Continuous Stern-Gerlach effect and the quantum-state diffusion model of state reduction

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A quantum-mechanical description of the continuous Stern-Gerlach effect is developed in which environmental effects, as well as effects arising from the continuous measurement of the electronic z motion, are taken into account. Master equations are presented that describe the behavior of a quantum statistical ensemble of continuous measurement processes. On the basis of these master equations and with the help of the quantum-state diffusion model a theoretical description of individual continuous measurement processes is developed.

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I. INTRODUCTION

The continuous Stern-Gerlach effect (CSGE) [1–6] has been designed as a method to measure continuously and nondestructively the spin state of an electron confined in a Penning trap in ultrahigh vacuum. In contrast to the classical Stern-Gerlach effect, which is one of its historical cornerstones, in the CSGE the spin detection process does not absorb the observed electron, so that the dynamics of an individual, single electron can be monitored continuously. This effect has been used in a series of impressive experiments to determine fundamental physical constants, such as the electronic g factor, with unprecedented precision. Furthermore, it is a near-ideal realization of a continuous quantum-mechanical measurement process. According to Dehmelt the CSGE demonstrates two important points [1,2]: (1) for reduction of the state of an individual quantum system to take place no animate observer is required; interaction of the continuously measured physical system with its environment is sufficient for this process to occur; and (2) the completion of an individual quantum measurement process requires a minimum, nonzero time T_m. Therefore the CSGE raises the question of how an individual continuous quantum measurement process should be described theoretically (here an "individual" measurement process is understood as a single member of a quantum statistical ensemble of measurement processes).

The quantum theory of measurement is motivated by the idea of the universality of quantum mechanics according to which this theory should be applicable, in particular, to the measurement process itself. It has been shown by von Neumann [7] and Lüders [8] that, as a key element, a dynamical model for an ideal quantum measurement process should yield the collapse of the quantum-mechanical state into an eigenstate of the measured observable as the result of a dynamical albeit stochastic process. Within the framework of traditional quantum mechanics this collapse of the quantum-mechanical state occurs instantaneously in the form of a "quantum jump." It is difficult to explain this stochastic collapse of the quantum state by the unitary time evolution implied by the Schrödinger equation [9]. This is the origin of the so called quantum measurement problem [10]. In quantum optical problems substantial progress has been achieved in the theoretical description of individual photoelectric detection processes by which the state of a photon source is monitored [11–15]. In particular, in this context it has been shown that contingent on those particular measurement records which involve homodyning or heterodyning with a classical intense photon source the time evolution of a continuously observed photon source can be described by particular types of nonlinear, stochastic Schrödinger equations [14]. Motivated by the more general question "what is actually happening during an (arbitrary, individual) quantum measurement" [16] recently various classes of nonlinear, stochastic Schrödinger equations have been proposed as general dynamical approaches to the process of state reduction [16–21]. These equations aim at describing arbitrary quantum measurement processes which do not necessarily involve photon detection processes at all. Typically, these latter, general approaches to the quantum measurement problem constitute generalizations of traditional quantum mechanics and it is still an open question which one of these approaches might ultimately turn out as being consistent with experiment. In particular, in one of these approaches, namely, the quantum-state diffusion model (QSDM) [20,21], Dehmelt's conjectures (1) and (2) are taken into account in a natural way. In this phenomenological model any continuous quantum measurement process is modeled, like any other interaction of a quantum system with an environment, by a continuous, nonlinear stochastic process which is consistent with a master equation for the density operator of the associated quantum statistical ensemble. Thereby the continuity of the stochastic process implies in a natural way that the completion of an individual quantum measurement process requires a characteristic time T_m [20].

In this paper the controversial discussion about which one of these general, stochastic approaches to the problem of state reduction might ultimately lead to a satisfactory description of arbitrary real quantum measurement processes is approached from a pragmatic point of view. In the following a theoretical description of the CSGE is developed with the help of QSDM in the hope that this application of QSDM and the comparison of its theoretical predictions with this real experiment might ultimately help to falsify or support QSDM as a general model of state reduction. The CSGE as performed by Dehmelt and Brown and Gabrielse [1–6] is an almost ideal example of a real continuous quantum measurement process which does not involve the detection of photons at all and which has already been realized in the laboratory. It should be mentioned at this point that there have already been investigations of optical analogs of the classical Stern-Gerlach effect [22] in which the problem of the theoretical description of individual photon detection processes is approached with the help of the quantum trajectory approach [11]. However, in the CSGE, which we are considering in the following, the dynamics of the electron in the Penning trap is monitored by purely electronic means by measuring the charge-induced currents in the end caps of the trap. Therefore this experiment does not involve any photodetection process and these quantum optical models do not have direct physical significance for our subsequent investigation. The use of QSDM in this context as a general model of state reduction is motivated by Dehmelt's points of view (1) and (2) which are taken into account by QSDM in a natural way. In particular, the purpose of this paper is twofold. First of all, quantum-mechanical master equations are derived which describe the dynamics of a quantum statistical ensemble of continuous measurement processes which are performed on an electron in the CSGE. Thereby effects which arise from the coupling of the trapped electron to its surrounding (macroscopic) environment and the measuring apparatus as well as the relevant nonlinear couplings between cyclotron, spin, and z motion are taken into account in a consistent quantum-mechanical way. Secondly, on the basis of these master equations and the associated QSDM a theoretical description of individual continuous quantum measurement processes is developed with particular emphasis on the behavior of the quantum fluctuations which are peculiar to QSDM.

This paper is organized as follows: In Section II the quantum-mechanical treatment of the CSGE is developed. In Sec. II A the general theoretical description of individual continuous quantum measurement processes within the framework of QSDM is summarized. In Sec. II B master equations are presented which describe the dynamics of a single electron in a Penning trap under the influence of effects originating from interaction with its environment and the measuring apparatus. These master equations are the starting point for the description of individual continuous measurement processes by QSDM. By adiabatic elimination of the electronic z motion a simplified description of the cyclotron and spin dynamics is obtained. Thereby an analytical expression for the characteristic measurement time T_m is derived which generalizes Dehmelt's early estimate [2]. With the help of QSDM in Sec. III numerical simulations of individual continuous quantum measurement processes are discussed.

II. QUANTUM THEORY OF THE CONTINUOUS STERN-GERLACH EFFECT

In the first part of this section basic aspects of the quantum-state diffusion model as a model for continuous quantum-mechanical measurement processes are summarized. In the second part master equations are derived which describe the dynamics of a quantum statistical ensemble of continuous quantum measurement processes in the CSGE. These master equations are the starting point for the description of individual measurement processes within the framework of QSDM.

A. Continuous quantum measurements and the quantum-state diffusion model

Recently QSDM has been developed by Gisin and Percival [20,21] as a theoretical model for describing continuous quantum measurement processes. In a phenomenological way this model aims at answering the question "what is actually happening during a quantum measurement" [16]. Thereby the term "individual" refers to a single member of a quantum statistical ensemble. QSDM is based on two main theoretical elements.

(1) *A realistic state concept*. Independent of acts of perception, the state of an individual quantum system is described completely by a ray in a Hilbert space $|\psi(t)\rangle$. Physical properties of a quantum system are described by self-adjoint operators in this Hilbert space. The value $\langle A\rangle_t = \langle\psi(t)|A|\psi(t)\rangle$ is interpreted as the value of the physical quantity A which is assumed at time t in the course of a single continuous quantum measurement process.

(2) *A stochastic dynamical law*. The time evolution of the state of an individual quantum system is described by a stochastic Itô equation of the form

$$d|\psi(t)\rangle = -\frac{i}{\hbar}H|\psi(t)\rangle dt - \frac{1}{2}\sum_j (L_j^\dagger L_j + \langle L_j^\dagger\rangle_t\langle L_j\rangle_t - 2\langle L_j^\dagger\rangle_t L_j)|\psi(t)\rangle dt + \sum_j (L_j - \langle L_j\rangle_t)|\psi(t)\rangle d\xi_j, \tag{1}$$

with the complex Wiener processes

$$M d\xi_j = M d\xi_j^* = 0, \quad d\xi_j^2 = (d\xi_j^*)^2 = 0, \quad d\xi_j d\xi_k^* = \delta_{jk} dt.$$

Thereby M represents the mean over the statistical probability distribution and the quantities $\langle L_j\rangle_t = \langle\psi(t)|L_j|\psi(t)\rangle$ refer to single realizations of the stochastic process.

In Eq. (1) two different types of time evolutions are introduced, namely, a deterministic one, characterized by a Hamiltonian H, and a stochastic one, characterized by Lindblad operators L_j and associated Wiener processes $d\xi_j$. The Hamiltonian dynamics applies to the microscopic world. The stochastic time evolution arises from the interaction of the quantum system with its (macroscopic) environment and leads to destruction of quantum coherence [23,24]. Within the theoretical framework of QSDM a measurement process is a special case of interaction with an environment. The influence of an apparatus which measures the physical quantity A continuously is modeled phenomenologically by a self-adjoint Lindblad operator $L_1 = \sqrt{\gamma}A$. Therefore an ideal quantum measurement process of observable A is described by the single Lindblad operator L_1 and $H = 0$. In this case Eq. (1) implies

$$|\psi(t=0)\rangle \to |\psi(t\to\infty)\rangle = P_\alpha|\psi(t=0)\rangle/\sqrt{p_\alpha}, \tag{2}$$

with P_α denoting the projection operators onto the eigenspaces of operator A with eigenvalues α. This collapse of the quantum state as a result of an individual quantum measurement process [8] takes place with probability $p_\alpha = \langle\psi(t=0)|P_\alpha|\psi(t=0)\rangle$. The continuity of the stochastic law of Eq. (1) also implies that the completion of this collapse of the quantum state requires a minimum measurement time T_m whose magnitude is determined by the quantity γ. In the absence of any coupling to environments Eq. (1) reduces to the ordinary Schrödinger equation of quantum mechanics.

The dynamical law of Eq. (1) implies that the density matrix $\rho(t)=M|\psi(t)\rangle\langle\psi(t)|$ of the associated quantum statistical ensemble fulfills the master equation [25]

$$\dot{\rho}(t)=-\frac{i}{\hbar}[H,\rho(t)]+\frac{1}{2}\sum_j \{[L_j,\rho(t)L_j^\dagger]+[L_j\rho(t),L_j^\dagger]\}. \tag{3}$$

The quantum-mechanical mean value of any observable A is given by $\mathrm{Tr}\{A\rho(t)\}$ and is equal to the $M\langle A\rangle_t$. In the case of the single Lindblad operator $L_1=\sqrt{\gamma}A$ with $H=0$ the master Eq. (3) implies the von Neumann postulate [7]

$$\rho(t=0)\rightarrow\rho(t\rightarrow\infty)=\sum_\alpha P_\alpha\rho(t=0)P_\alpha. \tag{4}$$

In this case coherences between orthogonal eigenspaces of different eigenvalues α decay exponentially with a rate whose magnitude is determined by γ. The master equation (3) is invariant under arbitrary unitary transformations in the linear space of Lindblad operators [20]. In contrast to other possible associated continuous stochastic dynamical laws Eq. (1) also exhibits this invariance property.

The realistic state concept of QSDM is independent of the actual form of the stochastic dynamical law. One might be tempted to think that a replacement of Eq. (1) by another stochastic, dynamical law which leads to the same master equation for the density operator $\rho(t)$ of the quantum statistical ensemble does not affect physically observable phenomena. However, the physical interpretation of $|\psi(t)\rangle$ used by QSDM implies that this is not the case. This may be seen by considering statistical means of nonlinear functions F of quantum expectation values of the form $MF(\langle A\rangle_t)$. According to the realistic state concept of QSDM these quantities are amenable to experimental observation. But they cannot be evaluated from the associated density operator because in general $MF(\langle A\rangle_t)\neq M[\langle F(A)\rangle_t]$. Therefore this physical interpretation implies that on the basis of nonlinear functions F of observables A, i.e., by investigating the quantum fluctuations of a measured quantity in a continuous quantum measurement process, different classes of stochastic dynamical laws can be distinguished physically even if they are associated with the same master equation for the density operator [29].

B. Master equations for the continuous Stern-Gerlach effect

In this section master equations are derived which describe the dynamics of an electron in the CSGE under the influence of a measuring apparatus and other temperature-dependent dissipative (macroscopic) environmental effects. First of all a detailed description is developed which includes the electronic cyclotron, magnetron, spin, and z motion and its coupling to the ω_z-shift spectrometer [1,2] which constitutes the measuring apparatus. Based on this description a simplified treatment of the cyclotron and spin degrees of freedom is developed by eliminating the electronic z motion adiabatically. Thereby in a quantum-mechanically consistent way an analytical expression is derived for the characteristic measurement time T_m which is required to complete a measurement of the electronic cyclotron and spin state.

1. Cyclotron, magnetron, spin, and z motion

In idealized form the physical system with which the spin state of an individual electron in a Penning trap has been observed continuously and nondestructively by Dehmelt and co-workers [1–6] consists of: (1) a Penning trap in which the electron is confined and (2) a measuring apparatus, i.e., an ω_z-shift spectrometer, with which the dynamics of the electronic z motion is monitored continuously and nondestructively. This continuous and nondestructive measurement process of the electronic spin state has been termed the continuous Stern-Gerlach effect [1,2].

The dynamics of the electron which is confined in a Penning trap is described by the Hamiltonian [6]

$$H_{\mathrm{Penning}}=\hbar\omega_+a_+^\dagger a_++\hbar\omega_z a_z^\dagger a_z-\hbar\omega_-a_-^\dagger a_-+\tfrac{1}{2}\hbar\omega_s\sigma_z, \tag{5}$$

with ω_+, ω_z, ω_-, and ω_s denoting the (modified) cyclotron, z, magnetron, and spin frequency. The creation (annihilation) operators of cyclotron, z, and magnetron motion are denoted $a_+^\dagger$ (a_+), $a_z^\dagger(a_z)$, and $a_-^\dagger(a_-)$ and σ_z is the Pauli spin operator parallel to the applied homogeneous magnetic field. The spectrum of H_{Penning} is shown in Fig. 1(a) schematically together with typical values for the characteristic trap frequencies [3]. In order to enable measurements of spin-dependent properties of the trapped electron such as the electronic g factor, spin flips are induced by a weak, inhomogeneous, time-dependent magnetic field [26]

$$\mathbf{B}_1(\mathbf{x},t)=B_1\cos(\omega_a t)(x\mathbf{e}_1+y\mathbf{e}_2-2z\mathbf{e}_3), \tag{6}$$

with $\omega_a=(\omega_s-\omega_+)$. The electronic dynamics is monitored by observing its z motion continuously. For this purpose its z motion has to be coupled to the spin motion. This is achieved by applying a weak, inhomogenous magnetic (bottle) field which can be modeled by [27]

$$\mathbf{B}_2(\mathbf{x})=B_2\left[-xz\mathbf{e}_1-yz\mathbf{e}_2+\left(z^2-\frac{x^2+y^2}{2}\right)\mathbf{e}_3\right]. \tag{7}$$

However, this magnetic field leads also to a coupling between the electronic cyclotron and z motion.

The z motion itself is measured continuously and nondestructively by an ω_z-shift spectrometer. In idealized form this spectrometer consists of two main elements [compare with Fig. 1(b)] [1,2].

(i) A (deterministic) almost resonant sinusoidal drive voltage

$$U(t)=U_0\sin(\omega t), \tag{8}$$

which is applied between the ring electrode and one of the end caps of the Penning trap and which forces the z motion into an approximately coherent state with high mean quantum number.

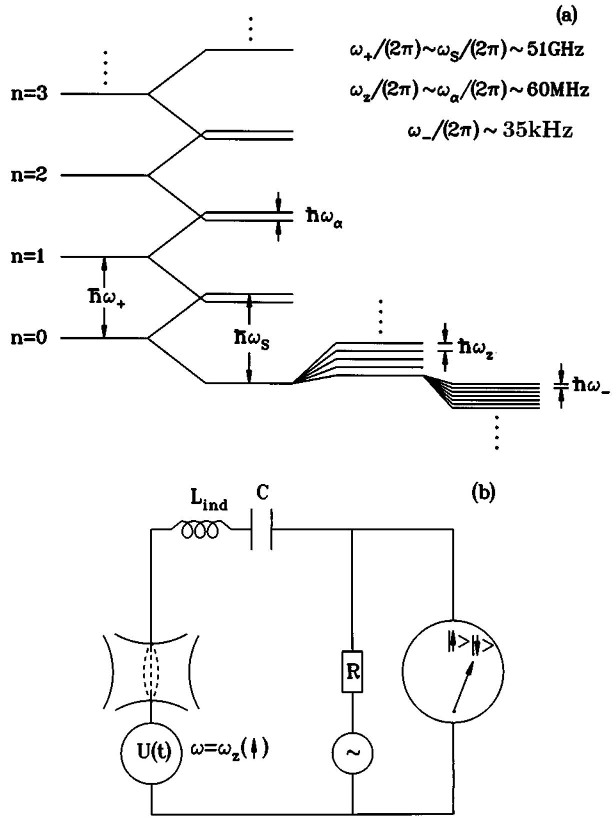

FIG. 1. Energy spectrum of an ideal Penning trap and typical values of the characteristic trap frequencies (a); schematic representation of the "ω_z-shift spectrometer" with which the spin- and cyclotron-quantum number is measured (b).

(ii) A resistor R together with an amplifier and a phase sensitive detector which measures continuously the out-of-phase quadrature component of the current which is induced in the other end cap of the Penning trap by the driven electronic z motion. As far as its electric properties are concerned, the electronic z motion in the Penning trap can be represented by an effective LCR circuit with inductance $L_{\rm ind}$, capacitance C, and resistance R.

Typically, the characteristic frequencies of the Penning trap differ at least by factors of the order of 10^3 (compare with Fig. 1). Therefore in an adiabatic approximation the description of the electronic dynamics inside the Penning trap can be simplified considerably. As outlined in Appendix A, on time scales of the order of Δt with $(2\pi)/\omega_- \ll \Delta t \approx 1 s$ the resulting total Hamiltonian, which describes the deterministic part of the electronic dynamics in the CSGE, can be approximated adiabatically by $H = H_0 + H_z + H_- + H_{0z}$ with

$$H_0 = \hbar\Omega_+ a_+^\dagger a_+ + \frac{1}{2}\hbar\Omega_s\sigma_z + \hbar\Delta\Omega(\sigma_+ a_+ e^{i(\Omega_+-\Omega_s)t} + \sigma_- a_+^\dagger e^{-i(\Omega_+-\Omega_s)t}),$$

$$H_z = \hbar\Omega_z a_z^\dagger a_z + \hbar\beta^* a_z e^{i\omega t} + \hbar\beta a_z^\dagger e^{-i\omega t},$$

$$H_- = -\hbar\omega_- a_-^\dagger a_-, H_{0z} = \hbar\Delta\omega\left(a_+^\dagger a_+ + \frac{g}{4}\sigma_z\right)a_z^\dagger a_z. \tag{9}$$

Thereby the Hamiltonian H_- describes the unperturbed magnetron motion which is still decoupled dynamically from the other degrees of freedom. The Hamiltonians H_z, H_0, and H_{0z} characterize the dynamics of the z motion under the influence of the external sinusoidal drive voltage, the dynamics of the cyclotron and spin degrees of freedom, and their anharmonic coupling by the weak, inhomogeneous magnetic fields. Furthermore, the characteristic trap frequencies are renormalized according to $\Omega_+ = \omega_+ + \Delta\omega/2$, $\Omega_z = \omega_z + \Delta\omega/2$, and $\Omega_s = \omega_s + g\Delta\omega/4$. The frequencies

$$\Delta\omega = (eB_2\hbar\omega_+)/[m^2\omega_z(\omega_+ - \omega_-)]$$

and

$$\Delta\Omega = (geB_1\sqrt{m\omega_+\hbar/2})/[4m^2(\omega_+ - \omega_-)]$$

characterize the strengths of the anharmonic couplings which are induced by the magnetic fields $\mathbf{B}_2(\mathbf{x})$ and $\mathbf{B}_1(\mathbf{x},t)$. The parameter $\beta = -iU_0/\sqrt{8\hbar\Omega_z L_{\rm ind}}$ describes the effects of the external (classical) sinusoidal voltage driving the z motion. The electric charge $Q(t)$ which is induced in the end cap of the Penning trap by $U(t)$ and the resulting current $I(t)$ through the resistor R are given in terms of the annihilation and creation operators of the z motion a_z and $a_z^\dagger$ by [28]

$$Q = \sqrt{2\hbar}\left(\frac{L_{\rm ind}}{C}\right)^{-1/4}{\rm Re}(a_z), \quad I = \frac{1}{L_{\rm ind}}\sqrt{2\hbar}\left(\frac{L_{\rm ind}}{C}\right)^{1/4}{\rm Im}(a_z). \tag{10}$$

Effects which arise from the continuous measurement process and coupling of the electron to the surrounding (macroscopic) environment will be characterized dynamically by Lindblad operators L_j. Thus the time evolution of the state $\rho(t)$, which describes the behavior of a quantum statistical ensemble in the CSGE, is described by a master equation of the form of Eq. (3). The dominant environmental effects which we shall take into account in the following are described as follows.

(1) The radiative exchange of energy between the cyclotron motion and the surrounding (thermal) radition field: It can be described by the Lindblad operators $L_1 = \sqrt{\kappa(\bar{n}+1)}a_+$ and $L_2 = \sqrt{\kappa\bar{n}}a_+^\dagger$ [28]. Thereby κ and $\bar{n} = [e^{\hbar\Omega_+/(kT)} - 1]^{-1}$ denote the spontaneous decay rate of the cyclotron motion and the mean number of quanta of the thermal radiation field at temperature T. Typically, it is found that $\kappa \approx 1\ {\rm s}^{-1}$ [6]. Radiative coupling of the other electronic degrees of freedom to the thermal radiation field can be neglected as the corresponding decay rates are vanishingly small compared to κ.

(2) The dissipative influence of the resistor R on the electronic z motion: It can be modeled by the Lindblad operators $L_3 = \sqrt{\kappa_z(\bar{n}_z+1)}a_z$ and $L_4 = \sqrt{\kappa_z\bar{n}_z}a_z^\dagger$ [28]. The resistor is characterized by the effective damping rate $\kappa_z = R/L_{\rm ind}$ and the effective occupation number $\bar{n}_z$ which depends on its temperature and describes the noise properties of this resistor within the rotating wave approximation.

(3) The continuous measurement of the out-of-phase quadrature component of the current induced in the resistor R: According to Eq. (10) the current can be decomposed into an in-phase and an out-of-phase quadrature component, i.e.,

$$I=\frac{1}{L_{\mathrm{ind}}}\sqrt{2\hbar}\left(\frac{L_{\mathrm{ind}}}{C}\right)^{1/4}[-\mathrm{Re}(a_z e^{i\omega t})\sin(\omega t)+\mathrm{Im}(a_z e^{i\omega t})\cos(\omega t)]. \tag{11}$$

Therefore the continuous measurement of the out-of-phase component of the current can be modeled phenomenologically by the self-adjoint Lindblad operator

$$L_5=\sqrt{\gamma}\mathrm{Im}(a_z e^{i\omega t}).$$

The parameter $1/\gamma$ may be interpreted as the mean measurement time [20,21]. However, it should be mentioned that in this phenomenological description it is not clear how the actual value of γ can be controlled physically.

The master equation (3) with the Hamiltonian (9) and the Lindblad operators $L_j (j=1,\ldots,5)$ yields a unified quantum-mechanical description of deterministic and stochastic effects which influence the dynamics of a quantum ensemble in the CSGE. It is the starting point for the theoretical description of the CSGE within the theoretical framework of QSDM. However, from this master equation it is not apparent that the dynamics of this physical system implies a continuous measurement of the cyclotron- and spin-quantum number. Furthermore, it is not clear which physical quantities determine the associated characteristic measurement time T_m. These two aspects are discussed in the next section.

2. Cyclotron and spin motion

In the limit in which the driven electronic oscillation along the z direction attains equilibrium almost instantaneously in comparison with all other time scales the z motion can be eliminated adiabatically. This corresponds to the so called "quantum Brownian motion limit" in which the fluctuations of the z motion, which is viewed as a reservoir, are fast and large [28]. This dynamical regime is realized in the limit

$$\lambda=\frac{\kappa_z}{|\Delta\omega|}\to\infty, \tag{12}$$

with $\Delta\omega$ and $[|\beta/\kappa_z|^2\Delta\omega/\kappa_z]$ being held constant.

According to Eq. (9) the magnetron motion is decoupled dynamically from the remaining degrees of freedom. As it can be taken into account in a simple, straightforward way it will no longer be considered explicitly in the subsequent treatment. Thus in the following the density operator $\rho(t)$ will refer to the spin, z, and cyclotron motion only. From the master Eq. (3) with Hamiltonian H of Eq. (9) and Lindblad operators L_j with $j=1,\ldots,5$ as described in Sec. II B 1 the relation

$$M\frac{d\langle\bar{a}_z\rangle_t}{dt}=-i\left((\Omega_z-\omega)-i\frac{\kappa_z}{2}\right)M\langle\bar{a}_z\rangle_t-i\beta-i\Delta\omega M\langle N\bar{a}_z\rangle \tag{13}$$

is obtained with $\bar{a}_z=a_z e^{i\omega t}$ and the cyclotron- and spin-number operator

$$N=a_+^\dagger a_+ +\frac{g}{4}\sigma_z. \tag{14}$$

Ensemble means are evaluated from the density operator $\rho(t)$, i.e., $M\langle\cdot\rangle=\mathrm{Tr}_{(z,+,s)}\{\cdot\rho(t)\}$ with the symbol M indicating the average over the quantum statistical ensemble. In the case of large and rapid fluctuations of the z motion to a good degree of approximation the density operator $\rho(t)$ factorizes, i.e., $\rho(t)=W(t)\otimes\rho_z^0+O(\lambda^{-1/2})$ [28]. Thereby ρ_z^0 denotes the stationary density operator of the z motion in the absence of the magnetic bottle field as defined in Eq. (B2) of Appendix B and $W(t)=\mathrm{Tr}_z\{\rho(t)\}$ is the reduced density operator of the cyclotron and spin motion. In the stationary limit, i.e., for $t\gg 1/\kappa_z$, this approximate factorization of $\rho(t)$ implies the relation

$$M\langle\mathrm{Im}(\bar{a}_z)\rangle_t=-\frac{\mathrm{Im}(\beta)}{(\kappa_z/2)^2}\Delta\omega M\langle N\rangle_t \tag{15}$$

for resonant driving of the z motion, i.e., $\Omega_z=\omega$. Therefore in this limit continuous measurement of the out-of-phase quadrature component, i.e., $\mathrm{Im}(\bar{a}_z)$, is equivalent to measurement of the cyclotron and spin-quantum number N.

As outlined in Appendix B, in this limit of rapid damping and large fluctuations of the z motion, a master equation can be obtained for $W(t)$ by adiabatic elimination of the z motion, namely,

$$\frac{dW(t)}{dt}=-i/\hbar[\tilde{H}_0,W(t)]+\frac{1}{2}\sum_{j=1,2}\{[L_j,W(t)L_j^\dagger]+[L_jW(t),L_j^\dagger]\}-\Gamma[N,[N,W(t)]]. \tag{16}$$

The Hamiltonian $\tilde{H}_0$ is obtained from H_0 by renormalizing the characteristic trap frequencies, i.e., $\Omega_+\to\tilde{\Omega}_+=\Omega_+ +\Delta\omega M\langle n_z\rangle_0$ and $\Omega_s\to\tilde{\Omega}_s=\Omega_s+(g/2)\Delta\omega M\langle n_z\rangle_0$. For an electronic g factor $g=2$ this renormalization does not affect the frequency difference $(\Omega_+-\Omega_s)$ which determines the frequency of the time-dependent, inhomogeneous magnetic field $\mathbf{B}_1(\mathbf{x},t)$. The mean number of quanta of the z motion in equilibrium is denoted $M\langle n_z\rangle_0$ and is given by Eq. (B3). As outlined in Appendix B, the characteristic measurement rate Γ and measurement time $T_m=1/\Gamma$ are determined by the fluctuations of the z motion around its equilibrium quantum number $M\langle n_z\rangle_0$. Explicitly it is given by [compare with Eq. (B9)]

$$\Gamma=\frac{(\Delta\omega)^2}{(\kappa_z/2)}\frac{|\beta|^2}{(\kappa_z/2)^2}\left(1+2\bar{n}_z+2\frac{\gamma}{\kappa_z}\right)[1+O(\lambda^{-1})]. \tag{17}$$

The master Eq. (16) together with the explicit expression for the characteristic measurement rate Γ of Eq. (17) are one of the main results of this paper. In the limit of negligible influence of the measurement of the out-of-phase component of the current through the resistor, i.e., $\gamma/\kappa_z\ll 1$, and negligible noise of the resistor, i.e., $\bar{n}_z\ll 1$, Eq. (17) yields Dehmelt's estimate for the minimum time T_m which is required for completing an individual quantum measurement process [2]. Thus for large damping of the z motion the actual value of the phenomenological rate γ, which characterizes the out-of-phase measurement of the current, becomes irrelevant for the dynamics of the cyclotron and spin measurement process. This implies that under these circumstances the relevant characteristic measurement time T_m can be varied in a con-

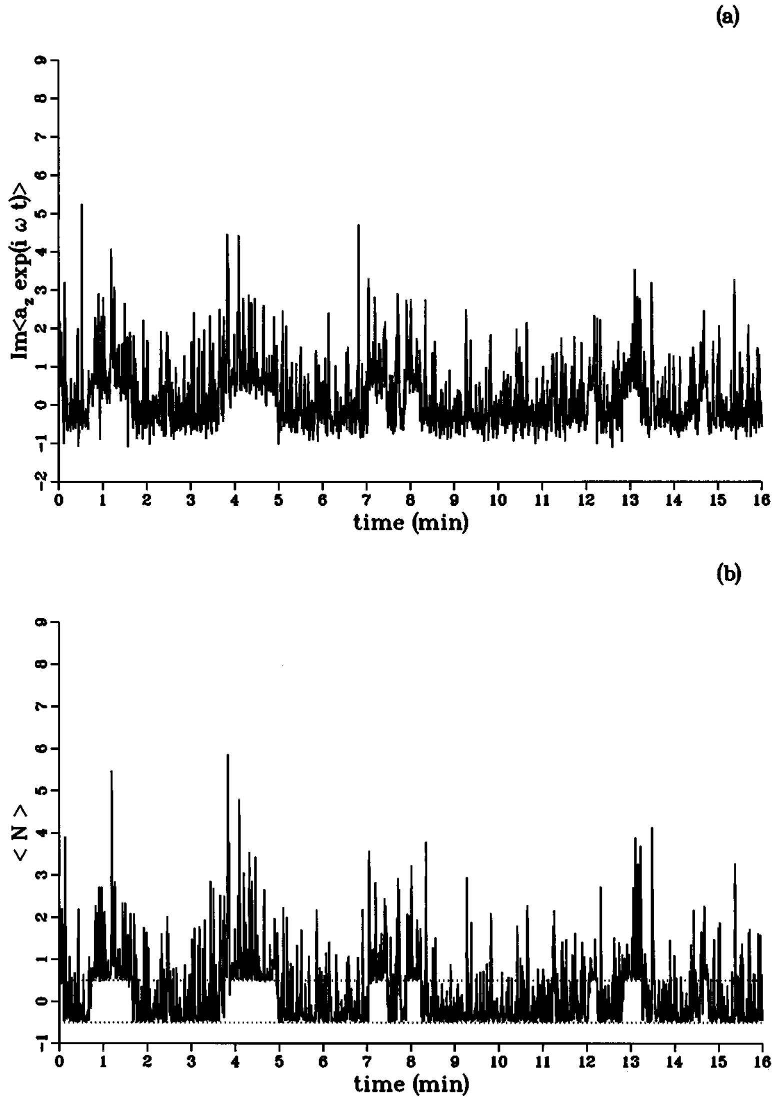

FIG. 2. A single realization of the stochastic law of QSDM [Eq. (1)]: $\langle \mathrm{Im}(a_z e^{i\omega t})\rangle_t$ (a) and $\langle N\rangle_t$ (b). The initial state is $|\psi\rangle_0 = |\alpha_+ = (1.0,0), \alpha_z = (0,10.0), s_z = +1\rangle$ and the characteristic parameters are $\Delta\omega = 1.0$, $\Delta\Omega = 0.1$, $\kappa = 2.0$, $\bar{n} = 0.5$, $\kappa_z = 20.0$, $\bar{n}_z = 0$, $\gamma = 2.0$, $\beta = (0, -100.0)$ (all rates and frequencies are in units of 1 s^{-1}).

trolled way over a large range of values by varying the external resistor R, the magnitude of the driving voltage U_0, or the magnitude of the inhomogeneous, static magnetic bottle field B_2 of Eq. (7). This fact will become important in the discussion of possible experimental tests of peculiar theoretical predictions of QSDM in Sec. III. However, it should be mentioned that in the context of the master Eq. (16) the time $T_m = 1/\Gamma$ characterizes only an ensemble property, namely, the time scale on which coherences between orthogonal eigenspaces of operator N are destroyed. Only if an individual measurement process is described by a continuous stochastic dynamical law, such as in QSDM, does this time scale attain physical significance also for an individual measurement process and determines the minimum time it takes to complete an individual measurement process in agreement with Dehmelt's point of view.

III. NUMERICAL SIMULATION OF INDIVIDUAL MEASUREMENT PROCESSES

Based on the stochastic dynamical law of Eq. (1), which is associated with the master equation of Sec. II B 1 or Eq. (16), in this part individual measurement processes are discussed for the CSGE.

In Fig. 2 a typical single realization of the stochastic dynamical law of Eq. (1) is shown which is associated with the master equation of the cyclotron, spin, and z motion as discussed in Sec. II B 1. In this example the orders of magnitude of the characteristic parameters are chosen similarly as in the experimental runs of Refs. [3,4]. In particular, the value of the characteristic measurement rate is $\Gamma = 12.0$ s^{-1}. This simulation exhibits the characteristic features of the experimental observation. The spin flips which take place on the time scale of minutes are clearly visible. The upwards spikes originate from thermal excitations of the cyclotron motion by thermal radiation (''cyclotron grass'' [4]). The resulting fluctuations take place on a time scale of the order of $1/[\kappa(1+\bar{n})] = (1/3)s$. The qualitative similarity between $\langle \mathrm{Im}(a_z e^{i\omega t})\rangle_t$ [Fig. 2(a)] and $\langle N\rangle_t$ [Fig. 2(b)] in this single realization indicates that measurement of the out-of-phase component of the current through the resistor is equivalent to measurement of the cyclotron- and spin-quantum number N.

In Fig. 3 the structure of the ''cyclotron grass'' of an individual measurement process is investigated in more detail. The measured quantity $\langle N\rangle_t$ is evaluated with the help of the QSDM which is associated with the master equation (16). In particular, in Fig. 3(a) fluctuations of the cyclotron

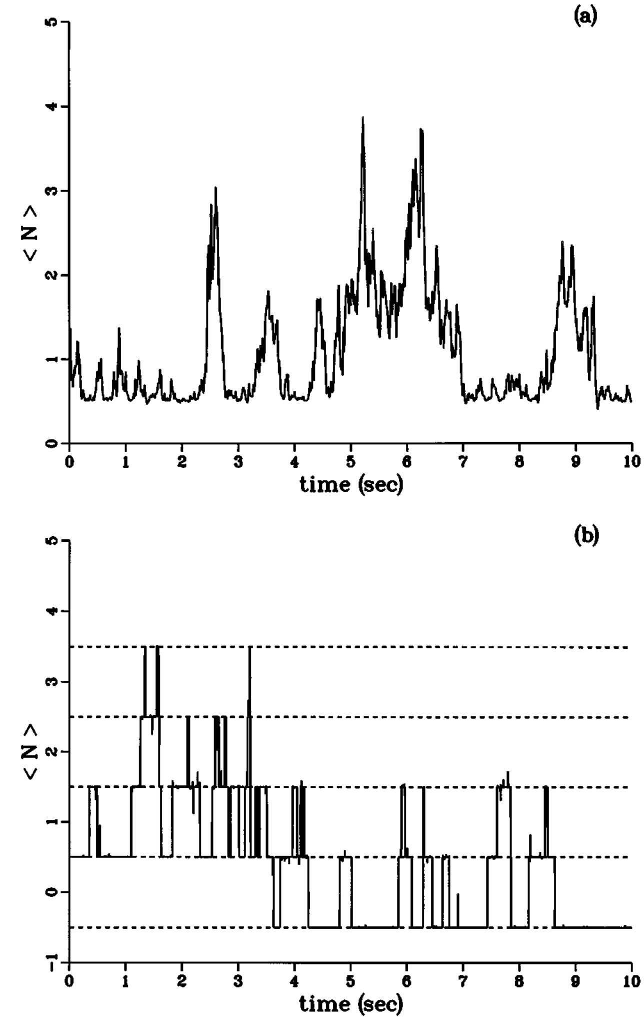

FIG. 3. Time-resolved structure of the "cyclotron grass" in an individual quantum measurement process with intial condition $|\psi\rangle_0 = |\alpha_+ = (1.0,0), s_z = +1\rangle$, and $\Delta\Omega = 0.1$, $\kappa = 2.0$, $\bar{n} = 0.5$; $\Gamma = 12.0$ (a), $\Gamma = 6400.0$ (b) (rates are in units of 1 s^{-1}).

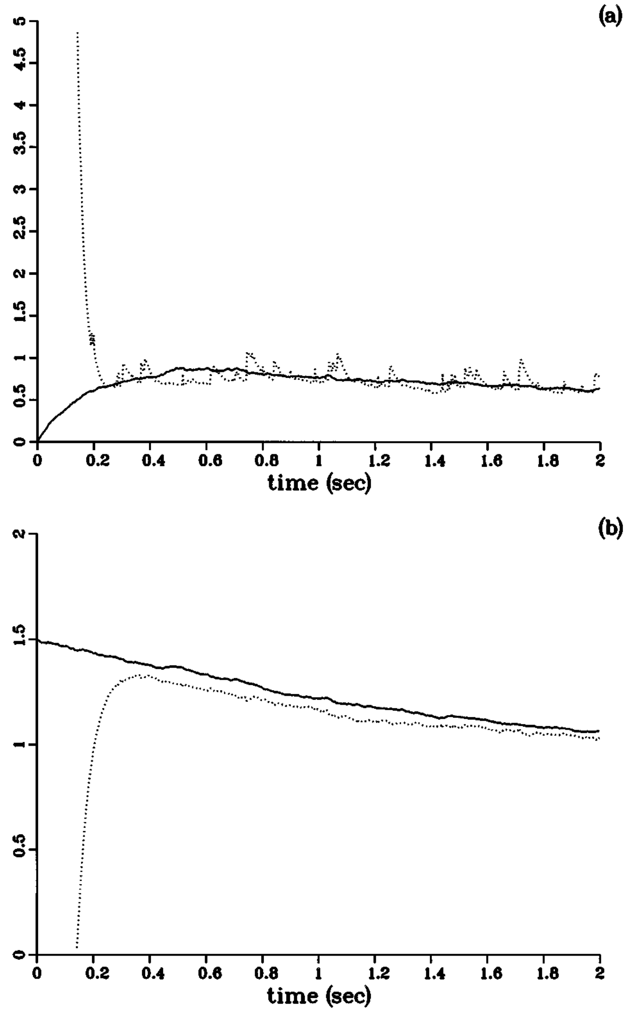

FIG. 4. Time dependence of $\Delta^{(2)}(N)$ (full curve) and $\Delta^{(2)}(\mathrm{Im}(a_z e^{i\omega t}))$ (dotted curve) as evaluated from Eq. (16) and Sec. II B 1 (a); analogous comparison between $M\langle N\rangle_t$ (full curve) and $M\langle \mathrm{Im}(a_z e^{i\omega t})\rangle_t$ (dotted curve) (b). The parameters are $\Delta\omega = 1.0$, $\Delta\Omega = 0.1$, $\kappa = 1.0$, $\bar{n} = 0.5$, $\kappa_z = 40.0$, $\bar{n}_z = 0$, $\gamma = 1.0$, $\beta = (0,100.0)$ (all rates and frequencies are in units of 1 s^{-1}). The statistical means have been estimated by 2000 realizations.

motion are resolved on the time scale of seconds for $\Gamma = 12.0\ s^{-1}$, i.e., for a case like the one shown in Fig. 2. In Fig. 3(b) the characteristic measurement time $T_m = 1/\Gamma$ is decreased significantly so that $T_m \ll 1/[\kappa(1+\bar{n})]$. Therefore measurement of the cyclotron quantum number $a_+^\dagger a_+$, which requires at least a time of the order of T_m, can be completed in the time intervals between successive thermal excitations of the cyclotron motion. In this case of a "complete" measurement of the cyclotron-quantum number almost instantaneous quantum jumps occur between the thermally activated cyclotron states.

The behavior of the fluctuations of $\langle N\rangle_t$ and $\langle \mathrm{Im}(a_z e^{i\omega t})\rangle_t$ is investigated in Fig. 4. According to the realistic state concept used in QSDM the fluctuations of these quantities are characterized by the associated variances $\Delta^{(2)}(N) = M\langle N\rangle_t^2 - (M\langle N\rangle_t)^2$ and $\Delta^{(2)}(\mathrm{Im}(a_z e^{i\omega t}))$ and all corresponding higher moments. In general these quantities cannot be evaluated from the associated density operator which describes a quantum statistical ensemble of measurements [18,29]. In Fig. 4(a) the time evolution of $\Delta^{(2)}(N)$ (for the cyclotron and spin motion) and $\Delta^{(2)}(\mathrm{Im}(a_z e^{i\omega t}))$ (for the cyclotron, spin, and z motion) are shown. For times $t \gg 1/\kappa_z$ for which the z motion has reached equilibrium both variances approach each other. This indicates that relation (15) between the out-of-phase component of the current through the resistor, namely, $\langle \mathrm{Im}(a_z e^{i\omega t})\rangle_t$, and the cyclotron- and spin-quantum number $\langle N\rangle_t$ is not only valid for ensemble means but also applies on the level of individual measurement processes provided condition (12) is fulfilled. Furthermore, it shows that for $\lambda \to \infty$ the behavior of the quantum fluctuations of the measured quantity can be described consistently either in the larger system which includes cyclotron, spin, and z motion or the smaller system which takes into account the cyclotron and spin degrees of freedom only. In Fig. 4(b) the quantity $M\langle \mathrm{Im}(a_z e^{i\omega t})\rangle_t$ of the larger system (cyclotron, z, and spin motion) is compared with $M\langle N\rangle_t$ of the smaller system (cyclotron and spin motion). It is apparent that after the establishment of equilibrium, i.e., for times $t \gg 1/\kappa_z$, both quantities approach each other. This demonstrates that in this example λ is large enough for Eq. (15) to be valid approximately.

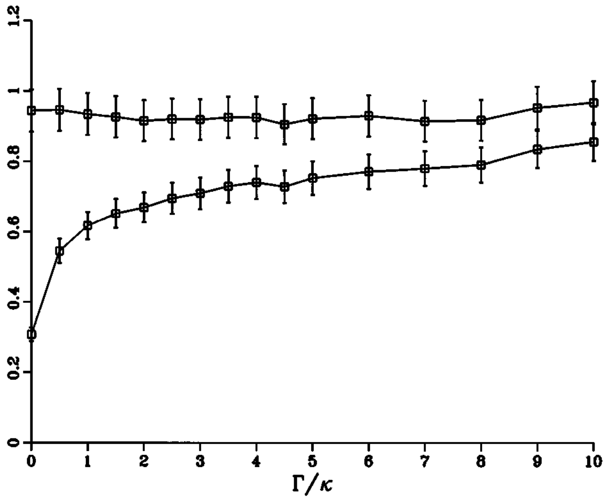

FIG. 5. Steady state dependence of $\Delta^{(2)}(N)=M\langle N\rangle_t^2-(M\langle N\rangle_t)^2$ (lower curve) and $\Sigma^{(2)}(N)=M\langle N^2\rangle_t-(M\langle N\rangle_t)^2$ (upper curve) on characteristic environmental time scale and measurement time, i.e., Γ/κ. The parameters are $\Delta\Omega=1.0$, $\kappa=1.0$, $\bar{n}=0.5$ (rates are in units of 1 s^{-1}). The bars indicate the 95%-confidence intervals.

In Fig. 5 the behavior of the fluctuations of $\langle N\rangle_t$ around its mean value is investigated in more detail. For the cyclotron and spin motion the dependence of $\Delta^{(2)}(N)$ on the ratio between the characteristic measurement time $T_m=1/\Gamma$ and the other relevant environmental time scale, namely, $1/\kappa$, is shown in steady state. This result is based on numerical solutions of the stochastic law of Eq. (1) which is associated with the master equation (16). For the sake of comparison also the quantity $\Sigma^{(2)}(N)=M\langle N^2\rangle_t-(M\langle N\rangle_t)^2$ is shown which can be evaluated from the density operator $W(t)$ of the corresponding quantum statistical ensemble directly. Figure 5 demonstrates that in the limit of "complete" continuous measurements in which T_m is much smaller than all other relevant characteristic time scales, i.e., $\Gamma\gg\kappa$, the variance $\Delta^{(2)}(N)$ approaches $\Sigma^{(2)}(N)$. Therefore it is expected that in this limit all stochastic dynamical laws of state reduction which are based on the realistic state concept used in QSDM and which are consistent with the master equation (16) will yield the same fluctuations as QSDM. However, for "incomplete" measurements in which T_m is of the same order as or larger than all other relevant time scales, i.e., $\Gamma\leqslant\kappa$, the variances $\Delta^{(2)}(N)$ differ significantly from $\Sigma^{(2)}(N)$. Therefore in this limit these variances cannot be evaluated from the density operator $W(t)$ and are a peculiar property of the particular stochastic dynamical law of QSDM. As in the CSGE the characteristic measurement time T_m can be varied in a controlled way over a large range of values, these specific theoretical predictions of QSDM concerning the fluctuation behavior of the measured quantity should be amenable to experimental tests.

IV. CONCLUSION

A consistent quantum-mechanical treatment of the CSGE has been developed in which the dominant interactions of an electron in the Penning trap with its environment and the measuring apparatus have been taken into account. Starting from the quantum-mechanical master equation for the cyclotron, magnetron, spin, and z motion it has been shown that it is the cyclotron- and spin-quantum number of Eq. (14) which is measured continuously in this experimental setup. Furthermore, an explicit expression has been derived for the characteristic measurement time T_m which determines the time scale on which reduction from a pure state into a mixed state takes place according to von Neumann's projection postulate [7]. This master equation has been used as a starting point for describing the dynamics of individual continuous quantum measurement processes within the theoretical framework of QSDM. Thereby the continuity of the relevant stochastic, nonlinear process for the quantum state implies in a natural way that T_m also determines the time scale required for completing an individual quantum measurement process and whose significance for the CSGE has already been recognized by Dehmelt [2]. As is apparent from Eq. (17), this characteristic measurement time T_m can be varied in a controlled way over a large range of values. Thus it is possible to realize both "complete" and "incomplete" continuous measurement processes depending on whether T_m is much smaller or much larger than all other relevant environmental time scales. In particular, incomplete continuous measurement processes might turn out useful for testing theoretical predictions of QSDM as far as quantum fluctuations of $\langle N\rangle_t$ are concerned. Due to the realistic state concept QSDM is based on, in such incomplete measurements these fluctuations cannot be evaluated from the density operator of the associated quantum statistical ensemble and are peculiar properties of QSDM.

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APPENDIX A

In this appendix the derivation of the adiabatic Hamiltonian of Eq. (9) is outlined. It describes the dynamics of an electron in the CSGE adequately on time scales Δt which are long in comparison with all oscillatory time scales in the Penning trap, i.e., $\Delta t\gg(2\pi)/\omega_+,(2\pi)/\omega_z,(2\pi)/\omega_-$.

The starting point is the total Hamiltonian H which describes the complete deterministic dynamics of an electron in a Penning trap under the additional influence of the inhomogeneous magnetic fields $\mathbf{B}_1(\mathbf{x},t)$ and $\mathbf{B}_2(\mathbf{x})$ of Eqs. (6) and (7), i.e.,

$$H=\frac{1}{2m}[\mathbf{p}-e\mathbf{A}(\mathbf{x},t)]^2+e\Phi(\mathbf{x})-\frac{g}{4}\frac{e\hbar}{m}\boldsymbol{\sigma}\cdot\mathbf{B}(\mathbf{x},t). \tag{A1}$$

The total vector potential is given by

$$\mathbf{A}(\mathbf{x},t)=\frac{B_0}{2}(-y\mathbf{e}_x+x\mathbf{e}_y)+B_1\cos[(\omega_s-\omega_+)t](2yz\mathbf{e}_x+xy\mathbf{e}_z)+\frac{B_2}{2}\left(z^2-\frac{x^2+y^2}{4}\right)\mathbf{e}_z\wedge(-x\mathbf{e}_x+y\mathbf{e}_y), \tag{A2}$$

with the corresponding total magnetic field $\mathbf{B}(\mathbf{x},t)=\mathrm{rot}\mathbf{A}(\mathbf{x},t)$. The electrostatic quadrupole potential of the Penning trap is given by

$$\Phi(\mathbf{x})=\Phi_0(2z^2-x^2-y^2). \tag{A3}$$

With the help of the relations

$$x=\sqrt{\frac{\hbar}{2m(\omega_+-\omega_-)}}(a_+^\dagger+a_++a_-^\dagger+a_-),$$

$$y=-i\sqrt{\frac{\hbar}{2m(\omega_+-\omega_-)}}(a_+^\dagger-a_+-a_-^\dagger+a_-),$$

$$z=\sqrt{\frac{\hbar}{2m\omega_z}}(a_z^\dagger+a_z),$$

$$p_x=\frac{i\hbar}{4}\sqrt{\frac{2m(\omega_+-\omega_-)}{\hbar}}(a_+^\dagger-a_++a_-^\dagger-a_-),$$

$$p_y=\frac{\hbar}{4}\sqrt{\frac{2m(\omega_+-\omega_-)}{\hbar}}(a_+^\dagger+a_+-a_-^\dagger-a_-),$$

$$p_z=\frac{i\hbar}{2}\sqrt{\frac{2m\omega_z}{\hbar}}(a_z^\dagger-a_z) \tag{A4}$$

between position and canonical momentum operators on the one hand and creation and annihilation operators on the other hand this Hamiltonian can be transformed to the normal coordinates of the ideal Penning trap. In typical cases of experimental interest the characteristic frequencies of the dynamics in the Penning trap differ at least by three orders of magnitude so that a simple Hamiltonian can be derived which describes the dynamics in the Penning trap properly on time scales Δt which are large in comparison with the slowest characteristic time scale, namely, the magnetron time scale $[(2\pi)/\omega_-]<10^{-3}$ s. If one keeps only terms in the Hamiltonian which are slowly varying on this time scale and neglects all terms of the order of (ω_z/ω_+) in comparison with 1 the adiabatic Hamiltonian of Eq. (9) is obtained [30].

APPENDIX B

In this appendix the derivation of the master equation for the reduced density operator of the cyclotron and spin dynamics $W(t)$ is outlined.

The starting point is the master equation of the CSGE effect as given in Sec. II B 1, i.e.,

$$\frac{d\rho}{dt}=(\mathscr{L}_0+\mathscr{L}_z+\mathscr{L}_{0z})\rho(t), \tag{B1}$$

with the Liouville operators

$$\mathscr{L}_0=-i/\hbar[H_0,.]+\tfrac{1}{2}\Sigma_{j=1,2}([L_j,.L_j^\dagger]+[L_j.,L_j^\dagger]),$$

$$\mathscr{L}_z=-i/\hbar[H_z,.]+\tfrac{1}{2}\Sigma_{j=3,4,5}([L_j,.L_j^\dagger]+[L_j.,L_j^\dagger]),$$

and

$$\mathscr{L}_{0z}=-i/\hbar[H_{0z},.].$$

For the sake of simplicity the magnetron motion, which decouples dynamically from the other degrees of freedom, will not be considered explicitly. For $H_{0z}=0$ the stationary state of the z motion ρ_z^0 is determined by the relation

$$\mathscr{L}_z\rho_z^0=0. \tag{B2}$$

In the case of exact resonance, i.e., $\Omega_z=\omega$, this stationary state gives rise to the following stationary values of the correlation functions characterizing the dynamics of the z motion:

$$M\langle a_z\rangle_0 e^{i\omega t}=-i\frac{\beta}{\kappa_z/2},\quad M\langle n_z\rangle_0=|\langle a_z\rangle_0|^2+\bar{n}_z+\frac{\gamma}{2\kappa_z},$$

$$M\langle a_z^2\rangle_0 e^{2i\omega t}=\frac{\gamma}{2\kappa_z}-\left(\frac{\beta}{\kappa_z/2}\right)^2,$$

$$\langle(\Delta n_z)^2\rangle_0=\left|\frac{\beta}{\kappa_z/2}\right|^2\left(1+2\bar{n}_z+\frac{\gamma}{\kappa_z}\right)+\left(\bar{n}_z+\frac{\gamma}{2\kappa_z}+\frac{1}{2}\right)^2-\frac{1}{4}+\frac{\gamma}{2\kappa_z}\left[\frac{\gamma}{2\kappa_z}-2\mathrm{Re}\left(\frac{\beta}{\kappa_z/2}\right)^2\right],$$

$$M\langle\!\langle(a_z-\langle a_z\rangle)\Delta n_z\rangle\!\rangle_0=\frac{\gamma}{2\kappa_z}M\langle a_z^\dagger\rangle_0 e^{-2i\omega t}+M\langle a_z\rangle_0\left(1+\bar{n}_z+\frac{\gamma}{2\kappa_z}\right),$$

$$M\langle\!\langle(a_z^\dagger-\langle a_z^\dagger\rangle)\Delta n_z\rangle\!\rangle_0=M\langle a_z^\dagger\rangle_0\left(\bar{n}_z+\frac{\gamma}{2\kappa_z}\right)+\frac{\gamma}{2\kappa_z}M\langle a_z\rangle_0 e^{2i\omega t}, \tag{B3}$$

with $M\langle\cdot\rangle_0=\mathrm{Tr}_z\{.\rho_z^0\}$, $n_z=a_z^\dagger a_z$, and $\Delta n_z=n_z-M\langle n_z\rangle_0$.

In the following we want to investigate the dynamical regime in which the anharmonic coupling between the z motion and the cyclotron and spin degrees of freedom does not disturb this equilibrium of the z motion significantly. In order to derive an equation of motion for the reduced density operator $W(t)=\mathrm{Tr}_z\{\rho(t)\}$ of the cyclotron and spin degrees of freedom we define the projection operator

$$\mathscr{P}\cdot=\rho_z^0\otimes\mathrm{Tr}_z\{\cdot\}. \tag{B4}$$

Furthermore, it will be convenient to include part of the interaction Hamiltonian H_{0z} in H_0 by redefining

$$\tilde{H}_0=H_0+\hbar\Delta\omega NM\langle n_z\rangle_0=\hbar\tilde{\Omega}_+a_+^\dagger a_++\frac{1}{2}\hbar\tilde{\Omega}_s\sigma_z,$$

$$\tilde{H}_{0z}=\hbar\Delta\omega N\Delta n_z, \tag{B5}$$

with the renormalized frequencies $\tilde{\Omega}_+=\Omega_++\Delta\omega M\langle n_z\rangle_0$, $\tilde{\Omega}_s+\Delta\omega\, g/2\, M\langle n_z\rangle_0$, and N as given by Eq. (14). The projection operator $\mathscr{P}$ fulfills the elementary relations

$$\mathscr{P}\tilde{\mathscr{L}}_0=\tilde{\mathscr{L}}_0\mathscr{P},$$

$$\mathscr{P}\mathscr{L}_z=\mathscr{L}_z\mathscr{P}=0,$$

and

$$\mathscr{P}\tilde{\mathscr{L}}_{0z}\mathscr{P}=0.$$

The resulting reduced density operator $W(t)$ obeys the (exact) Nakajima-Zwanzig equation [28]

$$\frac{dW}{dt}=\tilde{\mathscr{L}}_0W(t)+\int_0^\infty dt' K(t-t')W(t')+I(t), \quad \text{(B6)}$$

with the memory kernel

$$K(\tau)=\mathrm{Tr}_z\{\tilde{\mathscr{L}}_{0z}e^{(1-\mathscr{P})(\tilde{\mathscr{L}}_0+\mathscr{L}_z+\tilde{\mathscr{L}}_{0z})(1-\mathscr{P})\tau}\tilde{\mathscr{L}}_{0z}\rho_z^0\}.$$

The influence of initial correlations is described by the term

$$I(\tau)=\mathrm{Tr}_z\{\tilde{\mathscr{L}}_{0z}e^{(1-\mathscr{P})(\tilde{\mathscr{L}}_0+\mathscr{L}_z+\tilde{\mathscr{L}}_{0z})(1-\mathscr{P})\tau}(1-\mathscr{P})\rho(0)\}.$$

In the limit of strong damping of the z motion, i.e., $\lambda=\kappa_z/|\Delta\omega|\to\infty$ with $[|\beta/\kappa_z|^2(\Delta\omega/\kappa_z)]$ remaining constant, $I(\tau)$ and $K(\tau)$ decay so rapidly that the Markov approximation can be employed in Eq. (B6). Furthermore, in this limit $(\tilde{\mathscr{L}}_0+\mathscr{L}_z+\tilde{\mathscr{L}}_{0z})$ can be approximated by $\mathscr{L}_z$ in the memory kernel. This corresponds to the so called ''quantum Brownian motion limit'' of the Nakajima-Zwanzig equation [28] in which the rapidly damped z motion is eliminated adiabatically. Thus Eq. (B6) simplifies to

$$\frac{dW}{dt}=\tilde{\mathscr{L}}_0W(t) -[N,[N,W(t)]](\Delta\omega)^2\int_0^\infty d\tau M\langle\Delta n_z(\tau)\Delta n_z(0)\rangle_0 +[W(t),N]N(\Delta\omega)^2\int_0^\infty d\tau M\langle[\Delta n_z(\tau),\Delta n_z(0)]\rangle_0. \quad \text{(B7)}$$

With the help of the quantum fluctuation-regression theorem [28] it can be shown in a straightforward way that for $\Omega_z=\omega$ the relevant correlation functions of the z motion, which describe fluctuations of the cyclotron and spin degrees of freedom due to their coupling to the z motion, are given by

$$\int_0^\infty d\tau M\langle[\Delta n_z(\tau),\Delta n_z(0)]\rangle_0=0 \quad \text{(B8)}$$

and

$$\int_0^\infty d\tau M\langle\Delta n_z(\tau)\Delta n_z(0)\rangle_0 =\langle(\Delta n_z)^2\rangle_0/\kappa_z+|\langle a_z\rangle_0|^2\left(1+2\bar{n}_z+2\frac{\gamma}{\kappa_z}\right)\Big/\kappa_z. \quad \text{(B9)}$$

Thus the equation of motion for the reduced density operator $W(t)$ of Eq. (16) is obtained. We want to point out that Eq. (B8) is necessary for obtaining a physically acceptable master equation for $W(t)$ in canonical Lindblad form [25].

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