

Generation of spatially unpolarized light by a Cornu depolarizer: equator polarization states as type II unpolarized light

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Abstract: We demonstrate the generation of spatially unpolarized light by exploiting a quartz Cornu depolarizer. Linearly polarized light impinging on the depolarizer is spatially polarization-dispersed, and the output polarization state is analyzed by a Schaefer-Collett or Berry-Gabrielse-Livingston polarimeter and a CCD camera or a photo detector, respectively. The ensemble of the observed spatially resolved normalized Stokes parameters $s_1(x, y)$, $s_2(x, y)$, $s_3(x, y)$ and the spatially resolved degree of polarization $DOP(x, y)$ together with their counterparts spatially integrated over the beam profile show that the full beam is in fact spatially unpolarized light. The light consists of a spatial superposition of the manifold of all purely linearly polarized states encompassing completely the equator of the Poincaré sphere, thus so-called equator states. The accompanying invariance under the influence of a half-wave plate suggests that this is type II unpolarized light.

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1. Introduction

Polarization is even 170 years after Sir Gabriel Stokes a fascinating topic in optics. This is stimulated by recent developments in quantum optics and quantum technologies, e.g., in metrology and communication applications where novel fields with a new horizon for polarization emerged [1–4]. These works comprise both, either a classical or a full quantum description of polarization in terms of Stokes vectors or Jones vectors with well-defined polarization states of linear, circular, or elliptical polarization [5,6].

The point-of-view of our work here, is based on traditional, classical polarization which is described and discussed in terms of Jones or Stokes formalism [7] and depicted on the Poincaré sphere (PCS) [8–10]. On the other hand, besides fully polarized light, the rather surprising, amazing and at first glance, slightly counter-intuitive topic of unpolarized or randomly polarized light, both in the spatial and temporal domain, sometimes also called "natural light" due to its broad occurrence in nature, e.g. sunlight, has attracted attention accompanied by very interesting application perspectives, as e.g. ghost polarization communication (GPC) [11], ghost polarimetry [12] and in particular for Floquet engineering [13,14]. First references mentioning "unpolarized light" go back to the 1930s and 1940s [15–17]. In the 1990s Paul *et al.* postulated [18–20] that unpolarized light should exhibit particular correlations, and in fact, there should be several types of unpolarized light depending on their invariance and symmetry properties. Specifically, type I unpolarized light and type II unpolarized light are distinguished by their invariance with respect to phase changes. This invariance property is readily checked using wave plates.

Recently, the statistical properties of unpolarized light [21,22] have attracted new attention, also from the quantum optics point of view [3,23] and in the context of ghost metrology modalities [11,24,25]. Still, unpolarized light requires experimental investigations to achieve deeper understanding [26–31]. New insight into the physical nature of unpolarized light has been given by Shevchenko *et al.* who developed a new point of view of understanding both the

fundamental aspects of unpolarized light [24,32,33] and also insight into particular applications of unpolarized light [34,35].

Very recently, we have demonstrated by a Stokes parameter correlation analysis that unpolarized light emitted by an erbium-doped fiber amplifier at 1530 nm has the properties of type I unpolarized light [36]. This is in accordance with a more than 25 years old prediction by Paul *et al.* according to which this specific type I unpolarized light should exhibit these found Stokes parameter correlations and symmetries. According to theory there should be even more types of unpolarized light with other symmetry properties and thus characteristics [19,20,37,38]. Here, in this spirit of the generation of type I (temporally) unpolarized light we proceed towards the generation of type II spatially unpolarized light.

We demonstrate that the polarization of a fully linearly polarized light beam can be manipulated by exploiting the spatially dispersing polarization properties of an optically active classical polarization component, the so-called Cornu depolarizer (named after Marie Alfred Cornu [39–41]) such that spatially unpolarized light is generated. Spatially unpolarized light means here that it is fully polarized at one specific point in space, however, on space average it occupies a variety of polarization states, such that on spatial average it represents unpolarized light. This understanding is in analogy to temporally unpolarized light which has been characterized by an instantaneous temporal polarization state (IPS) which in time randomly occupies the full surface of the Poincaré sphere (PCS) [32], however which is fully polarized at one very short time slot (approximately on the fs timescale) by a single unique point on the PCS [33]. Another prominent class of light with an internally varying polarization structure are vortex beams, in particular beams with a radially or azimuthally dependent polarization structure [42]. A comprehensive review of this type of nonuniformly (totally) polarized (NU(T)P) beams is nicely summarized by Piquero *et al.* [43] emphasizing the theoretical properties, methods of synthesis and characteristics with all the current aspects of research and perspectives, where the so-called class of full Poincaré (FPB) beams containing all possible states of polarization across the profile exhibit very interesting beam properties and their generation is possible by using an uniaxial calcite crystal [44].

Here, we exploit the polarization properties of a Cornu depolarizer [45], transforming one polarization state into a tailored continuum of polarization states, here in space [46,47], therefore also often called "pseudo-depolarizer". We perform a spatially resolved Stokes parameter analysis of linearly y -polarized light after passage through the Cornu depolarizer by combining a polarimeter and a CCD camera, yielding spatially resolved, normalized Stokes parameters

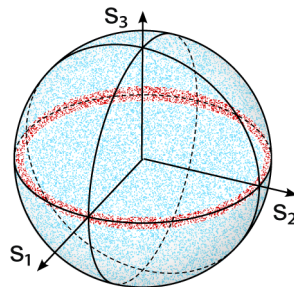


Fig. 1. Schematic depiction of the Poincaré sphere spanned up by the three Stokes vectors s_1 , s_2 , and s_3 which give the amount of horizontal/vertical (s_1), $\pm 45^\circ$ (s_2) polarization and right circular (RCP) and left circular (LCP) polarization (s_3) on the poles. The red-dotted curve visualizes the ensemble of all equator states, the type of polarization states which are here investigated whereas the ensemble of the blue dots on the complete surface of the Poincaré sphere containing all polarization states illustrates unpolarized type I light.

$s_1(x, y)$, $s_2(x, y)$, $s_3(x, y)$ and the degree of polarization $DOP(x, y)$. The results of the spatially resolved Stokes parameter analysis, the symmetry properties of s_n together with the numerically integrated spatially resolved Stokes parameters over a variable aperture demonstrate that we have experimentally realized spatially unpolarized equator light (in terms of the Poincaré sphere), i.e. light with a Stokes vector distributed randomly in space on all polarization states of the equator of the Poincaré sphere, containing only s_1 and s_2 contributions with a spatial ensemble average $\langle s_{1,2}(x, y) \rangle_{x,y} = 0$ as schematically illustrated by Fig. 1. According to the theorem by Paul *et al.* and the postulated half-wave plate (HWP) invariance this is type II unpolarized light [19].

2. Experimental setup

Our experimental setup is schematically depicted in Fig. 2. We use a linearly y -polarized HeNe laser ($\lambda = 632$ nm) as fundamental light source. A microscope objective in combination with a pinhole serves as a spatial filter. A collimated beam (diameter of 50 mm) illuminates the Cornu depolarizer (CD), a double-wedge crystal we here exploit for spatial depolarization. The CD has been manufactured by B. Halle Nachfl. GmbH (Berlin) and consists of quartz. Our crystal is schematically shown in Fig. 3 with an octagon-like geometry and the dimensions thickness (z) of 30 mm, height (y) of 50 mm and width (x) of 50 mm. It converts a fully polarized light beam into a beam with spatially nonuniform polarization distribution in space, however in the special sense that if we perform a spatial average of the Stokes parameters within a well-selected spatial domain they all compensate each other such that the spatial (random) distribution of the Stokes parameters (magnitude and direction of the polarization) leads to a cancellation and thus resulting in unpolarized light with an effective degree of polarization of zero.

Monochromatic, polarized light in form of a spatially expanded beam crossing a Cornu depolarizer results in a light beam with polarization varying in space across the diameter of the beam, thus on average it appears unpolarized. The CD consists of two crystalline quartz prisms attached such that they form an octagon. The first quartz prism (left-handed quartz) acts like a retarder plate whereby the induced phase shift is proportional to the distance over which the beam has to propagate within the prism such that at the interface the phase shift of a collimated beam is a function of the entrance height y of the beam. The second prism is arranged such that the phase shift has a negative sign compared to the first one, which is accomplished by switching the principal axes (right-handed quartz) resulting in a total phase shift which is proportional to the specific optical rotatory power of quartz. The beam emerging from the CD exhibits different polarization states at different heights (y -coordinate). However, by spatially averaging over the full beam cross section, we shall demonstrate that in fact, it is spatially unpolarized.

After passage through the CD the beam is compressed by a Galilean telescope to accommodate its beam diameter to the diameter of the polarization optics components and the imaged area of the CCD sensor, which has a surface area of roughly $8.6 \text{ mm} \times 6.9 \text{ mm}$. The subsequently

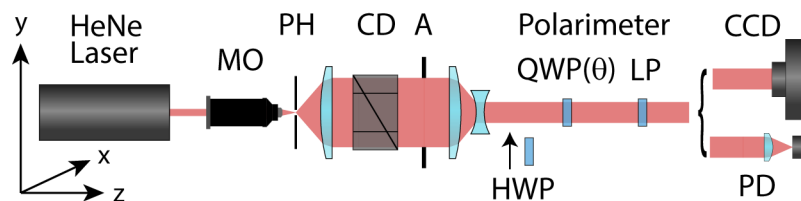


Fig. 2. Experimental setup: Helium-neon laser followed by a spatial filter consisting of a microscope objective (MO) and a pinhole (PH), Cornu depolarizer (CD), variable iris aperture (A), optional half-wave plate (HWP), rotatable quarter-wave plate (QWP, angle θ with respect to the LP), linear polarizer (LP) and CCD camera. The CCD is replaced by a spatially integrating Si photodetector (PD) for spatially integrated measurements.

collimated beam (plane wave) then passes a polarization analyzer consisting of a rotatable quarter-wave plate (QWP) and a linear polarizer (LP). Either a spatially integrating silicon photo detector or a spatially resolving CCD camera registers the transmitted beam profile's intensity distribution behind the polarimeter as a function of the QWP angle θ relative to the orientation of the LP (Schaefer-Collett or Berry-Gabrielse-Livingston polarimeter). The transmitted intensity measured behind the polarimeter and its Fourier analysis yields the four Stokes parameters s_0 , s_1 , s_2 and s_3 , from which finally the degree of polarization is calculated, such that all these characteristic polarization parameters can be obtained via both detector schemes either spatially integrated or spatially resolved [48].

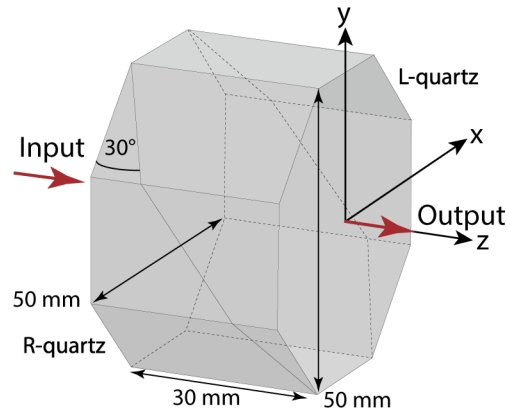


Fig. 3. Schematic depiction of our Cornu depolarizer consisting of two antiparallel oriented crystalline left-handed and right-handed quartz wedges with an outer octagonal geometry.

3. Experimental results and discussion

At first, we verify the imaging quality of our setup without the CD. Then, we insert the CD, and we determine the spatially resolved Stokes parameters of the transmitted light by using the polarimeter together with the CCD camera as spatially resolving detector as presented in Fig. 2, yielding spatially resolved Stokes parameters. Figure 4 shows the three spatially resolved Stokes parameters $s_1(x, y)$, $s_2(x, y)$ and $s_3(x, y)$ with the corresponding color-coding for positive and negative values, together with the $DOP(x, y) = \sqrt{s_1(x, y)^2 + s_2(x, y)^2 + s_3(x, y)^2}$ calculated from these s_n . For all four spatially resolved polarization distributions, a small degree of spatial barrel distortion is visible toward the edges of the beam profiles, which is due to a non-ideal positioning of the collimating lenses behind the CD. The nonuniform distribution of $DOP(x, y)$ is due to errors resulting from slight off-axis misalignment of the CD with respect to the beam's propagation axis.

Here, $s_1(x, y)$ and $s_2(x, y)$ reflect the structure and the spatially birefringent properties of the Cornu depolarizer due to the introduced spatially dependent phase shifts [46] when globally illuminating with linearly y -polarized light. These spatially dependent phase shift are due to the spatially dependent optical properties of the Cornu depolarizer (as explained in detail), its composition of right and left-handed material of varying thickness yielding finally in the polarization-dispersing character, which we exploit for the generation of the characteristic spatially-dependent polarization patterns behind the Cornu depolarizer. Here, s_1 has a mirror symmetry with respect to the x -axis (the abscissa ($y = 0$) lies on a Stokes parameter s_1 value of -1) whereas s_2 exhibits rotational symmetry with respect to the z -axis (the abscissa ($y = 0$) lies on a Stokes parameter value s_2 of 0 (white)), such that s_1 and s_2 can be considered as complementary or with a shift in space by half a period against each other. While s_1 and s_2 have a full modulation

between $+1$ and -1 , the Stokes parameter s_3 is consistently nearly equal to zero, independent of x and y (within the margin of error and optical beam deficiencies). The spatially resolved $\text{DOP}(x, y)$ reflects these dependencies and symmetries being mainly due to the complementarity and the large values of s_1 and s_2 . The alternating and complementary behavior in the sign of each of the two spatial Stokes parameters $s_1(x, y)$ and $s_2(x, y)$, their symmetry together with the negligible amount of $s_3(x, y)$ will now be further exploited towards the successful realization of spatially unpolarized light by spatial superposition of complementarily linearly polarized light.

First, we perform this integration numerically with the multi-pixel data of the CCD, i.e. the results depicted in Fig. 4. Secondly, we perform it via optical integration. An aperture A of variable diameter d is placed in the center ($x = 0$ and $y = 0$) of the beam behind the CD, the spatially selected beam diameter is focused onto the (single-pixel) photo detector behind the polarimeter and the spatially integrated Stokes parameters are measured as a function of d . This spatial ensemble average of the Stokes parameters $s_n(x, y)$ we denote by $\langle s_n(x, y) \rangle_{x,y}$. The results are shown in Fig. 5 for both methods. The results nicely coincide for both methods. The symmetries of the sign of $s_1(x, y)$ and $s_2(x, y)$ are obvious. For small aperture diameters (less than 10 mm) $\langle s_1(x, y) \rangle_{x,y}$ displays a clear large residual polarization being negative, also reflected in the $\langle \text{DOP}(x, y) \rangle_{x,y}$, whereas the rotational symmetry of $s_2(x, y)$ already results in full cancellation ($\langle s_2(x, y) \rangle_{x,y} = 0$) and $\langle s_3(x, y) \rangle_{x,y}$ is negligibly small. Already for integrating diameters larger than 14 mm all spatially ensemble-averaged Stokes parameters $\langle s_n(x, y) \rangle_{x,y}$ and also the spatially averaged $\langle \text{DOP}(x, y) \rangle_{x,y}$, are equal to zero (within the error bars). In fact, we have also applied the Mueller matrix formalism using the appropriate matrices for the polarization optics and the matrix of the Cornu depolarizer [46] to calculate the patterns shown in Fig. 4 and Fig. 5 for y -polarized input. These simulated results are in close agreement with the measurements.

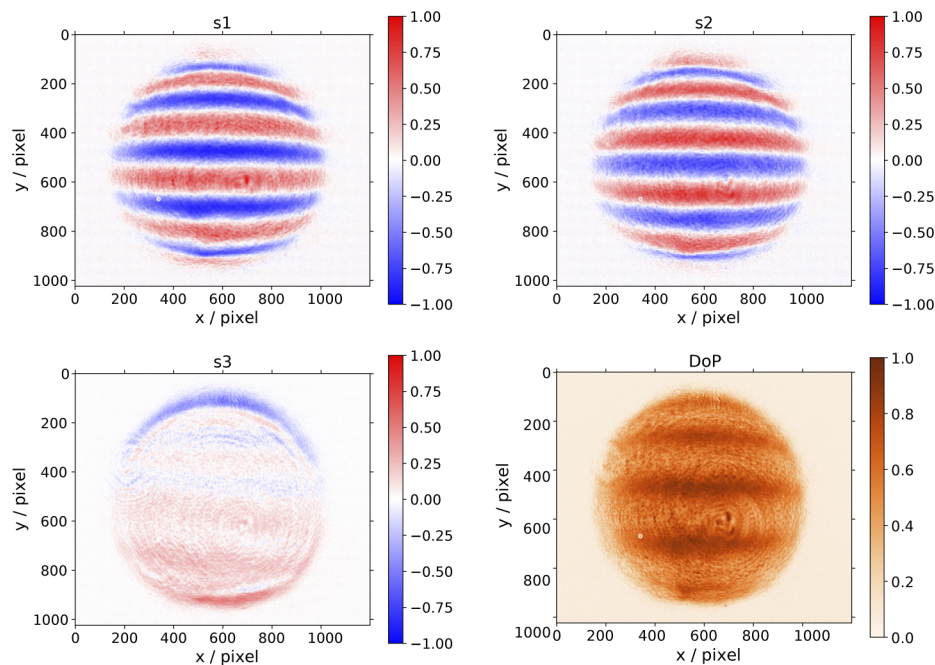


Fig. 4. CCD images illustrating the spatial distribution of the normalized Stokes parameters $s_1(x, y)$ (top left), $s_2(x, y)$ (top right), $s_3(x, y)$ (bottom left) and $\text{DOP}(x, y)$ (bottom right). Red colors indicate positive values, blue colors indicate negative values. A small degree of barrel distortion is visible towards the edges of the profiles.

The introduction of a HWP – only – behind the CD keeps all Stokes parameter results invariant, which is the confirmation for type II unpolarized light [19,37], now in the spatial domain. These results clearly confirm our understanding of spatially unpolarized light: At a specific coordinate in space we have fully polarized light, whereas averaging over the beam profile in space results in the superposition, – the ensemble average –, of all linearly polarized light states such that finally spatially unpolarized light is realized with all its Stokes parameters randomly on the equator, i.e. in the plane spanned up by s_1 and s_2 (c.f. Fig. 1), thus unpolarized equator light.

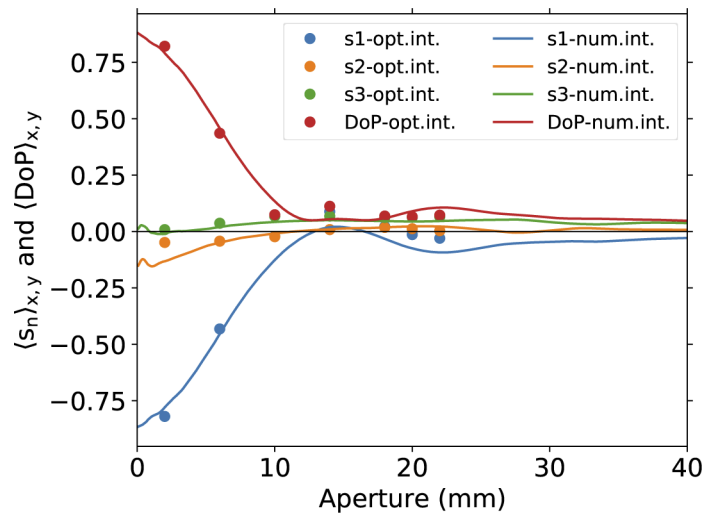


Fig. 5. Stokes parameters and DOP behind the Cornu depolarizer optically integrated (full dots) over a circular beam aperture with variable diameter d and numerically integrated via the CCD pixel data (full lines).

4. Conclusion

We have generated spatially unpolarized light by using a Cornu depolarizer and linearly polarized light. The analysis of the measurements of the spatially resolved Stokes parameters s_1 and s_2 in terms of their spatial symmetries with a global zero-value for s_3 and the zero-value for the spatial ensemble average of s_1 and s_2 over the full beam cross-section confirmed that we have generated equator polarization states as type II unpolarized light. These results complement the model of classical polarization leading to a better understanding but also to novel applications [13,49], e.g. in imaging.

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Disclosures. The authors declare no conflicts of interest.

Data availability. Data underlying the results presented in this paper are not publicly available at this time but may be obtained from the authors upon reasonable request.

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