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Ultra-fast Stokes parameter correlations of true unpolarized thermal light: type-I unpolarized light

FLORIAN KROH, MARKUS ROSSKOPF, AND WOLFGANG ELSÄSSER*

Institute of Applied Physics, Technische Universität Darmstadt, 64289 Darmstadt, Germany
*Corresponding author: elsaesser@physik.tu-darmstadt.de

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We measure Stokes parameter correlations in analogy to the intensity correlation measurements in the original Hanbury-Brown & Twiss configuration by realizing an experimental setup by combining a Schaefer–Collett or Berry–Gabrielse–Livingston polarimeter with a Hanbury-Brown & Twiss intensity interferometer. We investigate true unpolarized light emitted from a broadband thermal light source, which we realize by an erbium-doped fiber amplifier, thus being an ideal source of true unpolarized light. We find that all Stokes parameter correlations $\langle S_n S_n \rangle$, $n \in \{1, 2, 3\}$ are equal to $0.5\langle I \rangle^2$. The proven invariance of the Stokes parameter correlations against retardation by wave-plates clearly shows for the first time, to the best of our knowledge, that our true unpolarized thermal light represents type I unpolarized light in accordance with a theoretical prediction for a classification of unpolarized light postulated more than 20 years ago.

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Besides intensity or photon number and wavelength or frequency, polarization represents one important characteristic feature of electromagnetic radiation. With the historical and pioneering work by Sir Gabriel Stokes [1] and the corresponding understanding of polarization, this topic seemed to have become a mature completed topic with a lot of subsequent research implications in science, applications, and theory. Traditionally, polarization is described and discussed in terms of Jones or Stokes formalism [1] and depicted on the Poincaré sphere [2,3], directly visualizing the polarization states in terms of linear, elliptical, and circular polarization according to their position on the Poincaré sphere. The Stokes parameter S_1 describes the difference between linear horizontal and vertical polarization intensity, the Stokes parameter S_2 describes the difference between linear $+45°$ or $-45°$ polarization, and the Stokes parameter S_3 describes the difference between right- or left-circular polarization contained within the light beam. In some sense, circular polarization represents a particular interesting case due to the fact that there are only two ideal circular states on the north and south poles, whereas there is an infinite manifold of linear polarizations. Furthermore, circular polarization demands particular attention due to its particular interdisciplinary character and occurrence in a wide field of topics ranging from sensing and communication in biology and nature, particularly in vision [4–8], but also in the context of meta-surfaces [9]. Many impressive experimental techniques were conceived to measure Stokes parameters [10–12], thus yielding a full polarization state description of light, including spatiotemporal Stokes parameters [13] but also giving direct insight into the selection rules of the radiation-generating process in the light source [14].

With the advent of recent quantum optics and technologies, e.g., in metrology and communication applications via polarization-entangled states, novel interests with a new horizon for polarization emerged. Nearly all of this work concentrated on a description of polarization in terms of Stokes vectors or Jones vectors with well-defined polarization states of linear, circular, or elliptical polarization. On the other hand, unpolarized light is, at first glance, slightly counter-intuitive even though broadly occurring in nature, e.g., sunlight, with first references mentioning "unpolarized light" going back to the 1930s and 1940s [15–17]. In the 1990s Paul and coworkers postulated [18–20] that unpolarized light should exhibit particular correlations, and in fact, there should be several types of unpolarized light depending on their invariance and symmetry properties. Recently, the statistical properties of unpolarized light have also attracted new attention, also from the quantum optics point of view [21,22] and in the context of ghost metrology modalities [23–25]. Still, unpolarized light requires experimental investigations to achieve more insight [26–29]. Very recently, new insight into the physical nature of unpolarized light has been given by Shevchenko *et al.* who developed a new point of view of understanding from both the fundamental aspects of unpolarized light [23,30,31] and also insight into particular applications of unpolarized light [32,33], e.g., in a secure communication scheme [25]. Shevchenko *et al.* considered unpolarized light as a randomly moving diffusing Stokes vector on the surface of the Poincaré sphere with a characteristic time constant of this instantaneous polarization state (IPS) [31] in the 10–100 fs regime in which the Stokes vector remains fixed.

Here, we understand unpolarized light in terms of this IPS and its dynamics on the Poincaré sphere [31,34,35]. We investigate the ultra-fast polarization correlations performing an ultra-fast Stokes parameter correlation measurement of true unpolarized amplified spontaneous emission (ASE) thermal light emitted at 1550 nm by an erbium-doped fiber amplifier (EDFA). We conceive and realize an experimental setup consisting of a Schaefer–Collett (SC) or

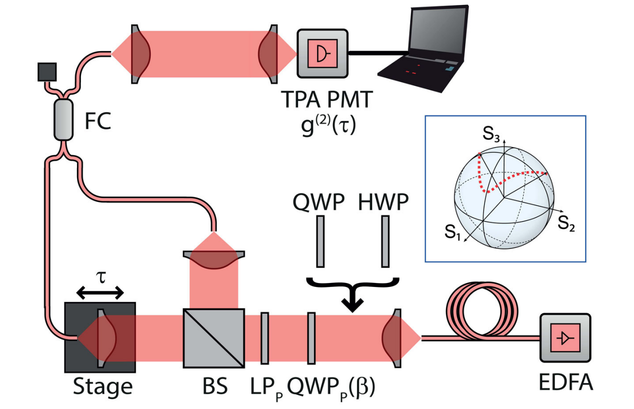

Fig. 1. Experimental setup for measurements of the Stokes parameter correlations by combining a Schaefer–Collett polarimeter (quarter-wave plate QWP_P and linear polarizer LP_P) with a HBT intensity interferometer. Beam splitter, BS; delay stage; fiber coupler, FC; two-photon absorption photomultiplier, TPA-PMT. The QWP and HWP inserted for proof of the polarization state invariance are also depicted. The inset below the notebook depicts schematically a trajectory of the Stokes vector of unpolarized light on the Poincaré sphere.

Berry–Gabrielse–Livingstone (BGL) polarimeter [10,12] subsequently followed by a Hanbury-Brown and Twiss (HBT) second-order intensity interferometer [36,37], with two-photon absorption (TPA) detection [36]. We derive a model yielding an expression for $g^{(2)}(\tau)$ now containing the Stokes parameter correlations $\langle S_n S_m \rangle$, $n, m \in \{0, 1, 2, 3\}$, where the brackets indicate a generalized ensemble average over time. We find that the unpolarized light emitted by the EDFA exhibits equal values of the three Stokes parameter correlations $\langle S_1 S_1 \rangle = \langle S_2 S_2 \rangle = \langle S_3 S_3 \rangle = 0.5\langle I \rangle^2$, with $\langle I \rangle$ being the intensity. The second-order intensity correlation coefficient amounts to 1.5, as expected for unpolarized light without selecting one specific polarization [38]. The measured Stokes parameter correlations remain unchanged when modifying the unpolarized light state by introducing a half-wave plate (HWP) or a quarter-wave plate (QWP). This invariance against retardation is the proof for type-I unpolarized light [19,20,39].

Our experimental setup as depicted in Fig. 1 is in principle a HBT Mach–Zehnder setup comparable to a ghost imaging or a ghost polarimetry setup complemented by a SC or BGL polarimeter [3,10].

The second-order intensity correlation coefficient $g^{(2)}(\tau)$ is measured by introducing a time delay (τ) in the object arm. In order to have the necessary time resolution for measuring second-order coherence of spectrally broadband sources, we exploit the ultra-fast TPA in a photomultiplier (TPA-PMT) [36]. The $g^{(2)}(\tau = 0)$ values are then determined from the fringe-resolved interferometric autocorrelation. As a light source for "true unpolarized" light, we use a standard EDFA emitting spectrally broadband ASE light with a central wavelength of 1530 nm and a spectral width (full-width half maximum) of 4 nm. This is an ideal thermal unpolarized light source [24,40] with a second-order intensity correlation coefficient $g^{(2)}(\tau = 0) = 1.5$ when measured without any polarization-selective element in the HBT setup and $g^{(2)}(\tau = 0) = 2.0$ when selecting a linear polarization state using polarizers [38].

The emitted light, collimated by a lens, passes a SC polarimeter consisting of a revolvable QWP QWP_P and a fixed linear polarizer LP_P and is detected by a photodetector connected to a computer that records the measured intensity as a function of the angle β in degrees between the vertical axis of the polarizer and the fast axis of the QWP [10,12]. The intensity of the light that impinges on the detector can be calculated by taking the influence of the QWP_P and a linear polarizer LP_P on the Stokes vector of the light into account by using the Mueller matrix formalism. This polarization state analysis for our EDFA light yields the normalized Stokes parameters $s_n = \langle S_n \rangle / \langle S_0 \rangle$, with $s_1 = (-8.0 \pm 235) \cdot 10^{-4}$, $s_2 = (-7.8 \pm 24) \cdot 10^{-3}$, and $s_3 = (-3.7 \pm 6.1) \cdot 10^{-3}$, leading to a degree of polarization $\mathrm{DOP} = \frac{\sqrt{\langle S_1 \rangle^2 + \langle S_2 \rangle^2 + \langle S_3 \rangle^2}}{\langle S_0 \rangle} = (8.6 \pm 21) \cdot 10^{-3}$.

According to the concept of unpolarized light represented as an IPS with ultra-fast dynamics developed by Shevchenko *et al.* and schematically depicted in the inset of Fig. 1, we describe the Stokes vector of unpolarized light with the following time dynamics:

$$\mathbf{S}_{\mathrm{in}}(t) = \mathbf{S}_{\mathrm{in}}(r, \theta, \phi) = \begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} = \begin{pmatrix} r(t) \\ r(t)\cos(\theta(t)) \\ r(t)\sin(\theta(t))\cos(\phi(t)) \\ r(t)\sin(\theta(t))\sin(\phi(t)) \end{pmatrix}, \quad (1)$$

where r, θ, and ϕ are time dependent, such that the Stokes vector moves randomly on the surface of the Poincaré sphere with an IPS, as illustrated by the inset in Fig. 1.

In this framework, we would now like to discuss the Stokes parameters and the Stokes parameter correlations of unpolarized light. First, we start by characterizing the polarization properties of the true thermal light emitted by the EDFA in terms of Stokes parameters and DOP by using a SC-type polarimeter consisting of a linear polarizer in combination with a revolvable quarter-wave plate QWP_P (angle β with respect to the orientation of the linear polarizer LP_P). At the output of the polarimeter, we measure the intensity $I(\beta)$ according to

$$I(\beta) = \frac{1}{2}\left[S_0 + S_1\cos^2(2\beta) + S_2\cos(2\beta)\sin(2\beta) + S_3\sin(2\beta)\right], \quad (2)$$

which can be written as

$$I(\beta) = \frac{1}{2}\left[A + B\sin(2\beta) + C\cos(4\beta) + D\sin(4\beta)\right], \quad (3)$$

where

$$A = S_0 + \frac{S_1}{2}, \quad B = S_3, \quad C = \frac{S_1}{2}, \quad D = \frac{S_2}{2}. \quad (4)$$

The Stokes parameter S_n can be obtained by a fit according to Eq. (3) to the experimental data or via Fourier analysis according to Eqs. (20) and (21) [10].

In order to measure the Stokes parameter correlations $\langle S_n S_m \rangle$, $n, m \in \{0, 1, 2, 3\}$, we inject the light with intensity $I(\beta)$ from the output of the polarimeter into a HBT interferometer, where via a Glauber protocol [41] the second-order or intensity correlation $g^{(2)}(\tau = 0)$ between the intensity in the so-called reference arm I_{ref} and object arm I_{obj} within the ghost metrology nomenclature is determined. We describe the polarization correlations by extending a Glauber ansatz for ghost imaging to polarization correlations measured by the HBT interferometer [25]:

$$g^{(2)}(\tau=0, \mathbf{S}_{\rm ref}, \mathbf{S}_{\rm obj}) = \frac{\langle \mathbf{S}_{\rm ref}\mathbf{S}_{\rm obj}\rangle}{\langle \mathbf{S}_{\rm ref}\rangle\langle \mathbf{S}_{\rm obj}\rangle} = \frac{\langle I_{\rm ref} I_{\rm obj}\rangle}{\langle I_{\rm ref}\rangle\langle I_{\rm obj}\rangle}, \tag{5}$$

$$I_{\rm ref} = I_{\rm obj} = \frac{1}{2} I_{\rm output\ polarimeter}(\beta), \tag{6}$$

where $\mathbf{S}_{\rm ref}$ and $\mathbf{S}_{\rm obj}$ are the Stokes vectors of the reference and object arms, respectively and $I_{\rm output\ polarimeter}(\beta)$ is the intensity of Eq. (2). Finally, inserting the intensities from Eq. (2) into Eq. (5) and assuming that the Stokes parameter cross-correlations $\langle S_n S_m\rangle = 0$ for $m \neq n$ [42], we obtain the following expression for the central ($\tau = 0$) second-order correlation coefficient $g^{(2)}(\tau = 0, \beta)$:

$$\begin{aligned} g^{(2)}(\tau=0,\beta) &= \frac{\frac{1}{4}[\langle S_0^2\rangle + \langle S_1^2\rangle \cos^4(2\beta)]}{\frac{1}{4}\langle S_0\rangle^2} \\ &+ \frac{\frac{1}{4}[\langle S_2^2\rangle \cos^2(2\beta)\sin^2(2\beta) + \langle S_3^2\rangle \sin^2(2\beta)]}{\frac{1}{4}\langle S_0\rangle^2} \\ &= \frac{8\langle S_0^2\rangle + 3\langle S_1^2\rangle + \langle S_2^2\rangle + 4\langle S_3^2\rangle + 4\langle S_1^2\rangle\cos(4\beta)}{8\langle S_0\rangle^2} \\ &+ \frac{-4\langle S_3^2\rangle\cos(4\beta) + \langle S_1^2\rangle\cos(8\beta) - \langle S_2^2\rangle\cos(8\beta)}{8\langle S_0\rangle^2} \\ &= \frac{1}{8\langle S_0\rangle^2}\left[A + B\cos(4\beta) + C\cos(8\beta)\right], \end{aligned} \tag{7}$$

with

$$\begin{aligned} &A = 8\langle S_0^2\rangle + 3\langle S_1^2\rangle + \langle S_2^2\rangle + 4\langle S_3^2\rangle, \\ &B = 4\langle S_1^2\rangle - 4\langle S_3^2\rangle, \quad C = \langle S_1^2\rangle - \langle S_2^2\rangle. \end{aligned} \tag{8}$$

This result shows that in analogy to the SC formula where the analysis of $I(\beta)$ behind the polarimeter yields the Stokes parameters, an analogue analysis of $g^{(2)}(\tau = 0, \beta)$, where β ($\mathrm{QWP_P}$) is the polarimeter angle, results in the Stokes parameter correlations $\langle S_n S_n\rangle$.

The experimental results for $g^{(2)}(\tau = 0)$ as a function of the polarimeter angle β ($\mathrm{QWP_P}$) are shown in Fig. 2. We find that $g^{(2)}(\tau = 0)$ remains constant at $g^{(2)}(\tau = 0) = 2.0$, independent of β for all angles of β. According to Eq. (8), this condition can be fulfilled only if the coefficients $B = C = 0$. This implies that all Stokes parameter correlations values are equal: $\langle S_1 S_1\rangle = \langle S_2 S_2\rangle = \langle S_3 S_3\rangle$. Furthermore, the second-order intensity correlation of the unpolarized thermal light without any polarization selecting element in the light beam path has been determined by the HBT interferometer to a value of $\langle S_0 S_0\rangle = 1.5\langle I\rangle^2$. This results in $A = \langle S_0^2\rangle + \langle S_n^2\rangle$, leading to the final result for the Stokes parameter correlations $\langle S_n S_{\rm n}\rangle = 0.5\langle I\rangle^2$, for $n \in \{1, 2, 3\}$. And in fact, these values of the temporal Stokes parameter correlations (Stokes moments) have been calculated by Eliyahu [42] for unpolarized thermal light originating from a Gaussian emission process

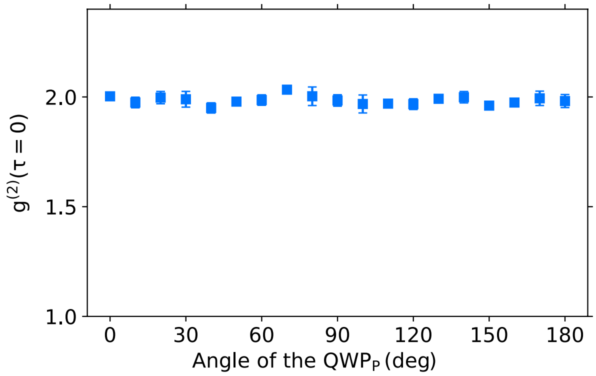

Fig. 2. Measured ultra-fast second-order correlation coefficient at the output of the combined Collett polarimeter and Hanbury-Brown & Twiss interferometer. The precision of the depicted $g^{(2)}(\tau = 0)$ measurement results is defined by the statistical error resulting from five subsequent measurements and is indicated in all experimental data sets by the error bars.

[43]. He gave results for the joint probability distribution function of the Stokes variables [44] with a universal form for the four S_n variables, leading to rather simple expressions for the second moments of the Stokes variables for unpolarized light (DOP = 0). Furthermore, these values of Stokes parameter correlations have been observed experimentally by Ellis and Dogariu for laser speckles of polarized laser light scattered by specifically designed scatterers, i.e., spatially unpolarized light [45,46], an unpolarized light source in some sense comparable to a pseudo-thermal light source [47,48]. They demonstrated the existence of various types of non-classical, globally unpolarized light, and suggested experimental means for discriminating between such field distributions. We refer here to the work of Paul and coworkers [18–20] who postulated that unpolarized light should exhibit particular correlations, leading to a classification of several types of unpolarized light depending on their invariance and symmetry properties. They also suggested methods for how to generate them and how to differentiate between them. The intuitive idea of unpolarized light can be formalized by postulating specific invariance properties under transformations. Two classes of unpolarized light have been defined, type-I and type-II [39], depending on the set of symmetries and invariances they satisfy. Type-II unpolarized light satisfies the following properties:

(1) Rotational invariance with respect to the propagation direction. These rotations can be implemented, for example, via optical activity or the Faraday effect.
(2) Symmetry with respect to left- and right-handed circular polarization. This interchange can be achieved by the action of HWPs.

Type-I unpolarized light or natural light has to fulfill a further property, in addition to criterion (1) and criterion (2):

(3) Invariance with respect to phase changes between linearly polarized components. These phase changes can be implemented by standard phase plates.

According to the classification introduced by Lehner *et al.* [18–20] for the symmetry classification of unpolarized light, the phase retardation invariance (3) has to be investigated [39]. Therefore, we now repeat our Stokes parameter correlation

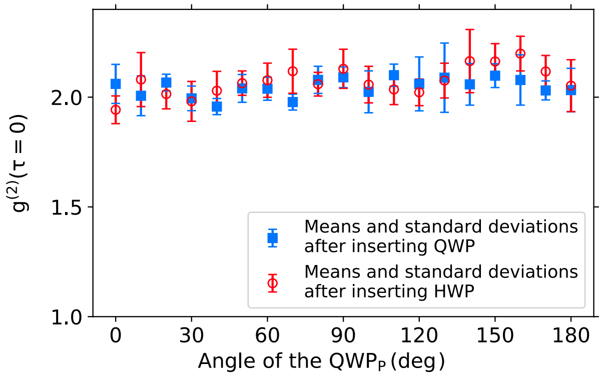

Fig. 3. Second-order intensity correlations measured when introducing a quarter-wave or a half-wave plate.

investigations by introducing a HWP or QWP in front of the polarimeter, as depicted in Fig. 1. The results are depicted in Fig. 3 for introducing a HWP or a QWP. Obviously, the second-order correlation results remain unchanged, irrespective of the rotation angles of the HWP or QWP, the same as in Fig. 2 without introduced wave plates. Therefore, the derived Stokes parameter correlations remain unchanged as well. This invariance against modifying the polarization properties of unpolarized light by wave plates is a clear signature of type I and confirms that our true thermal EDFA light is of type-I unpolarized light.

In conclusion, we have investigated the Stokes parameter correlations of true unpolarized ASE thermal light emitted by an EDFA at 1530 nm. We found that all Stokes parameter correlations $\langle S_n S_n \rangle$ for $n \in \{1, 2, 3\}$ are equal to $0.5\langle I \rangle^2$ and that the emitted light is invariant against modifications of the polarization state by a HWP or QWP, confirming in fact type I unpolarized light with Gaussian emission statistics.

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