lambda - \hbar\omega_0 - \mathrm{i}\hbar\Gamma/2}{\Lambda - \hbar\omega_0 + \mathrm{i}\hbar\Gamma/2}$$

describes scattering of the photon during its returns to the center of the cavity. So, during its first return to the center of the cavity, for example, the photon is either not scattered at all or it is scattered by the qubit due to absorption and subsequent spontaneous emission. These two possibilities manifest themselves in the terms of Eq. (29.15) with $M = 2, r = 0$ and $M = 2, r = 1$, for example. In particular, if the photon was not scattered during its first return the qubit can be excited at the photon's second return already at time $t = 4R/c$. If the photon was scattered during its first return it experiences a time delay thus leading to a corresponding later excitation of the qubit at time $t \approx 4R/c$. The terms of Eq. (29.15) which are associated with higher returns of the photon to the center of the cavity can be interpreted in an analogous manner with M enumerating the number of returns and the index r enumerating the number of previous scattering processes. In particular, the binomial coefficient $\binom{M-1}{r}$ counts the indistinguishable possibilities to scatter r times during $(M-1)$ previous returns.

According to Eqs. (29.14) and (29.15) the dynamics of the qubit depend significantly on the number of cavity modes which are coupled resonantly to the qubit by the spontaneous energy exchange.

1. *The large cavity limit*. In this case, the number of cavity modes significantly participating in the spontaneous decay process is large, i.e. $\Gamma dn/d\omega_{nL=1} \equiv \Gamma R/(c\pi) \gg 1$. Thus, the spontaneous emission time $1/\Gamma$ is much shorter that the time $T = 2R/c$ required by a photon to travel from the center of the spherical cavity to its boundary and back again. The resulting short interaction times between the qubit and the spontaneously emitted photon imply that the contributions to $f_0(t)$ of subsequent returns of the photon are well separated in time at least for sufficiently small numbers of returns. A typical time dependence of the initial-state probability $P(t)$ for such a case is depicted in Fig. 29.1(a). Apart from the initial approximately exponential decay for times $0 \leq t < T$, one also notices the contributions of $M \geq 1$ repeated returns which lead to an increase of $P(t)$. Nevertheless, for the reasons discussed above, the initial-state probability does not rise again to a value of unity at the first return $(M = 1)$ of the spontaneously emitted photon to the center of the cavity. Furthermore, the contribution of the M-th return is split into M distinct peaks which are associated with all possible previous scatterings of the photon at the center of the cavity. According to Eq. (29.15) each of these scatterings leads to a time delay and a resonant phase shift of magnitude π so that these peaks are always separated by zeros of $P(t)$. Eventually, for sufficiently large values of M contributions

of repeated returns overlap in time thus giving rise to a complicated interference pattern of the quantum probability amplitude.

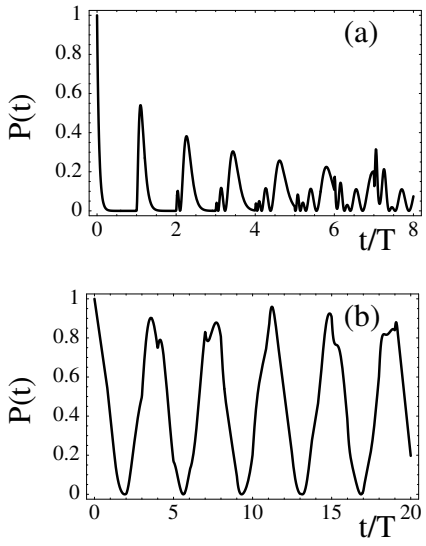

Figure 29.1. Initial-state probability $P(t)$ as a function of time t in units of the photon period $T = 2R/c$ in a spherical metallic cavity of radius R. (c is the speed of light in vacuum.) The spontaneous decay rates (in the infinite cavity limit) are given by $\Gamma = 20/T$ (a) and $\Gamma = 0.75/T$ (b). The transition between the large (a) and small (b) cavity limit is apparent.

2. *The small cavity limit*. In this opposite limit, only one cavity mode is significantly coupled to the qubit, i.e. $\Gamma dn/d\omega_{nL=1} \equiv \Gamma R/(c\pi) \ll 1$. Thus the spontaneous decay time $1/\Gamma$ is much larger than the time $T = 2R/c$ which is required for a photon to travel from the center of the spherical cavity to its boundary and back again. In this case, the contributions of numerous repeated returns in Eq. (29.15) overlap in time and an analysis of the spontaneous decay process in terms of the semiclassical path representations of Eqs. (29.14) and (29.15) is no longer practical. However, a straightforward evaluation of the probability amplitude $f_0(t)$ is still possible in the framework of the dressed-state representation of Eq. (29.11). In fact, there are only two relevant dressed energies, namely $\Lambda_\pm = \hbar(\overline{\omega}_0 + \omega_\mathrm{C})/2 \pm \sqrt{\hbar^2(\overline{\omega}_0 - \omega_\mathrm{C})^2/4 + \hbar^2 c\Gamma/(2R)}$, where ω_C denotes the frequency of the almost resonant cavity mode. So, in this limiting case one obtains from Eq. (29.11) the corresponding results of the Jaynes–Cummings–Paul model, i.e.

$$f_0(t) = e^{-\mathrm{i}(\overline{\omega}_0+\omega_\mathrm{C})t/2}\left\{\frac{\Omega - (\omega_\mathrm{C} - \overline{\omega}_0)/2}{2\Omega}e^{-\mathrm{i}\Omega t} + \frac{\Omega + (\omega_\mathrm{C} - \overline{\omega}_0)/2}{2\Omega}e^{\mathrm{i}\Omega t}\right\}$$

with the detuning-dependent vacuum Rabi frequencies $\Omega = \sqrt{(\omega_\mathrm{C} - \overline{\omega}_0)^2/4 + c\Gamma/(2R)}$.

29.2 Suppression of Radiative Decay of a Qubit in a Photonic Crystal

In this section, we discuss basic physical aspects of the radiative decay of a single qubit which is embedded in a photonic crystal [23, 24]. Inside a photonic bandgap, the small density of states of the electromagnetic field modes may lead to a significant suppression of spontaneous decay of such a qubit even in the continuum limit. Furthermore, the possibility of forming bound qubit–field states is discussed for which the electromagnetic field is localized around the position of the qubit.

29.2.1 Photonic crystals and associated density of states

Nowadays it is possible to engineer materials in such a way that the possible modes of the electromagnetic field propagating in such a medium exhibit bandgaps in their frequency spectrum. Such materials are referred to as photonic bandgap (PBG) materials, PBG crystals, or photonic crystals (PCs). Typically, the density of states (DOS) of the electromagnetic field inside such a PGB material is singular. The original idea of PCs is due to John and Yablonovitch, who both suggested independently that materials with periodic variation in the dielectric constant could influence the properties of photons in much the same way as semiconductors affect the properties of electrons [14]. In contrast to semiconductors, however, PCs do not exist naturally and therefore need to be fabricated. More precisely, one has to create a periodic lattice of dielectric matter with periodicity on the scale of the wavelengths of light. Typically this dielectric matter involves rods, spheres, slabs, etc. which are sometimes also referred to as "dielectric atoms." As a result, under appropriate conditions a complete PBG may arise so that for frequencies inside this gap regime electromagnetic wave propagation of any polarization is forbidden in any direction.

It is also possible to create *point defects* in a PC by destroying the periodicity of the lattice of the crystal locally. Such imperfections may involve changes of the dielectric constant (or equivalently of the refractive index) of one of the "dielectric atoms." Alternatively, they may also arise from a modification of the size or even from the removal of a "dielectric atom" from the lattice of the crystal. By destroying the perfect periodicity such a point defect can then "pull" a mode (or a group of modes) inside an otherwise forbidden bandgap. The resulting photonic state known as *defect mode* is strongly localized and decays exponentially in the bulk, while its frequency and symmetry can be controlled. The crystal surrounding a defect acts as a highly reflecting mirror. Clearly, if losses can be controlled, a high-Q microcavity (with a size of the order of the cubic wavelength of light λ^3) can be obtained. Moreover, this microcavity may operate at optical or even near-infrared wavelengths, where ordinary cavities which are used in typical quantum optical experiments are already very lossy. Alternatively, instead of a point defect one can also introduce *line defects* in an otherwise perfect photonic crystal structure which may act as a "lossless" waveguide. Finally, combining both line and point defects, the creation of channel-drop filters and other components necessary for the construction of all-optical circuits is possible. A thorough and rather readable account of the fabrication and the optical properties of PCs can be found in [25, 26], for example.

Let us now focus on the ability of photonic crystals to suppress the spontaneous emission of photons. The discussion of quantum optical phenomena, such as spontaneous emission, in PCs requires a suitable DOS incorporating all the essential physical features associated with these materials. Neglecting the vectorial nature of electromagnetic waves one may obtain a simple isotropic model where a propagating photon experiences the same periodic potential, irrespective of its polarization or direction of propagation [27,28]. Thus, the propagation of an electromagnetic wave in such an ideal structure can be described by a scalar one-dimensional wave equation. The dispersion relation of this electromagnetic wave exhibits forbidden gaps and allowed bands. Typically, for frequencies close to the upper band-edge frequency $\omega_{\rm e}$ the dispersion relation can be approximated by the effective-mass dispersion relation i.e., $\omega_k \approx \omega_{\rm e} + A(k - k_{\rm e})^2$, where A is a material-specific constant and $k_{\rm e}$ is the wave-vector corresponding to $\omega_{\rm e}$. Accordingly, the isotropic DOS $\rho_{\rm I}(\omega)$ $(\equiv dn/d\omega)$ for frequencies close to $\omega_{\rm e}$ is approximately given by

$$\rho_{\rm I}(\omega) = \frac{V}{(2\pi)^3}\frac{k_{\rm e}^2}{2\sqrt{A}}\frac{\Theta(\omega-\omega_{\rm e})}{\sqrt{\omega-\omega_{\rm e}}} \tag{29.16}$$

where $\Theta(\omega - \omega_{\rm e})$ is the unit step function indicating that there is a frequency gap below $\omega_{\rm e}$ and V is the volume. In a finite one-dimensional PC, however, the singular behavior of Eq. (29.16) is smoothened [29, 30]. This effect can be incorporated into the isotropic model by an appropriate smoothing parameter in Eq. (29.16) [31, 32].

Band-structure studies have shown that the vectorial nature of electromagnetic waves has to be taken into account in order to achieve good agreement with experiments. Quantum optical phenomena, however, are expected to depend mainly on the local DOS (LDOS), i.e., the DOS in the neighborhood of the relevant embedded qubit, rather than on the global DOS. Furthermore, according to band-structure calculations, even if a PC does not possess a complete PBG, its LDOS may exhibit pseudogaps as well as Van-Hove singularities for which an isotropic DOS is a good local approximation [33, 34]. Finally, it is worth noting that a highly peaked behavior analogous to that of Eq. (29.16) appears also in an ideal waveguide close to its fundamental frequency [13].

Besides the isotropic model, also an anisotropic one has been proposed [28] which preserves the vectorial nature of electromagnetic waves. The corresponding dispersion relation close to the upper band-edge frequency is of the form $\omega_k = \omega_{\rm e} + A(\mathbf{k} - \mathbf{k}_{\rm e})^2$, while the associated DOS differs from Eq. (29.16) as the square-root factor now appears in the numerator instead of the denominator, i.e. $\rho_{\rm A}(\omega) \sim \sqrt{\omega - \omega_{\rm e}}\Theta(\omega-\omega_{\rm e})$. Although the anisotropic model is closer to realistic three-dimensional PCs, it is mainly the isotropic DOS of Eq. (29.16) which has been used in quantum-optical problems so far.

What should be kept in mind is that both isotropic and anisotropic models are valid for frequencies around the band-edge and for relatively large gaps. This is apparent from the fact that none of these models exhibits the correct behavior for relatively large frequencies i.e., none of them approaches the open-space value for $\omega \gg \omega_{\rm e}$. Moreover, in a realistic PBG material the gap does not necessarily mean a true zero but a range of frequencies over which the DOS is several orders of magnitude smaller than that of open space. The essential point therefore is that, an appropriate model of DOS for the description of a PBG continuum must exhibit a dip over a range of frequencies and also it has to tend to the open-space DOS as

the frequency becomes much larger or much smaller than the midgap frequency. A rather simple model of such a DOS is an inverted Lorentzian of higher order (p), such as given by the expression

$$\rho_L(\omega) = \rho_{\rm o}(\omega)\left[1 - \frac{\mathcal{K}^p}{(\omega-\omega_{\rm c})^p + \mathcal{K}^p}\right], \tag{29.17}$$

where $\omega_{\rm c}$ is the midgap frequency, $\mathcal{K}$ is the width of the gap and $\rho_{\rm o}(\omega)$ denotes the open-space DOS which is a smooth function of ω [32, 35–38].

29.2.2 "Photon + atom" bound states

Let us consider an initially excited qubit which is placed in a material exhibiting gaps in the spectrum of the electromagnetic field it supports [31, 39, 40]. Such a qubit may be realized by an atom which is placed inside a PC or by an appropriate "dielectric atom," for example. Clearly, for transition frequencies of this qubit around the band edge of the PC, i.e. $(\omega_0 \sim \omega_{\rm e})$, for both isotropic and anisotropic models we have an unconventional DOS which is not a smoothly varying function of frequency. In fact, the Fourier transforms (memory kernels) of the isotropic and anisotropic DOS, i.e. $G_{I(A)}(\tau) = \int_{-\infty+{\rm i}0}^{\infty+{\rm i}0} d\omega \rho_{I(A)}(\omega) \exp[-{\rm i}\omega\tau]$, reflect long-range correlations in time of the form $G_{\rm I}(\tau) \sim \tau^{-1/2}$ and $G_{\rm A}(\tau) \sim \tau^{-3/2}$ for $\tau > 0$, respectively [41].

For the sake of illustration let us focus on the spontaneous emission by such a qubit embedded in a PC which can be described by the isotropic model. In terms of the resolvent operator of this system, i.e. $\mathcal{G}(z) = \int_0^{\infty+{\rm i}0} dt\, \exp({\rm i}zt)\hat{U}(t)$, the probability amplitude $\mathcal{A}_{\rm e}(t) \equiv \langle e|\hat{U}(t)|e\rangle$ of observing the qubit at time t after its preparation in its excited state (and the electromagnetic field in its ground state again) is determined by the matrix element [39]

$$\mathcal{G}_{ee}(z) = \frac{\sqrt{z-\omega_{\rm e}}}{(z-\omega_0)\sqrt{z-\omega_{\rm e}} + {\rm i}C}. \tag{29.18}$$

The constant C represents the strength of the coupling between the qubit and the continuum of field modes of the electromagnetic field. For the isotropic model it is given by

$$C = \frac{|\langle e|\hat{\mathbf{d}}|g\rangle|^2 k_{\rm e}^2 \omega_{\rm e}}{12\pi\epsilon_0\sqrt{A}}.$$

One can easily verify that the expression for $\mathcal{G}_{ee}(z)$ has three poles. Whether they are complex- or real-valued will be determined by the detuning $\delta = \omega_0 - \omega_{\rm e}$. In general, the poles with positive imaginary parts fall outside the contour of integration which is relevant for determining the time evolution of the system. Poles with negative imaginary parts describe the irreversible spontaneous decay of the qubit and purely real-valued poles are responsible for asymptotic long-time oscillations of the probability amplitude $\mathcal{A}_{\rm e}(t)$. These latter oscillations may be associated with stable bound states of the atom–field system within the PC.

In Fig. 29.2, the time evolution of the atomic population $|\ \mathcal{A}_{\rm e}(t)\ |^2$ is depicted for the isotropic model and for various detunings $\delta = \omega_0 - \omega_{\rm e}$ of the transition frequency of the qubit ω_0 from the band-edge frequency $\omega_{\rm e}$. As expected for transition frequencies well inside the

bandgap, i.e. $\delta = -10C^{2/3} \ll 0$, the qubit remains in the excited state forever. The periodically modulated dielectric host prevents the energy exchange between the qubit and the modes of the electromagnetic field inside the PC. Thus a significant part of the spontaneously emitted radiation remains localized close to the position of the qubit. Typically, such localized photonic states may extend over many wavelengths around the qubit [28]. As a result of the strong interaction between the atom and its own localized radiation the population $| \mathcal{A}_e(t) |^2$ exhibits oscillations for $\delta < 0$, while in the long-time limit we have the formation of a *"photon + atom" bound state* . This bound state consists of an excited-state and a ground-state component of the qubit and of an electromagnetic field component which cannot propagate in the PC. The possibility of formation of such "photon + atom" bound states in PCs has already been predicted in the early 1970s by Bykov [42]. For transition frequencies sufficiently outside the bandgap, i.e. $\delta = 10C^{2/3} \gg 0$, the dynamics of the coupling to the electromagnetic field is governed by an exponential decay of the initially excited qubit. However, the decay rate Γ depends on the detuning from the band edge, i.e. $\Gamma \sim C/\sqrt{\delta}$. This is due to the fact that the isotropic DOS does not approach its open-space value even for detunings $\delta \gg 0$.

In the language of dressed states, the coupling of the atom to the strongly modified radiation reservoir causes a strong vacuum Rabi splitting which is reflected by the vacuum Rabi oscillations in the atomic populations. One of the two components of the doublet created by the splitting is pushed inside the gap, where it is protected against spontaneous decay, while the other one is pushed outside where it decays. Depending on the magnitude and the sign of δ, the relative magnitude of the two components changes. This relative magnitude determines which fraction of the initial excitation remains trapped at the position of the qubit in the long-time limit.

As depicted in Fig. 29.2, in the isotropic model the qubit exhibits a nonzero steady-state population even for moderate positive detunings. This, however, is an artifact originating from the divergence of the isotropic DOS as described in Eq. (29.16). For the anisotropic model and for the DOS of Eq. (29.17) the component of the doublet outside the bandgap decays much faster. Thus, even for small positive detunings the "photon + atom" bound state decays and the asymptotic oscillations in the population are not so pronounced. In general, the dynamics of a qubit coupled to a PBG continuum depend mainly on the width of the gap (as compared with the atomic linewidth) as well as on the "band-edge" behavior of the continuum. In addition, they slightly depend on the particular profile of the DOS one may adopt (see, for instance, Fig. 29.3).

Finally, in contrast to the decay rate, the Lamb shift of an atom which is embedded in a PC is not affected by the unconventional radiation reservoirs significantly [43]. This is due to the fact that the Lamb shift originates from virtual photons of all frequencies up to an effective cut off of the order of the rest mass energy of an electron [18]. Compared to this huge frequency regime, a PC modifies the spectrum of the electromagnetic field in a small frequency interval only.

29.2.3 Beyond the two-level approximation

Besides single-photon spontaneous emission also other quantum optical phenomena involving collections of two-level systems and few-level systems have been addressed in the context of PBG continua. For an extensive review see [23], for example. In general, as long as the

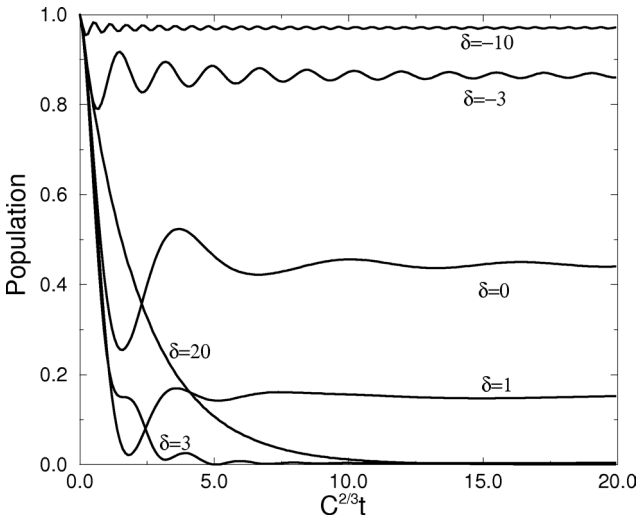

Figure 29.2. Spontaneous decay of an initially excited qubit embedded in a PC (isotropic model of DOS): The time evolution of the population of the excited state of the qubit is plotted for various detunings $\delta \equiv \omega_0 - \omega_e$ of the transition frequency of the qubit from the band-edge frequency. All the detunings are in units of $C^{2/3}$.

problem under consideration involves the exchange of a single photon between the embedded system and the PC-continuum, it can be handled in a straightforward way by direct solution of the appropriate time-dependent Schrödinger equation. Nevertheless, the direct extension of this approach to situations involving more than one photon in a PBG continuum of an arbitrary DOS does not seem tractable. On the other hand, in view of the non-smooth frequency dependences of typical DOS standard tools of quantum optics, such as Markovian master equations and quantum Monte Carlo wavefunctions, are not able to describe the essential physical effects involved. The description of such cases has been attracting increasing interest recently as problems of this kind keep emerging also in other branches of physics. As a result a number of new techniques applicable to strongly interacting dissipative systems have been developed during the last years [36, 44–48].

29.2.4 Exercises

1. *Hamiltonian diagonalization*. Diagonalize the Hamiltonian for spontaneous emission (29.6). In particular show that the elements of the related unitary transformation are given by Eq. (29.10), whereas the dressed energies are determined by Eq. (29.9).

2. *Excitation probability*. Show that the probability for an initially $(t = 0)$ excited qubit to be excited also at time t is given by (29.11). Assuming that the cavity modes significantly coupled to the qubit are highly excited, derive Eq. (29.12). Finally, derive Eqs. (29.14) and (29.15) from Eq. (29.12).

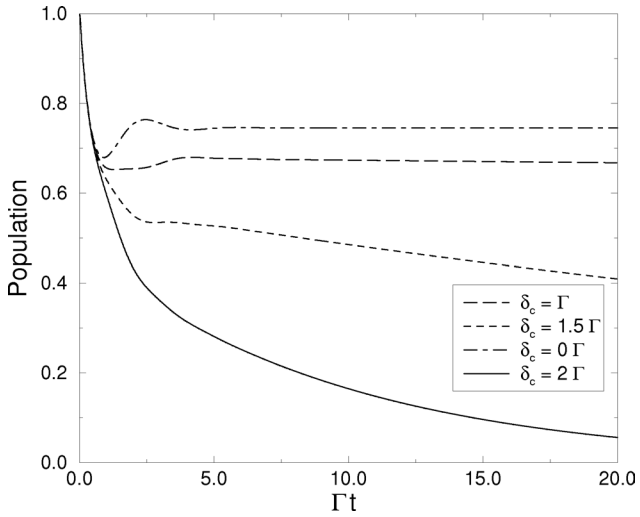

Figure 29.3. Spontaneous decay of an initially excited qubit embedded in a PC (inverted-Lorentzian profile of DOS for $p = 8$ and $\mathcal{K} = 2\Gamma$): The time evolution of the population of the excited state of the qubit is plotted for various detunings $\delta_c \equiv \omega_0 - \omega_c$ of the transition frequency of the qubit from the midgap frequency.

3. *Resolvent operator*. The resolvent of a Hamiltonian $\hat{H}$ is defined by $\mathcal{G}(z) = (z - \hat{H})^{-1}$. Consider a system with a total Hamiltonian $\hat{H} = \hat{H}_0 + \hat{V}$, where $\hat{H}_0$ and $\hat{V}$ are the unperturbed part and the interaction, respectively. Let also $\mathcal{S} \equiv \{|a\rangle, |b\rangle, |c\rangle, \ldots\}$ be the set of eigenstates of $\hat{H}_0$ with respective energies $\omega_a, \omega_b, \omega_c, \ldots$, in units with $\hbar = 1$. Show that, if initially the system is in state $|a\rangle$, the matrix elements $\mathcal{G}_{aa}$ and $\mathcal{G}_{ab}$ are determined by

$$(z - \omega_a)\mathcal{G}_{aa} = 1 + \sum_{j \in \mathcal{S}} \hat{V}_{aj}\mathcal{G}_{ja},$$
$$(z - \omega_b)\mathcal{G}_{ba} = \sum_{j \in \mathcal{S}} \hat{V}_{bj}\mathcal{G}_{ja}.$$

4. *Spontaneous emission in the resolvent-operator formalism*. Let $\hat{H}$ be given by Eq. (29.6) and $|a\rangle = |e\rangle \otimes |\{0\}\rangle$, $|b\rangle = |g\rangle \otimes |1_l\rangle$ with respective energies $\omega_a = \omega_0$, $\omega_b = \omega_l$. Thereby, l is an index running over all the field modes and $|1_l\rangle$ denotes an one-photon state. Show that

$$\mathcal{G}_{aa} = \left(z - \omega_0 - \sum_l \frac{|\alpha_l|^2}{z - \omega_l} \right)^{-1}.$$

Starting from this equation, derive Eq. (29.18), using the density of states (29.16).

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