

A Quantum-Defect Theory of Rydberg Electrons in Laser Fields

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1. Introduction

A theory of Rydberg states in intense laser fields [1,2] is developed based on the observation that the effect of laser radiation can be described in a scattering formulation as a *finite volume interaction* coupling Coulomb type dissociation channels. Physically speaking, this picture emerges because the long range effect of the laser corresponds to elastic forced oscillations without real absorption or emission of laser photons. The identification of radiative interaction as a short range coupling opens the route for a theoretical approach which treats separately the short and long range processes and connects them in a further step. In particular, laser induced couplings may be incorporated in a quantum defect (QDT) treatment [3,4], leading us to define a set of dressed channels with intensity dependent quantum defects and mixing coefficients. These quantities may be derived by solving a system of close-coupling equations for the electron wave function (in a frame where the asymptotic electron oscillations have been transformed away) [5,6]. This allows us to read off a reaction matrix, which is a smooth function of energy for an energy range small compared with the photon energy.

Section two below is devoted to a discussion of Rydberg-free and free-free transitions (above threshold ionization) of hydrogen in circularly polarized laser fields from a Quantum Defect Theory (QDT) point of view [7].

Section three gives an outline how the full time dependence of transition probabilities from low-lying bound states to Rydberg states close to threshold may be calculated within a QDT [8].

2. Rydberg-Free and Free-Free Transitions (Above threshold ionization) [7]

2.1 The Interaction Hamiltonian

We consider an electron moving in a potential $V(\vec{x})$, which behaves asymptotically like a Coulomb potential, and under the influence of a laser field described by an electric field $\vec{E}(\vec{x}, t)$. In the dipole approximation the Schrödinger equation is

$$i\hbar \frac{\partial}{\partial t} \Psi_L(\vec{x}, t) = \left[\frac{1-\partial^2}{2m} + V(\vec{x}) - e\vec{x} \cdot \vec{E}(0, t) \right] \Psi_L(\vec{x}, t). \quad (2.1)$$

The subscript L for Ψ_L indicates that in first order perturbation theory (in

the light field) Eq.(2.1) leads to transition matrix elements in the length form. For a discussion of Rydberg electrons it is more convenient to work in a frame, where the asymptotic oscillations of the electron are transformed away. In this "space translated frame" [5,6] the Schrödinger equation is

$$i\hbar \frac{\partial}{\partial t} \Psi_A(\vec{x}, t) = \left[\frac{1-\beta^2}{2m} p^2 + V(\vec{x}+\vec{\alpha}(t)) \right] \Psi_A(\vec{x}, t) \quad (2.2)$$

with $\vec{\alpha}(t)$ the solution of Newton's equation $m \frac{d^2}{dt^2} \vec{\alpha}(t) = e \vec{E}(0, t)$ for a free electron oscillating in the laser field. The subscript A for the wave function refers to the fact that Eq. (2.2) leads to the acceleration form for first order transition matrix elements.

Approximate methods for solving the Schrödinger equation in a time dependent field are usually based on separating the Hamiltonian $H(t)$ into an unperturbed Hamiltonian $H_0 = \frac{1-\beta^2}{2m} p^2 + V(\vec{x})$, and an interaction part $H_1(t)$. In the cases (2.1) and (2.2) the "length" and "acceleration" interaction Hamiltonians have the form $H_{1L}(t) = -e\vec{x} \cdot \vec{E}(0, t)$ and $H_{1A}(t) = V(\vec{x}+\vec{\alpha}(t)) - V(\vec{x})$, respectively. A central point we wish to emphasize is that the two forms of the interaction Hamiltonians put the weight of the interaction into different regions of space: close to the atomic core it is preferable to work with the dipole interaction Hamiltonian $H_{1L}(t)$, which there becomes a perturbation (in

the sense that $H_{1L} \rightarrow 0$ for $r = |\vec{x}| \rightarrow 0$). On the other hand $H_{1A}(t)$ goes to zero asymptotically as it behaves for large r like the potential of an oscillating dipole; thus this form is the most convenient when dealing with electrons in extended Rydberg orbitals or in the continuum.

In the present exploratory work the calculations will be restricted to hydrogen and we shall use the Hamiltonian $H_{1A}(t)$ in whole space. For complex atoms or molecules it would be preferable to solve the Schrödinger equation using the length form $H_{1L}(t)$ in an inner region, and the acceleration form in an outer region, with the wave functions in both regions connected by a time-dependent unitary transformation. This procedure is reminiscent of (but not identical to) the frame-transformation technique [4] in MQDT and has - in the special case of calculating dipole matrix element - been used by Peach [9] and Seaton [10].

The physical picture emerging from this discussion is the following. The electronic motion is governed by different forces in different regions of configuration space: close to the core the atomic forces dominate; in the asymptotic domain the electron vibrates rapidly in the time-dependent optical laser field while the weak atomic force is responsible for a slow mean motion of the electron in the asymptotic Coulomb potential. Transition from Rydberg states to different bound or free orbits by absorption or induced emission of laser photons occur in the transition zone between the two regions.

2.2 Close-Coupling Equations and Radiative Reaction Matrix

We proceed in our discussion by studying the electron wave function in the presence of a laser field. To be specific, we study the simplest possible system, namely a Rydberg electron in a hydrogen atom under the influence of circularly polarized light $\vec{E}(0, t) = \xi \vec{e} e^{-i\omega t} + c.c.$ ($\vec{e} = -(\vec{e}_1 + i\vec{e}_2)/\sqrt{2}$). The potential $V(\vec{x}+\vec{\alpha}(t))$ is now a Coulomb potential which is moving with angular

frequency ω on a circle in the xy -plane of our coordinate system. We have $\vec{\alpha}(t) = \alpha_0(\vec{e}_1 \cos(\omega t + \delta) + \vec{e}_2 \sin(\omega t + \delta))$ with the radius $\alpha_0 (\geq 0)$ and phase δ defined by $\sqrt{2}e\mathcal{E}/m\omega^2 = \alpha_0 e^{-i\delta}$. In this case the Floquet-Ansatz

$$\Psi_A(\vec{x}, t) = \sum_{N=-\infty}^{\infty} \sum_{\ell m} F_{\ell m}^{(N)}(r)/r Y_{\ell m}(\theta, \phi) e^{-iN(\omega t + \delta) - iEt/\hbar}, \quad (2.3)$$

with $F_{\ell m}^{(N)}(r)$ a radial wave function and E a quasi-energy, reduces the time-dependent Schrödinger equation (2.2) to a system of time independent close-coupling equations (see also Ref. [5,6]),

$$\left\{ E + N\hbar\omega + \frac{\hbar^2}{2\mu} \left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} \right) - V_{\ell m, \ell m}^{(0)}(\alpha_0, r) \right\} F_{\ell m}^{(N)}(r) - \sum_{N' \ell' m'} V_{\ell m, \ell' m'}^{(N-N')}(\alpha_0, r) F_{\ell' m'}^{(N')}(r) = 0. \quad (2.4)$$

Here the potential terms $V_{\ell m, \ell' m'}^{(N-N')}(\alpha_0, r)$ are defined as the Fourier coefficients of matrix elements of the potential $V(\vec{x} + \vec{\alpha}(t))$ between spherical harmonics. Using the multipole expansion we find

$$V_{\ell m, \ell' m'}^{(N-N')}(\alpha_0, r) = \frac{e^2}{4\pi\epsilon_0} \sum_{k=|N-N'|}^{\infty} \langle \ell m | C_{k, N-N'} | \ell' m' \rangle (-1)^{N-N'} C_{k, -(N-N')}(\pi/2, 0) r_{<}^k r_{>}^{k+1}. \quad (2.5)$$

Here we have defined $r_{<} = \min(\alpha_0, r)$, $r_{>} = \max(\alpha_0, r)$. The parameter α_0 plays the role of an intensity parameter in our problem. When the close-coupling equations (2.4) are rewritten in atomic units, the dimensionless parameter which determines the coupling between the channels in Equ. (2.4) is $\bar{\alpha}_0 = \alpha_0/a_0 = \sqrt{2} (2\mathcal{R}_y/\hbar\omega)^2 (I/I_0)^{1/2}$ with a_0 the Bohr radius, \mathcal{R}_y the Rydberg constant, $I = 2c\epsilon_0 |\mathcal{E}_0|^2$ the light intensity and $I_0 = 2c\epsilon_0 \mathcal{E}_{a.u.}^2 = 1.4 \cdot 10^{17} \text{ W/cm}^2$ the atomic unit light intensity. In the close-coupling equations (2.4) each channel is identified by the parameters $\{N, \ell, m\}$ with N the Floquet (photon-) index and ℓ, m angular momentum quantum numbers of the electron. The prime on the sum in Equ. (2.4) indicates that diagonal terms should be left out in performing the sum.

The potential terms in Equ. (2.5) have the following properties. First of all, the potential $V_{\ell m, \ell' m'}^{(0)}$ goes asymptotically like $\delta_{\ell\ell'} 1/r$, i.e. behaves like a Coulomb potential; asymptotically, degenerate channels with different ℓ and ℓ' are at most coupled by an α_0^2/r^3 potential. Eqs. (2.4) and (2.5) identify the thresholds of our system as $-N\hbar\omega$. The asymptotically dominant inter-channel potential goes like α_0/r^2 . It is essential that the interaction

terms which asymptotically behave as $1/r^2$ couple only channels differing by one photon energy $\hbar\omega$, i.e. channels with different thresholds (this asymptotic structure of the potential terms is in complete analogy to what one finds in close-coupling equations for low energy electron scattering from ions). Indeed, the asymptotic oscillations of the Coulomb functions in non-degenerate channels tend to interfere destructively, thus limiting the effective range of the coupling to a finite region. A crude estimate shows that for energies near threshold we have $r_s \approx \omega^{-2/3}$ (in atomic units). Furthermore, the size of the interaction zone also grows with the oscillation radius $\bar{\alpha}_0$. For the range of $\bar{\alpha}_0$ and ω values below, we have $\bar{\alpha}_0 < r_s$, so that the frequency is the dominating factor to determine the boundary of the reaction zone.

The possibility of truncating the set of close-coupling equations (2.4) is mainly a question of light intensity for a given laser frequency. For $\bar{\alpha}_0 \ll 1$ the coupling is weak and we expect the results of perturbation theory (with respect to the field intensity) to emerge in this limit. On the other hand for $\bar{\alpha}_0 \geq 1$ the inter-channel coupling becomes strong and one expects that the size of the truncated system should be increased until convergence is found for the given light intensity.

For a given energy E and N_{tot} channels included in the calculation, there are N_{tot} real independent solutions of the close-coupling system which may be written for $r > r_c$ as

$$F_{ij}(r) = s_i(r) \delta_{ij} + c_i(r) \cdot \mathcal{K}_{ij} \quad (r > r_c), \quad (2.6)$$

where $j = \{N_j, \ell_j, m_j\}$ denotes the index of the solution and i the channel components. $s_i = s(\epsilon_i, \ell_i, r)$ and $c_i = c(\epsilon_i, \ell_i, r)$ are energy normalized regular and irregular Coulomb wave functions for the energy $\epsilon_i = E + N_1 \hbar\omega$ [3]. \mathcal{K} is a radiative reaction matrix which varies slowly with the asymptotic electron energy since it is built at short range [4].

The physical interpretation of the smooth \mathcal{K} -matrix depends on the energy range considered. If all channels are open ($\epsilon_i > 0$ for all i), \mathcal{K} is the usual reaction matrix describing laser induced transitions between alternative fragmentation channels (e.g., for inverse Bremsstrahlung). If only N_o channels are open, the $N_c = N_{\text{tot}} - N_o$ closed channels ($\epsilon_i < 0$) must be eliminated from the asymptotic wave function ($r \rightarrow \infty$) since the corresponding Coulomb functions s_i and c_i have exponentially diverging components. This elimination leads to the MQDT expression of the effective reaction matrix R (dimensions $N_o \times N_o$), restricted to the open channel space [3],

$$R = \mathcal{K}_{oo} - \mathcal{K}_{oc} [\tan\pi\nu_c + \mathcal{K}_{cc}]^{-1} \mathcal{K}_{co} \quad (2.7)$$

with the subscripts o and c referring to a partitioning of \mathcal{K} with respect to open and closed channels; $(\tan\pi\nu_c)_{ij} = \tan\pi\nu_i \delta_{ij}$ is an $N_c \times N_c$ diagonal matrix with ν_i effective quantum numbers defined by $\epsilon_i = -\mathcal{K}_y/v_i^2$. Contrarily to

the matrix R varies rapidly with the energy (via the energy variables ν_i) in the vicinity of the Rydberg resonances in the closed channels. These resonances have a finite width due to the decay (ionization) of the Rydberg electron into the continuum (see also Ref. [6]).

2.3 Solution of the radiative close-coupling equations for hydrogen in circularly polarized laser light

To illustrate the method we have studied photoionization of ns Rydberg states of hydrogen by circularly polarized light as a function of light intensity. The discussion will be centered on the cases where single

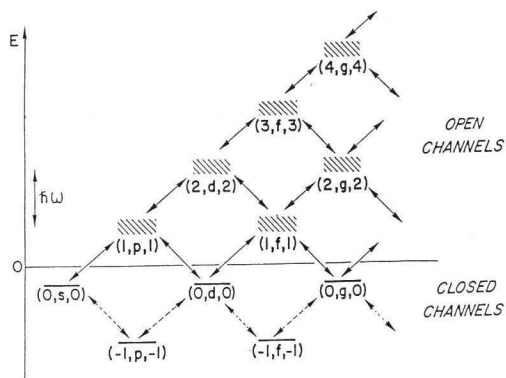


Fig. 1: Absorption and stimulated emission processes for an s electron in circularly polarized light. Only the processes involving up to five partial waves are indicated. Stimulated emission to the strongly closed channels (dotted arrows) is not included in the present work.

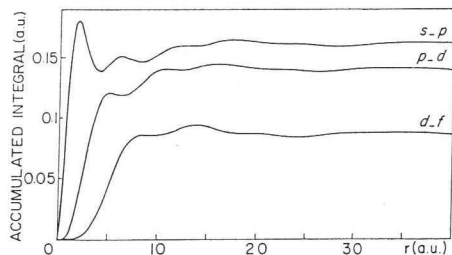


Fig. 2: Convergence of the radiative reaction matrix for a weak field one-photon transition (proportional to a radial integral in the acceleration form) $\ell \rightarrow \ell+1$ as a function of the size of the reaction zone r .

photoionization is energetically possible ($n > (\mathcal{H}_y/\hbar\omega)^{1/2}$) so that an increasing light intensity will cause absorption of additional photons in the continuum (above threshold ionization). In its simplest form the system of close-coupling equations must include a single closed channel with an s electron and a number of open channels with higher ℓ -values (Fig. 1). At high intensities additional closed channels reached by stimulated emission must be introduced, as indicated in Fig. 1. In the present calculations we have included up to twenty-five channels with ℓ -values up to $\ell = 8$, and Floquet indices $0 \leq N \leq 8$. The corresponding close-coupling system (2.4) is solved by propagating the solution typically out to $r_c \approx 30 - 50$ au. The calculations have been performed slightly above threshold ($E = 0.01 \mathcal{H}_y$). In this preliminary study we have not included strongly closed channels corresponding to laser induced transitions down to low-lying bound states (dotted arrows in Fig. 1; $N < 0$).

To demonstrate the finite range character of the interaction we have plotted in Fig. 2 for a weak laser intensity the reaction matrix element, which is accumulated in an interaction region of radius r , for a one-photon transition (i.e., essentially an accumulated radial integral) for $\epsilon_i = 0$, $\epsilon_j = \omega = 0.3 \mathcal{H}_y$ and $\ell_j = 0, 1, 2$ ($\ell_j = \ell_i + 1$). Note that in the present example the matrix elements have essentially converged in a region $r \leq 15 a_0$.

The total and partial ionization probabilities of the $6s$ states are plotted in Fig. 3 as a function of the laser intensity for the frequency $\omega = 0.35$ Rydberg. These results were obtained by calculating the reaction matrix \mathcal{K} for an energy $E = 0.01 \mathcal{H}_y$ and extrapolating \mathcal{K} to the bound region to extract the ionization widths of the Rydberg states. The straight lines in Fig. 2 correspond (in logarithmic coordinates) to perturbative results obtained in lowest order of perturbation theory for each partial rate $\gamma^{(k)}$ (proportional to I^k). The close-coupling values hardly depart from the perturbative ones below 10^{13} W/cm^2 . A quantitative comparison with accurate perturbation calculations ($\bar{\alpha}_0 = 0.1$) of above threshold two-photon ionization [11,12] is given in Table 1 for some ns levels. We have varied the radius r_c of the interaction zone (matching point to Coulomb functions) in order to demonstrate once again the finite range character of the radiative interaction. Note that the three n -values for which comparison with previous perturbation results is possible at $\omega = 0.2531 \mathcal{H}_y$ are obtained from a single close-coupling calculation and multiplication by a scaling factor n^{-3} . The agreement is very satisfactory for the higher n -values ($n=5,6$) and as expected becomes worse with decreasing n .

Above 10^{13} W/cm^2 ($\bar{\alpha}_0 > 0.5$) the partial rates for the lowest order processes saturate (curves $\gamma^{(1)}$ and $\gamma^{(2)}$ on Fig. 3). The total rate is slightly larger than the linear first order perturbation theory result (golden rule result; uppermost straight line on Fig. 3) in the intensity range below $5 \cdot 10^{13} \text{ W/cm}^2$ and then becomes smaller. An important point to be noted is that the total ionization probability per unit time remains very close to the golden rule result up to intensities for which the partial rates already differ notably from their lowest order perturbation values.

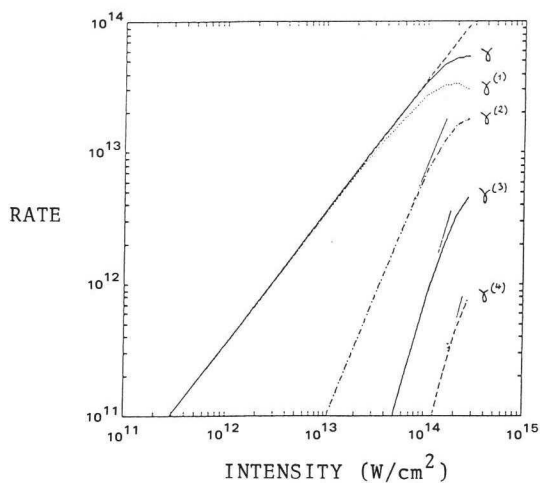


Fig. 3: Ionization rate of hydrogen 6s state by circularly polarized light ($\omega = 0.35 \text{ Ry}$). γ denotes the total ionization rate and $\gamma^{(k)}$ the partial rate for k-photon ionization.

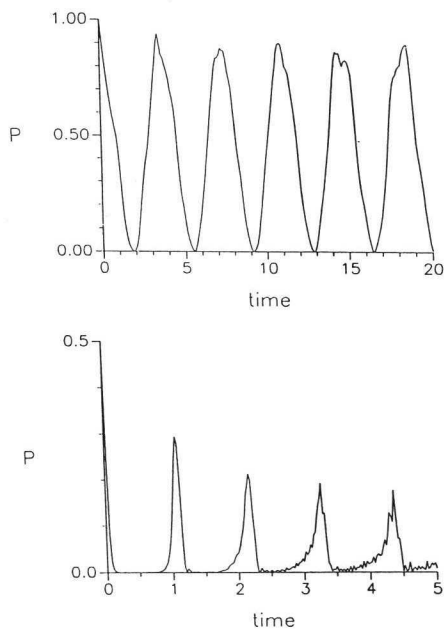


Fig. 4: Ground state probability as a function of time (in units of $T_{\bar{e}}$) for $\gamma = 10^{-6}$ (a.u.) and (a) $\bar{\epsilon} = -2 \cdot 10^{-4}$ (a.u.), (b) $\bar{\epsilon} = -1.25 \cdot 10^{-5}$ (a.u.) ($\alpha = 0$).

3. Near threshold excitation of Rydberg series from a low-lying bound state [8]

The purpose of this section is to discuss the problem of near threshold excitation of Rydberg series from one of the low-lying atomic states by an intense laser field. In particular the list of questions to be addressed includes the time-evolution, intensity and frequency dependence of transition probabilities close to threshold when both an infinite number of Rydberg levels and continuum states participate in the laser induced dynamics [13,14].

Usually, laser excitation of atoms and molecules is described within models where the Schrödinger equation is solved in a finite basis of (resonant) atomic states in the rotating wave approximation (in the simplest case a two-level system), while the coupling to non-resonant levels is treated in perturbation theory. Diagonalizing the Hamiltonian of the atom in this finite basis then leads to a set of dressed atomic states. The time dependence of the transition probabilities in a monochromatic coherent field exhibits the familiar Rabi oscillations. Obviously, the validity of this approach is confined to an energy region far below threshold as long as the Rabi frequency, which characterizes the laser induced coupling, is much smaller than the energy separation between adjacent excited states [16]. On the other hand, transitions from bound states to the flat electron continuum are usually formulated within a pole (or Markov) approximation [15], which describes the (exponential) decay of a discrete state into a continuum with a rate given by Fermi's golden rule and thus serves to define an ionization cross section. Again this approximation becomes invalid when a threshold is approached [13,14].

In the present section we give a brief outline of a theory of near threshold excitation starting from the observation that this problem can be formulated in QDT as the decay of bound channels (the low-lying atomic states) to a set of free channels (Rydberg or continuum states). The new aspect in comparison with the standard QDT formulation is the necessity to study the full time dependence of the transition probabilities.

For simplicity we confine ourselves to the model case of a single bound state and a single Rydberg series. Generalization to more channels is straightforward [8]. For the wave function $|\Psi(t)\rangle$ of the atomic electron which is excited from the ground state $|g\rangle$ with energy ϵ_g to Rydberg states with energies ϵ_n and continuum states we make the ansatz $|\Psi(t)\rangle = |g\rangle a_g(t) + |F(t)\rangle$, where $a_g(t)$ is the ground state amplitude and $|F(t)\rangle$ is the wave function of the Rydberg and continuum states. We assume that the electric field of the laser with frequency ω is turned on instantaneously at $t = 0$ and for $t \geq 0$ has constant amplitude ξ . Taking the Laplace transform of the Schrödinger equation we find the following system of close coupling equations (we use atomic units)

$$\begin{aligned} (z - \epsilon_g) a_g(z) + \langle g | d^* \xi^* | F(z+\omega) \rangle &= i \\ (z + \omega - H_A) |F(z+\omega)\rangle + d \xi |g\rangle a_g(z) &= 0 \end{aligned} \quad (3.1)$$

with z the Laplace transform variable, H_A the atomic Hamiltonian and d the dipole operator. Using QDT the solution for $a_g(z)$ is (see for example Ref [3])

$$a_g(z) = i [z - \epsilon_g - \Sigma(z+\omega)]^{-1}. \quad (3.2)$$

with

$$\Sigma(\epsilon) = \begin{cases} \delta\omega - i\gamma/2 & (\epsilon > 0) \\ \delta\omega + \gamma/2 \cot\pi(\nu+a) & (\epsilon < 0) \end{cases} \quad (z = \epsilon + i\eta, \eta \rightarrow 0^+) \quad (3.3)$$

a self-energy of the ground state. Here $\alpha, \delta\omega$ and γ are slowly varying QDT parameters: α is a quantum defect of the Rydberg series, $\delta\omega$ is a quadratic Starkshift of the ground state and γ is the ionization rate of $|g\rangle$ in agreement with Fermi's Golden rule (both are proportional to the light intensity). The rapid energy dependence of $\Sigma(\epsilon)$ below threshold ($\epsilon < 0$) is reflected in the variation with $\nu = (-2\epsilon)^{-1/2}$. Note that the poles of $a_g(z)$

(the quasi energies $\tilde{\epsilon}_n$) are determined by $\tilde{\epsilon}_n = -1/2 (n - \alpha - \mu(\tilde{\epsilon}_n))^{-2}$ with the intensity dependent quantum defect $\mu(\epsilon) = -1/\pi \arctan(\gamma/2 (\epsilon - \bar{\epsilon})^{-1})$ in analogy to the perturbation of a Rydberg series by a single interloper.

We now turn to study the time evolution of the ground state amplitude. Inverting the Laplace transform by contour integration and using the theorem of residues we find the ground state amplitude in the dressed state representation

$$a_g(t) = \sum_n e^{-i(\tilde{\epsilon}_n - \omega)t} \frac{1}{\pi} \tilde{\nu}_n^{-3} \frac{\gamma/2}{(\tilde{\epsilon}_n - \bar{\epsilon})^2 + (\gamma/2)^2} \left(1 + \frac{2}{\gamma} \frac{2}{\pi} \tilde{\nu}_n^{-3}\right) + \tag{3.4}$$

$$+ i/2\pi e^{-i(\bar{\epsilon} - \omega)t} \left\{ e^{-\gamma t/2} [E_1(-i\bar{\epsilon}t - \gamma t/2) - 2i\pi \text{ST}(\bar{\epsilon})] - e^{\gamma t/2} E_1(-i\bar{\epsilon}t + \gamma t/2) \right\}.$$

$E_1(x)$ is the exponential integral and ST a unit step function. Equ.(3.4) shows that all quasi-energies in an energy interval of width $\gamma(1 + \frac{2}{\pi\gamma} \nu^{-3})^{1/2}$ around the mean excited energy $\bar{\epsilon}$ are excited and are coupled back to the ground state. We can, therefore, distinguish between two different dynamical regimes, namely the (i) two-level limit characterized by $\gamma \ll \bar{\nu}^{-3}$ (or a weak intensity limit) and (ii) excitation close to threshold (or a strong field limit) where $\gamma \gg \bar{\nu}^{-3}$, which is equivalent to $T_{\bar{\epsilon}} = 2\pi (-2\bar{\epsilon})^{-3/2} \gg 2\pi/\gamma$ with $T_{\bar{\epsilon}}$ the classical orbit time of the Coulomb problem associated with energy $\bar{\epsilon}$ [16]. In the time domain this last inequality expresses the fact that the depletion time of the ground state corresponding to the 'ionization rate' γ as induced by the strong laser field is shorter than the classical orbit time for the electron excited into the Rydberg series.

Under the condition $\gamma \gg \bar{\nu}^{-3}$ a direct evaluation of Equ.(3.4) is inconvenient. Instead we prefer to represent the ground state amplitude in the form of a multiple scattering expansion [17]

$$a_g(t) = e^{-i(\bar{\epsilon} - \omega)t} e^{-\gamma t/2} + \tag{3.5}$$

$$\sum_{m=1}^{\infty} \int_{-\infty}^0 d\epsilon e^{-i(\epsilon - \omega)t} [i(\epsilon - \bar{\epsilon} + i\gamma/2)^{-1} \mathfrak{P}_{\bar{\epsilon}}^{(-)}] e^{2i\pi\nu} [\chi(\epsilon) e^{2i\pi\nu}]^{m-1} [\mathfrak{P}_{\bar{\epsilon}}^{(-)} i(\epsilon - \bar{\epsilon} + i\gamma/2)^{-1}]$$

with the complex photoionization amplitudes $\mathfrak{P}_{\bar{\epsilon}}^{(-)} = -i e^{i\pi\alpha} d_{eg}(\epsilon)$ and the laser-assisted electron-ion scattering matrix $\chi(\epsilon) = e^{2\pi i(\alpha + \mu(\epsilon))}$. In view of $\gamma \gg \bar{\nu}^{-3}$, $e^{2i\pi\nu}$ (which involves the classical action $2\pi\nu$ along a closed Coulomb trajectory) is a rapidly oscillating function of energy. Therefore Equ. (3.5) can be evaluated by stationary phase methods. This leads us to the

following interpretation:
 For short times $t \lesssim 1/\gamma \ll T_{\epsilon}^{-}$ (i.e., for times much shorter than the classical orbit time) the ground state amplitude decays exponentially with the "ionization" rate γ (like in a "true" ionization process above threshold). The depletion of the ground state $|g\rangle$ on a time scale $t \approx 1/\gamma \ll T_{\epsilon}^{-}$ implies the generation of a radial Rydberg wave packet [16]. The exponential decay law for the ground state reflects the fact that during the formation of the wave packet the electron does not "see" the outer turning point of the Coulomb potential of the ionic core, and therefore for times $t \ll T_{\epsilon}^{-}$ behaves like in a true ionization process above threshold. For times $t \gtrsim T_{\epsilon}^{-}$ the dominant contribution in Equ.(3.5) stems from the term with $m = 1$ which describes the first return of the wave packet to the inner turning point where it can be deexcited back to the initial state $|g\rangle$ (stimulated recombination). Similar arguments can be presented for times $t \approx m T_{\epsilon}^{-}$ ($m = 2,3,\dots$), so that the ground state will show population pulsations with the period of the classical orbit time.

In Figs. 4 we have plotted the ground state probability as a function of time for different frequencies ω and a fixed value of γ . Fig.2a represents a case where only a few quasi energies contribute to Equ.(3.4) giving rise to slightly modified Rabi oscillations (i.e. the system behaves like a two-level atom). As soon as the mean ionization time $1/\gamma$ becomes smaller than the mean classical orbit time T_{ϵ}^{-} the time dependence of the ground state probability reflects the dynamics of excitation and deexcitation of the generated radial wave packet. The broadening of the recombination peaks with increasing time corresponds to the spreading of the excited wave packet.

Table 1: Two-photon ionization rates (10^9s^{-1}) for ns Rydberg states of H in circularly polarized light. For a given frequency the close coupling equations are solved at an energy $E = 0.01\text{Ry}$ with four channels ($0 \leq N, l \leq 2$), $\bar{\alpha}_0 = 0.1$ and different values r_c of the size of the reaction zone. In extrapolating to the bound state region the energy matrix is ignored, i.e. the ionization rates for all ns states are obtained from a single calculation above threshold.

$\omega(\text{Ry})$	$I(\text{w/cm}^2)$	n	close-coupling calculations		perturbation theory	
			$r_c = 30 a_0$	$r_c = 100 a_0$	(a)	(b)
0.2531	$1.79 \cdot 10^{11}$	6	0.285	0.282	0.285	0.286
		3	2.28	2.27	2.39	2.40
		2	7.70	7.66	-	9.12
0.3375	$5.67 \cdot 10^{11}$	3	3.09	3.07	3.26	3.27
0.3645	$7.72 \cdot 10^{11}$	5	0.721	0.717	0.732	0.734

(a) from Klarsfeld and Maquet (Ref. 11)
 (b) from Aymar and Crance (Ref. 12)

4. Summary

We have shown that QDT can be applied to formulate a theory of Rydberg states in laser fields. The essential points we wish to emphasize are: (i) Our theory gives a unified description of an entire Rydberg series and adjoining continuum in a laser field; and more generally of radiative coupling between several Rydberg series and continua in terms of a few energy independent parameters. (ii) Strong field effects in continuum-continuum transitions are taken into account from the outset, limited only by the number of channels included and beyond the validity of (lowest) order perturbation theory. These results are relevant also for low-energy scattering in laser fields and above threshold ionization [1,2]. Furthermore the time dependence of transition probabilities from low-lying atomic states to Rydberg states can be calculated. (iii) Electron correlation effects (such as auto-ionization and perturbation of Rydberg series) can be included in the theory and can be treated on the same level as the radiative interactions [18].

Acknowledgment: This work was supported by the Österreichische Fonds zur Förderung der wissenschaftlichen Forschung P6008P and Jubiläumsfonds der Österreichischen Nationalbank under project number 2604.

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