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# Nonlinear Josephson-type oscillations of a driven, two-component Bose-Einstein condensate

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We propose an experiment that would demonstrate nonlinear Josephson-type oscillations in the relative population of a driven, two-component Bose-Einstein condensate. An initial state is prepared in which two condensates exist in a magnetic trap, each in a different hyperfine state, where the initial populations and relative phase between condensates can be controlled within experimental uncertainty. A weak driving field is then applied, which couples the two internal states of the atom and consequently transfers atoms back and forth between condensates. We present a model of this system and investigate the effect of the mean field on the dynamical evolution. [S1050-2947(99)50801-9]

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An interesting property of a weakly interacting Bose-Einstein condensate is that it can be ascribed an overall phase that can be measured relative to another condensate [1–5]. This is a quantum-mechanical effect exhibited on a macroscopic scale. Some recent experiments on Bose-Einstein condensation (BEC) in dilute alkali-metal vapors have investigated the relative phase of two overlapping condensates. For example, in the experiment reported in Ref. [6], interference fringes in the density of two overlapping condensates were observed. More recently, the authors of Ref. [7] measured the relative phase of two condensates in different hyperfine states using a technique based on Ramsey's method of separated oscillating fields [8].

A classic experiment that investigates the role of coherence on the dynamical evolution of two coupled macroscopic quantum systems is the Josephson-junction experiment, in which a superconducting current of Cooper pairs exhibits coherent oscillations [9]. There have been several proposals suggesting an analogous experiment in the context of the more recent work being conducted on BEC of dilute atomic gases [10–19]. One can imagine preparing two initially isolated condensates in a double-well potential and then lowering the central barrier to allow coupling between condensates to arise from tunneling. One could then observe the time rate of change of the relative population, which is the analogous quantity to the current of Cooper pairs in the usual Josephson-junction experiment. Further interesting effects could then also be studied, such as the nonlinear effect of the mean field on the system's behavior [14,16].

We propose a different experiment that would exhibit much the same physics as in the proposed double-well tunneling experiment but is based on the work done recently on two-component condensates, where two different hyperfine states can be populated and confined in the same trap [7,20–22]. We restrict our attention to the situation described in Refs. [7] and [21] where the authors trapped and cooled $^{87}$Rb atoms in a magnetic trap below the critical point for BEC. The trapped atoms were initially in the $|f=1,m_f=-1\rangle$ hyperfine state, but after condensation the $|f=2,m_f=1\rangle$ state could be populated through a two-photon transition. After this first $\pi/2$ pulse, which lasts a fraction of the period of the trap, the relative motion of the two condensates oscillates and damps out to a stationary situation [21]. After applying a second $\pi/2$ pulse, the authors observed a well-defined relative phase that persists even beyond these damped oscillations [7,21].

In light of this observed ''phase rigidity,'' we envision the following experiment. An initial stationary state is first prepared as described above by applying a short drive pulse that produces condensates in both $|1,-1\rangle$ and $|2,1\rangle$ states with known populations. When the transient relative motion has damped out, the two condensates each sit in different shifted harmonic traps due to their different magnetic moments, with an overlap region that can be controlled experimentally. A weak driving field that couples these two internal states is then applied so that the condensates are coupled in the overlap region. The time at which this sustained drive is turned on determines the initial relative phase accumulated by the condensates, which is measured relative to the accumulated phase of the driving field [7]. Oscillations in the relative population will occur, which depend on the initial relative phase and populations, and will exhibit nonlinear behavior due to the mean field.

The true system described in Refs. [7] and [21] has axial symmetry, with the two components separated along the $z$ axis. We simplify the problem by treating the system in only one dimension along the $z$ axis. We make another simplification by treating temperature zero and neglecting the fluctuations about the mean field that would give rise to damping.

In order to simplify our notation, we label the hyperfine states as $|2\rangle=|2,1\rangle$ and $|1\rangle=|1,-1\rangle$. In the presence of a weak external magnetic field these hyperfine states are separated in frequency by $\omega_0$. The system is driven by a two-photon pulse, the strength of which we characterize by the two-photon Rabi frequency $\Omega$, which we take to be real-valued. We label the frequency of the two-photon drive by $\omega_d$, which can be varied to give a finite detuning $\delta=\omega_d-\omega_0$. In the following we have made the rotating-wave approximation by dropping the high-frequency terms in the atom-field interaction. Finally, we assume that both states have a long lifetime compared to the period of the trap.

After making the above approximations, we carried out standard mean-field theory on this coupled, two-component

 

system to obtain the following equations for the time evolution of the condensates in the rotating frame:

$$i\begin{pmatrix}\dot{\psi}_2\\ \dot{\psi}_1\end{pmatrix}=\begin{pmatrix}H_2^0+H_2^{\mathrm{MF}}-\delta/2 & \Omega\\ \Omega & H_1^0+H_1^{\mathrm{MF}}+\delta/2\end{pmatrix}\begin{pmatrix}\psi_2\\ \psi_1\end{pmatrix}. \tag{1}$$

The frequency of each trap is $\omega_z$. We work in the ''natural'' units of the problem, so that time is in units of $1/\omega_z$, energy is in terms of the trap level spacing $\hbar\omega_z$, and position is in units of the harmonic-oscillator length $z_{\mathrm{sho}}=\sqrt{\hbar/m_{\mathrm{Rb}}\omega_z}$. The complex functions $\psi_i(z,t)$ are the mean-field amplitudes of each condensate, where $i=1,2$. They are normalized to give the populations $N_i(t)$, where the total number $N=N_2+N_1$ is constant.

The Hamiltonians appearing in Eq. (1) describe the free evolution $H_i^0$ and the mean-field interaction $H_i^{\mathrm{MF}}$ for each component

$$H_i^0=\frac{1}{2}p^2+\frac{1}{2}(z+\gamma_i z_0)^2,$$
$$H_i^{\mathrm{MF}}=\lambda_{ii}|\psi_i|^2+\lambda_{ij}|\psi_j|^2, \tag{2}$$

where $\gamma_1=1$ and $\gamma_2=-1$, and $z_0$ is the shift of each trap from the origin. The mean-field strength is characterized by $\lambda_{ij}=a_{ij}/z_{\mathrm{sho}}$, which depends on the scattering length $a_{ij}$ of the collision. In general there will be three different values, one for each type of collision in this two-component gas: $a_{22},a_{11},a_{21}$.

Before presenting results from numerical calculations of Eq. (1), it is useful to obtain two different forms of Eq. (1) that link this problem to two well-known physical systems in the literature: the standard Rabi problem in quantum optics [23], and the Josephson-junction problem in condensed-matter physics [9].

We obtain the equations of motion for the populations $N_i$ and the coherences $N_{ij}=\int dz\,\psi_i^*(z)\psi_j(z)$ from Eq. (1) by forming the appropriate products and integrating over space to yield

$$\dot{N}_2=-i\Omega(N_{21}-N_{12}),$$
$$\dot{N}_1=+i\Omega(N_{21}-N_{12}), \tag{3}$$
$$\dot{N}_{21}=-i\,\delta\,N_{21}+i\Lambda(t)-i\Omega(N_2-N_1),$$

where we define the time-dependent term $\Lambda(t)$ as

$$\Lambda(t)=-2z_0\int dz\; z\psi_2^*(z)\psi_1(z)+\int dz(H_2^{\mathrm{MF}}-H_1^{\mathrm{MF}})\psi_2^*(z)\psi_1(z). \tag{4}$$

The equations in Eq. (3) resemble the Bloch equations describing an undamped, driven two-level atom [23]. However, because the center-of-mass motion is correlated to the internal states of the atom, the extra term $\Lambda(t)$ appears, which includes the difference in external potentials between the two states.

The first term in Eq. (4) arises from the difference in the shifted harmonic traps, which is just linear in $z$. This term acts as a time-dependent detuning. As population is transferred from one condensate to the other, the position of the overlap region changes due to the mean-field repulsion. This will cause the system to move in and out of resonance resulting in a suppression of the transfer of atoms. The second term comes from the difference in mean-field interactions and would vanish if all three scattering lengths were exactly degenerate, which can be seen from

$$H_2^{\mathrm{MF}}-H_1^{\mathrm{MF}}=(\lambda_{22}-\lambda_{21})|\psi_2|^2-(\lambda_{11}-\lambda_{21})|\psi_1|^2. \tag{5}$$

In order to make a link to the standard dc Josephson effect, we must make some approximations in order to put Eq. (1) in a simpler form. For a very weak coupling ($\Omega\ll 1$) and an initial state that is the self-consistent solution of the uncoupled system, we can make the ansatz $\psi_i(z,t)=\sqrt{N_i(t)}e^{i\phi_i(t)}\Phi_i(z)$. Here we put the explicit time dependence into the population $N_i$ and the phase $\phi_i$ of each condensate, while putting the spatial dependence into an adiabatic solution $\Phi_i(z)$ to the undriven system

$$(H_i^0+H_i^{\mathrm{MF}})\Phi_i(z)=\mu_i\Phi_i(z), \tag{6}$$

where $\mu_i$ is the chemical potential for each condensate and the solutions $\Phi_i(z)$ are taken to be real. The chemical potentials $\mu_i$ and functions $\Phi_i(z)$ vary slowly in time, being ''slaved'' by the populations.

If we substitute this ansatz into Eq. (1), we obtain the following equations of motion for the relative population $\eta=(N_2-N_1)/N$ and the relative phase $\phi=(\phi_2-\phi_1)$:

$$\dot{\eta}=-k(1-\eta^2)^{1/2}\sin\phi,$$
$$\dot{\phi}=-[(\mu_2-\mu_1)-\delta]+k\eta(1-\eta^2)^{-1/2}\cos\phi, \tag{7}$$

where $k=2\,\Omega\int dz\,\Phi_2(z)\Phi_1(z)$ is proportional to the overlap of the condensates and so also varies slowly in time. These are nonlinear versions of the usual Josephson-junction equations [9] and are nearly identical in form to those obtained in Refs. [16] and [17] describing the double-well tunneling problem. The major difference is that in the double-well trap, the condensates are well separated, allowing the authors in Refs. [16] and [17] to neglect the mean field in the interaction region of the barrier. In contrast, the interaction between condensates due to their significant overlap plays an important role in the evolution of the system described in this paper. In particular, it is this mutual interaction that causes the system to move out of resonance.

We now show some results of calculations of both the exact solution given by Eq. (1) and the approximate solution given by Eq. (7). The main purpose of the present calculations is to use realistic parameters to investigate the effect the mean field has on the evolution of the system. These parameters are listed in Table I.

In Fig. 1 we show four curves that are described in the caption. As a point of reference, we plot the solution of Eq. (1) with the mean-field terms set to zero. This is the standard Rabi solution, but here the Rabi frequency is given by $\omega_R$

TABLE I. The values used for the various physical parameters appearing in our calculations are shown. The scattering lengths are taken from Ref. [21]. Since we are treating the system in one dimension, $N$ is not the actual population, but has been chosen to produce a realistic mean-field interaction for $5\times10^5$ atoms [7].

| | | | |
|---|---|---|---|
| $N$ | $2.3\times10^4$ | $\nu_z$ | 60 Hz |
| $a_{21}$ | 5.5(3) nm | $\Omega/2\pi$ | $1/20\nu_z$ |
| $a_{22}$ | $0.97a_{21}$ | $z_{\rm sho}$ | 1.4 $\mu$m |
| $a_{11}$ | $1.03a_{21}$ | $z_0$ | $0.15z_{\rm sho}$ |

$=2\Omega\int dz\,\Phi_2(z)\Phi_1(z)$, which includes the Frank-Condon-type overlap of the condensate wave functions. However, when we turn on the mean-field interaction using the parameters given in Table I, and set $\delta=-0.39$ so that the system is initially being driven resonantly, the amplitude is suppressed and the frequency has increased (the dashed line). As $\phi(0)$ is decreased, the amplitude increases, as illustrated by the solid curve where $\phi(0)=3\pi/16$. Also, as $\phi(0)$ is decreased, the presence of higher harmonics becomes stronger, as one can see in the shape of the solid line.

We also plot the adiabatic solution given by Eq. (7) for the case $\phi(0)=3\pi/16$ (the dotted line), where Eq. (6) is solved self-consistently in each time step. In this case the adiabatic solution agrees quite well with the exact solution (solid line) given by Eq. (1). The validity of the two-level, adiabatic solution depends on the structure of the evolving spectrum of this nonlinear system. In particular, one must compare the time rate of change of the Hamiltonian to the spacing between the instantaneous eigenmodes of the dressed basis [24]. These quantities will depend on the size of the mean-field interaction, the strength of the coupling, given by $2\Omega\int dz\,\Psi_2(z)\Psi_1(z)$, and also on the detuning $\delta$. It should also be noted that, in the true system, the confining potential along the $xy$ plane is weaker than that along the $z$ axis, which should imply a more stringent criterion for adiabaticity than in the one-dimensional case considered.

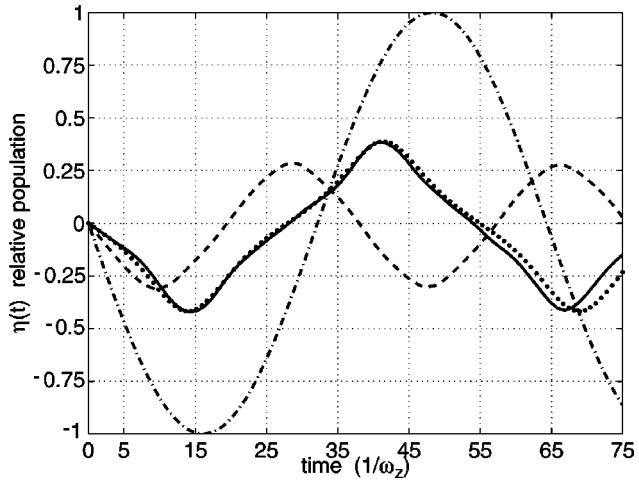


FIG. 1. Plot showing oscillations of the relative population. The dashed-dot line corresponds to the case where the mean-field interaction has been set to zero, the initial relative phase is $\phi(0)=\pi/2$, and $\delta=0$. The dashed line is for the same initial relative phase of $\pi/2$, but with the mean-field interaction turned on and $\delta=-0.39$. The solid line is for $\phi(0)=3\pi/16$ and $\delta=-0.39$. The three lines described are solutions of Eq. (1), whereas the dotted line is a solution of Eq. (7) with the same parameters as for the solid line.

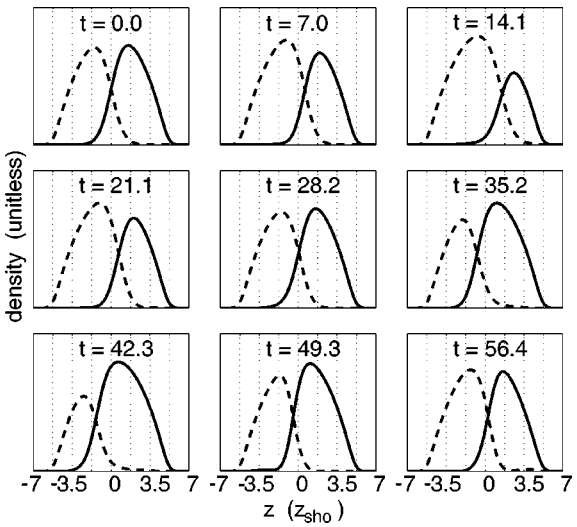


FIG. 2. Plot showing the time-evolution of the density of each component. The $x$ axis is the position $z$, and the time $t$ of each snapshot is shown. This case corresponds to the solid line plotted in Fig. 1. The detuning is $\delta=-0.39$, chosen so that the system is initially being driven on resonance.

We plot a snapshot evolution of the densities of the condensates in Fig. 2 in order to show that the effect of the mean field is to push the system out of resonance as population is transferred between condensates. The case considered in Fig. 2 corresponds to the solid line in Fig. 1. The detuning was chosen so as to compensate for the initial value of the term $\Lambda(0)$ in Eq. (3), which represents the difference in external potentials, so that initially the system is being driven on resonance. However, as the system evolves, the first term in Eq. (4) gets larger since the $|1\rangle$ condensate is pushing the $|2\rangle$ condensate away from the center of the trap. This causes the region of overlap to be displaced from the origin so that the

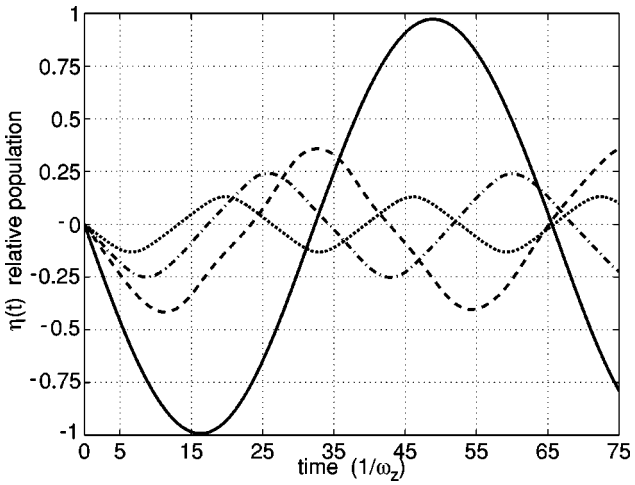


FIG. 3. Plot showing that the effect of the displacement $z_0$ on the system is to suppress the amplitude of oscillation. The four curves represent increasing values of $z_0$: 0 (solid), 0.1 (dashed), 0.2 (dash-dotted), and 0.5 (dotted). In each case, the detuning $\delta$ is chosen so that the system is initially being driven resonantly, and the initial phase is $\phi(0)=\pi/2$.

system is no longer being driven on resonance. At $t=14.1$ the region of overlap is centered at about $z=1.5$, at which time $|\Lambda(t)/N_{21}(t)|=0.63$, compared to the initial value $|\Lambda(0)/N_{21}(0)|=0.39$. This reduces the effectiveness of the drive and accounts for the suppression of the amplitude of oscillation in the relative population plotted in Fig. 1.

In Fig. 3 we vary the displacement $z_0$ for four different values. The amplitude and period of oscillation both decrease as the displacement between the traps is made larger, while the overall shape does not vary much. In other words, the effective detuning caused by $\Lambda(t)$ becomes more pronounced for larger trap displacements. There is another effect of increasing $z_0$: the region of overlap between the condensates decreases as the traps are pulled apart, which weakens the coupling. One might expect the period of oscillation to increase in this case; however, the plot displays a decreasing period as $z_0$ is increased. This indicates that the time-dependent detuning due to $\Lambda(t)$ has a stronger effect.

In this paper we have described a physical system based on the experiments reported in Refs. [20,21,7] that would exhibit nonlinear Josephson-like oscillations in the relative population between a driven two-component condensate. From our calculations we have observed the effect of the mean field, which acts to suppress the transfer of atoms between overlapping condensates in separated harmonic traps and gives rise to nonlinear oscillations in the relative population. We have also found that, for equal populations initially $\eta(0)=0$, the spectrum of Fourier components comprising the oscillation depends on the initial relative phase $\phi(0)$.

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