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Three level atom optics in dipole traps and waveguides

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We dedicate this paper to Bruce Shore on the occasion of his 70th birthday.

Abstract

An analogy is explored between a setup of three atomic traps coupled via tunneling and an internal atomic three-level system interacting with two laser fields. Within this scenario we describe a STIRAP like process which allows to move an atom between the ground states of two trapping potentials and analyze its robustness. This analogy is extended to other robust and coherent transport schemes and to systems of more than a single atom. Finally it is applied to manipulate external degrees of freedom of atomic wave packets propagating in waveguides.

1. Introduction

Exploring the wave nature of massive particles has become possible through the enormous experimental advances in the cooling of neutral atoms, ions, and molecules to temperatures where the de Broglie wavelength becomes comparable to or larger than optical wavelengths. These achievements have stimulated great interest into the field of quantum *atom* optics as an analogue of quantum optics with light [1,2]. A major objective within this field is to develop elements for the manipulation of the spatial wavefunction of atoms or atomic ensembles, as beam splitters, mirrors, lenses, etc. Applications are broad, ranging from a fundamental interest in probing the wave nature of particles to the manipulation of neutral atoms for implementing quantum gates and to the construction of atom interferometers for precision measurements of physical constants or as inertial sensors. In all these cases, a crucial requirement is – as in quantum optics – to preserve the coherence of the matter wave.

Of special interest to atom interferometers as well as to quantum information processing are concepts to trap, manipulate, or guide atomic matter waves. For trapped atoms, the interaction with external fields can be precisely controlled, spreading of the wave packet can be inhibited in some or all spatial dimensions, and the effect of gravity can be compensated. Trapping and guiding of neutral atoms is usually based either on the interaction of the atom's permanent magnetic dipole moment with magnetic fields [3,4] or on the coupling of laser fields to the atom's induced dipole moment [5]. Arrangements of current-carrying wires [3,4,6], standing light-waves [7,8], and optical lattices and superlattices [9–11], microsized dipole traps [12], time-dependent holograms [13], or appropriately shaped microlenses [14,15] allow to design and control a variety of potential shapes. Examples are Y-shaped guiding geometries to split a wave packet [16–20], cold atoms storage rings from guides forming a closed loop [21,22], or traps whose separation can be controlled in time [15,23–25]. For a Bose–Einstein condensate trapped in a double-well

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potential, interactions play a crucial role leading to anharmonic oscillations and self-trapping, as demonstrated experimentally [24]. On the other hand, a single atom initially located in one of two traps oscillates in a Rabi-type fashion between the two potentials in the presence of tunneling. This is in close resemblance to a two-level atom interacting with a laser field, but in contrast the 'Rabi-frequency' is controlled via tuning the tunneling. Such a process, if implemented correctly, is coherent as it does not introduce uncontrollable phases, and it indeed allows for a simple realization of quantum bits and quantum gates [26]. This technique however is not robust under variations of the system parameters and thus requires precise temporal control of the potentials. The same problem is of course present in optical two-level system, where a variety of robust techniques have been developed which are based on controlling couplings in multi-level systems, and which are nowadays standard techniques in experiments.

Here we will provide a theoretical analysis of atom optical analogues to three-level techniques, especially discussing a process reminiscent of stimulated Raman adiabatic passage (STIRAP, [27]) to coherently move atoms between traps. Other types of processes, as, e.g., coherent population trapping (CPT, [28]) to create spatial superpositions of atomic wavefunctions are discussed in [29]. We will furthermore provide simulations showing that this technique is also applicable to wave packets propagating in guiding structures.

2. Time-dependent trapping potentials

To obtain an analogy between external degrees of freedom of an atom in a system of traps coupled via tunneling and an electronic three-level system coupled via the electric dipole-interaction with two laser fields, consider a linear arrangement of three atom traps. We assume strong confinement in the orthogonal directions, such that the dynamics can be restricted to the one-dimensional (1D) Hamiltonian

$$\hat{\mathscr{H}}_{\text{free}} = \int \mathrm{d}x \hat{\psi}^{\dagger}(x)\left[\frac{p_x^2}{2m} + V(x,t)\right]\hat{\psi}(x) \equiv \int \mathrm{d}x \hat{\psi}^{\dagger}(x) H_{\text{free}} \hat{\psi}(x), \tag{1}$$

where $V(x,t)$ describes the potential consisting of three traps with tunable distances. At each time t, H_{free} can be diagonalized to obtain the instantaneous eigenstates. For large distance between the traps, these are localized at the centers of the traps. In the general case, states $\phi_{\text{L}}(x,t)$, $\phi_{\text{M}}(x,t)$, and $\phi_{\text{R}}(x,t)$, localized around the centers of the left, middle, and right trap, respectively, can be constructed from suitable combinations of the eigenstates with lowest energy. Restricting to these states, we can expand $\hat{\psi}(x) = \sum_{\alpha=\text{L,M,R}} \hat{b}_{\alpha}\phi_{\alpha}(x)$ to obtain [30]

$$\hat{\mathscr{H}}_{\text{free}} = -J_{\text{LM}}(t)\hat{b}_{\text{L}}^{\dagger}\hat{b}_{\text{M}} - J_{\text{MR}}(t)\hat{b}_{\text{M}}^{\dagger}\hat{b}_{\text{R}} - \frac{1}{2}\sum_{\alpha=\text{L,M,R}} \mu_{\alpha}(t)\hat{b}_{\alpha}^{\dagger}\hat{b}_{\alpha} + \text{c.c.} \tag{2}$$

Here $J_{\alpha\beta}(t) = -\int \mathrm{d}x \phi_{\alpha}^{*}(x,t) H_{\text{free}}(x,t)\phi_{\beta}(x,t)$ describes nearest-neighbor tunneling and $\mu_{\alpha}(t) = -\int \mathrm{d}x \phi_{\alpha}^{*}(x,t) H_{\text{free}}(x,t)\phi_{\alpha}(x,t)$ are the on-site energies. Interactions of next-nearest neighbors have been neglected. Considering just a single atom and shifting the ground state energy, we arrive at the following Hamiltonian, isomorphic to the Hamiltonian of a three-level system coupled by two laser fields in the rotating wave approximation [27,31]:

$$H = -J_{\text{LM}}(t)(|\phi_{\text{L}}\rangle\langle\phi_{\text{M}}| + |\phi_{\text{M}}\rangle\langle\phi_{\text{L}}|) - J_{\text{MR}}(t)(|\phi_{\text{M}}\rangle\langle\phi_{\text{R}}| + |\phi_{\text{R}}\rangle\langle\phi_{\text{M}}|) - (\mu_{\text{M}}(t) - \mu_{\text{L}}(t))|\phi_{\text{M}}\rangle\langle\phi_{\text{M}}| - (\mu_{\text{R}}(t) - \mu_{\text{L}}(t))|\phi_{\text{R}}\rangle\langle\phi_{\text{R}}|. \tag{3}$$

The couplings $-J_{\text{LM}}$ and $-J_{\text{MR}}$ correspond to the Rabi frequencies of the pump and the Stokes laser. $\mu_{\text{M}} - \mu_{\text{L}}$ and $\mu_{\text{R}} - \mu_{\text{L}}$ correspond to the detuning from the single- and two-photon transition, respectively (see Fig. 1).

Optical three-level systems have been extensively analyzed. Exploiting the different possible configurations of detunings and variations of the Rabi frequencies gives rise to a large number of coherent manipulation schemes of the underlying three-level system, among them stimulated Raman adiabatic passage (STIRAP, [27]), coherent population trapping (CPT, [28]), and electromagnetically induced transparency (EIT, [32,33]). The analogy to the system of three coupled traps, as demonstrated by the Hamiltonian equation (3), suggests to explore the application of these effects to coherently manipulate *external* degrees of freedom of a trapped neutral atom, given the ability to control the corresponding trap parameters. Such a control should be possible in various trapping configurations such as magnetic [3,4,6,25] and optical [12–15] microtraps as well as in optical lattices by exploiting superlattice techniques [10].

In the following we will especially refer to neutral atoms trapped in arrays of optical microtraps created by illuminating a set of microlenses with a red detuned laser beam [15], such that in each of the foci neutral atoms can be stored by the dipole force in a gaussian shaped trapping potential. By illuminating the microlenses using independent laser beams under different angles, it is possible to generate various sets of traps whose distance can be tuned by changing the angle between lasers [15]. As several potentials are superimposed, potential depths and trap

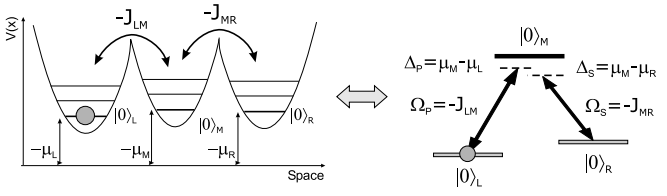

Fig. 1. Illustration of the analogy between a system of three coupled trapping potentials arranged linearly and an atomic three-level system in Λ-configuration. The tunneling matrix elements J correspond to the optical Rabi frequencies, the detunings are given by the difference between on-site energies.

frequencies change. For this reason it is difficult to control the coherent and adiabatic evolution of a trapped atom, and it is usually necessary to vary the potential depth in an appropriate way as the traps are approached, e.g., through controlling the laser intensity or by adding a blue detuned laser to produce a compensating extra potential [34]. Furthermore, optimal control techniques to suppress non-adiabatic excitations could be employed [26,35]. To keep the description general and more clear, here we will however assume a simplified trapping potential $V(x) = \frac{1}{2}m\omega_x^2 \times \min\{[x + a_L(t)]^2, x^2, [x - a_R(t)]^2\}$, where $a_L(t)$ and $a_R(t)$ fix the centers of the traps. Throughout the paper distances and times are measured in units of the harmonic oscillator length $\alpha = \sqrt{\hbar/(m\omega_x)}$ and of the inverse trap frequency ω_x^{-1}, respectively. We will later comment on a more realistic type of potentials. Hamiltonian (3) neglects contributions from non-adiabatic couplings to excited vibrational states as well as direct couplings from the left to the right trap. In what follows we will take into account the full Hamiltonian (1) through a numerical integration of the 1D Schrödinger equation to simulate the dynamics of a neutral atom in the three-trap potential.

2.1. STIRAP – robust shifting of atoms between traps

For zero detunings, one of the eigenstates of Hamiltonian (3), the *dark state*, only involves the states localized in the left and the right trap: $|D(\Theta)\rangle = \cos\Theta|L\rangle - \sin\Theta|R\rangle$. Here Θ is the mixing angle which depends on the couplings through $\cos\Theta = J_{LM}/J_{MR}$. To implement a robust method to move an atom from the leftmost to the rightmost trap using tunneling, the counter-intuitive STIRAP sequence can be applied: first the right and middle traps are approached and separated, and, with an appropriate delay time t_{Delay}, the same sequence is used for the left and middle trap [Fig. 2(a)]. This changes the mixing angle from $\Theta = 0$ to $\Theta = \pi/2$ [Fig. 2(b)], and if the atom initially is located in the left trap and the process is adiabatic, then the state is at all times identical to the dark state. This moves the atom directly from $|L\rangle$ to $|R\rangle$ [Fig. 2(c)].

The advantage of such a STIRAP-like process, as compared to a direct transport via Rabi-type oscillations, is its robustness with respect to the variation of certain experimental parameters. As shown in Fig. 3(a), the scheme works for a large range of delay times t_{Delay} and minimum distances d_{min}. A similar robustness is found for variations of, e.g., the duration t_r of the approaching/separation process and the time t_i for which the traps are kept at constant distance, the only requirements being the adiabaticity of the process and the order of approaching and separating the traps. In an experimental realization, certainly a shaking of the centers of the trapping potentials provides an important source of decoherence. It might be caused by a mechanical vibration of the microlenses, or by changes in the laser phases for optical lattices. Here we anticipate a periodic variation of the distance of the traps with frequency well below the trapping frequency, namely $\omega_{Shake} = 10^{-2}\omega_x$. As Fig. 3(b) shows, the transport efficiency is not significantly degraded even for shaking amplitudes on the order of a few percent of the minimal trap distance if the delay time is appropriately chosen.

A parameter difficult to control experimentally is the exact horizontal alignment of the traps. If the potential is tilted, gravity changes the relative depth of the potential minima. In this case, as has been reported in Bose–Einstein condensation in a double-trap potential [36], after a sufficiently adiabatic evolution, the atom(s) will eventually be found in the trap with lower energy. To allow for a transport to the desired state, the evolution should be explicitly non-adiabatic [37]. The STIRAP-like transport is within a large range not affected by gravity, i.e., by a potential $\Delta V_{tilt}(x) = \gamma\hbar\omega_x\alpha^{-1}x$. Here γ determines the slope of the ramp, and we will use $\gamma > 0$, such that the right trap is shifted up in energy with respect to the left one. For the parameters of our simulations, a value of $\gamma = 10^{-2}$ corresponds to a difference of the potential energies of $3 \times 10^{-2}\hbar\omega_x$ between the outer traps at the minimal distance. For $\gamma \ll 1$ such a tilt affects only the on-site energies $-\mu_{\alpha\beta}$ in the Hamiltonian (3). In the picture of an optical Λ-system this corresponds to a shift from the one- as well

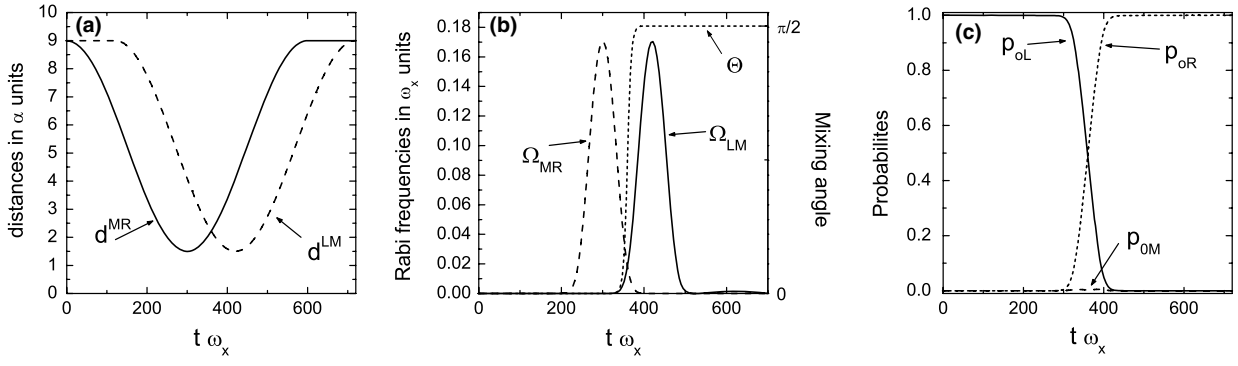

Fig. 2. (a) Approaching sequence for a STIRAP-like process, (b) the evolution of the tunneling matrix elements $-J_{LM}(t)$ and $-J_{MR}(t)$, calculated from the tunneling splitting energy of two traps, and the mixing angle $\Theta(t) = \arctan(J_{LM}(t)/J_{MR}(t))$; and (c) the corresponding ground state populations; The parameters are $d_{max}^{LM} = d_{max}^{MR} = 9\alpha$ (maximal distances), $d_{min}^{LM} = d_{min}^{MR} = 1.5\alpha$ (minimal distances), $t_r^{LM} = t_r^{MR} = 300\omega_x^{-1}$ (time used to approach/separate the traps), $t_i^{LM} = t_i^{MR} = 0$ (time at which the traps are at the minimal distance), and $t_{Delay} = 120\omega_x^{-1}$ (delay between the approaching processes).

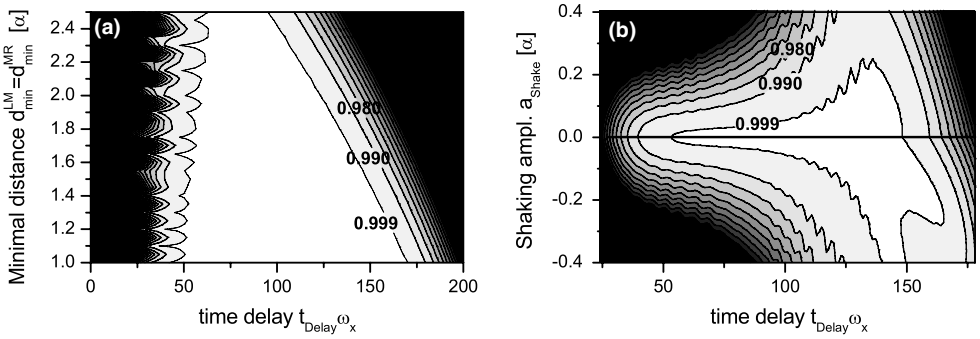

Fig. 3. Robustness of the atom optics version of STIRAP, i.e., the transfer efficiency from $|L\rangle$ and $|R\rangle$, measured by the population $\rho_R = |\langle R|\psi(t_{final})\rangle|^2$. All parameters not varied are as in Fig. 2. In (a) the delay time t_{Delay} between the two approaches (horizontal axis) and the minimal distances between traps (vertical axis) are modified. (b) shows the transfer efficiency as a function of t_{Delay} (horizontal axis) and of the amplitude a_{Shake} of a shaking in the positions of the outer traps (vertical axis) with $\omega_{Shake} = 10^{-2}\omega_x$. For $a_{Shake} > 0$ (<0) the shaking of the outer traps is in phase (out of phase by π).

as from the two-photon resonance. In this case there exist no adiabatic path from $|L\rangle$ to $|R\rangle$ [38]. This is exemplified in Fig. 4(a), which shows the energies of the eigenstates of the Hamiltonian (3) for a STIRAP sequence with $\gamma = 2\times10^{-2}$. To obtain transport from the left to the right trap, the process has to be designed as a combination of diabatic (circles in the graph) and adiabatic (arrows) processes. However, the conditions to obtain a diabatic crossing at the points indicated by the circles are usually fulfilled, such that the transfer efficiency is dominated by the adiabaticity requirement. Thus for a given γ the fidelity improves as the traps are moved slower, see Fig. 2. As should be stressed again, this is in contrast to the Rabi-type transport between two traps, where a faster process has larger fidelity of the atom ending up in the initially empty trap. In Fig. 4(c) the two schemes are compared.

To demonstrate the applicability to experimentally achievable setups of dipole traps generated from microlenses or using holographic techniques, we assume gaussian potentials whose distance and depth can be changed in time: $V(t) = V_L\exp\{-[x + d_L(t)]^2/2w^2\} + V_M(t)\exp\{-x^2/2w^2\} + V_R\exp\{-[x - d_R(t)]^2/2w^2\}$. As the traps are approached, the depth $V_M(t)$ of the center trap is adapted to facilitate an adiabatic process. Fig. 5(a) and (b) shows the time dependence of the trap parameters, (c) shows that also in this case the process is robust as long as adiabaticity is maintained. In this case fidelities >0.99 can be obtained. For a trap frequency of $\omega_x = 2\pi\times100$ kHz, a corresponding process takes around $T = 5$ ms. Applying optimal control techniques [39] to suppress non-adiabatic processes should allow to reduce this duration.

From the isomorphism between the Hamiltonians, it is obvious that also other processes from three-level optics can be exploited in the context of atomic wavepackets. As an example, consider a process reminiscent of coherent population trapping (CPT, [28]), for which the parameters are chosen such that after the process $\Theta = \pi/4$, corresponding to a symmetric, i.e., simultaneous, separation of the traps. As $|D(\pi/4)\rangle = (|L\rangle - |R\rangle)/\sqrt{2}$, this generates a spatial superposition state with maximum atomic coherence, which, e.g., could be useful for interferometry. Such a process is discussed in more detail in [29]; we will come back to a CPT-like process in Section 3 in the context of the manipulation of matter waves in guiding structures.

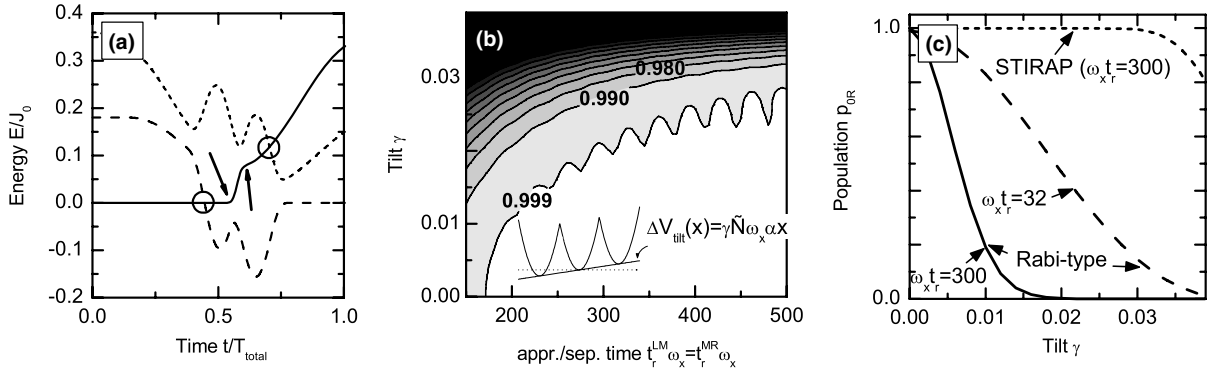

Fig. 4. Transfer efficiency with an additional potential $\Delta V_{tilt}(x) = \gamma\hbar\omega_x\alpha^{-1}x$. (a) Temporal variation of the energy levels obtained from a diagonalization of Eq. (3) for parameters of Fig. 2 and $\gamma = 0.02$. Circles indicate diabatic crossings, arrows show points with larger probability of non-adiabatic transitions. (b) Transfer efficiency as a function of the time t_r needed to approach/separate the traps and of γ; (c) The dependence of the transfer efficiency on the tilt of the potentials is compared for STIRAP for the parameters from (b) with $t_r = 300\omega_x^{-1}$ and for Rabi-type transfer between two traps for $t_r = 300\omega_x^{-1}$ (the time t_i for which the traps are kept at the minimal distance is chosen such that full population transfer occurs for $\gamma = 0$: $t_i = 12\omega_x^{-1}$) and for $t_r = 32\omega_x^{-1}$ ($t_i = 25\omega_x^{-1}$), i.e., for approaching the traps ten times faster (see text for details).

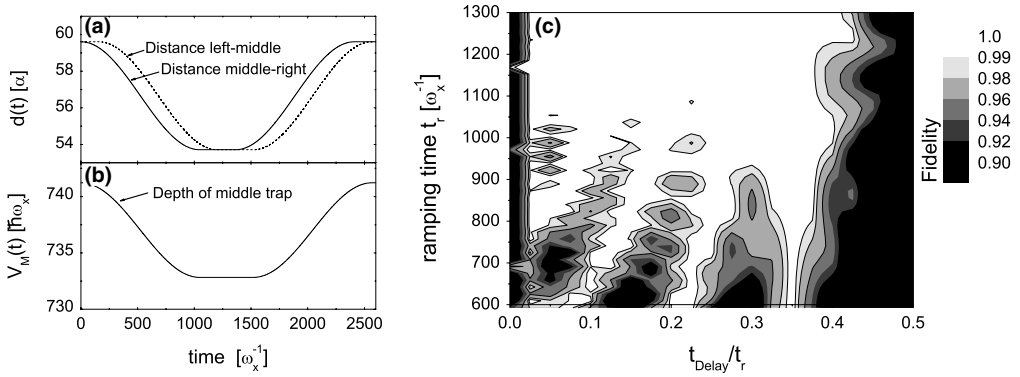

Fig. 5. STIRAP procedure for gaussian-shaped traps. Variation of the (a) trap distances and (b) the depth $V_{\mathrm{M}}(t)$ of the center trap. (c) Fidelity, i.e., final population of the right trap, as the approaching time t_r and the delay time t_{Delay} are varied. Here $V_{\mathrm{L,R}} = 828.25\hbar\omega_x$, $w = 32.75\alpha$, $V_{\mathrm{M}}(t)$ and distances $d_{\mathrm{L,R}}(t)$ as in (a) and (b), respectively, with t_r and t_{Delay} changed accordingly. The time at which the traps are at the minimal distance is $t_i = 0.3t_r$. For ^{87}Rb atoms and a trap frequency of $\omega_x = 2\pi \times 100$ kHz, $\alpha = 24$ nm, $\omega_x^{-1} = 1.6$ μs, and $\hbar\omega_x = 10\ \mu\mathrm{K} \times k_{\mathrm{B}}$.

2.2. Effects of atom–atom interaction

In the Hamiltonian (3) the restriction to a single particle can be released, and interaction can be explicitly added. For sufficiently low temperatures, interaction between bosonic atoms is dominated by *s*-wave scattering, and restricting again to states $\phi_\alpha(x)$, the Hamiltonian describing the system is modified as follows [40,41]:

$$\hat{\mathscr{H}} = \hat{\mathscr{H}}_{\mathrm{free}} + \frac{1}{2}\sum_{\alpha=\mathrm{L,M,R}} U_\alpha \hat{b}_\alpha^\dagger \hat{b}_\alpha^\dagger \hat{b}_\alpha \hat{b}_\alpha. \tag{4}$$

Here $U_\alpha = 4\pi\hbar\tilde{a}_{\mathrm{sc}} \int \mathrm{d}x |\phi_\alpha(x)|^4$, where $\tilde{a}_{\mathrm{sc}}$ is the 1D scattering length which can be changed via changing the orthogonal confinement or exploiting a Feshbach resonance. For small atom numbers, it could be possible to have $|U_\alpha|$ sufficiently large to separate sectors in energy space with different particle number within one trap. Then, if initially all atoms are confined in the left trap, several atoms can be moved at once using a STIRAP sequence. This scenario also would allow to generate an equal superposition of all atoms being in the left or all atoms being in the right trap, thus creating a 'Schrödinger cat'-like state. To this aim, the traps have to be separated symmetrically as in the CPT sequence described at the end of Section 2.1. On the other hand, starting from a system of three traps and two bosonic atoms initially in different traps allows to coherently and robustly transport the 'hole', i.e., the empty site. An optical analogue of such a system has been studied in [42], where coherent population trapping has been analyzed for two electrons with aligned spins in a three-level system. Also in this case a dark state exists which can be interpreted as the dark state of a 'hole'. A similar effect can be achieved in the atom optical system. As an example, Fig. 6 demonstrates the corresponding STIRAP process which moves the hole between the outer traps.

3. Manipulation of matter waves in guiding structures

In the previous part we have, in close analogy to the three-level processes for internal atomic states, manipulated the external wavefunction of a trapped atom by a temporal variation of the coupling between traps. We will demonstrate that similar methods allow to manipulate an atomic wave packet propagating in an appropriately designed *fixed* guiding structure. We will assume a system of three waveguides oriented in the *y*-direction, with *y*-dependent distances (see Fig. 7(a) for an example), and a corresponding Hamiltonian $H_{\mathrm{free}} = (p_x^2 + p_y^2)/2m + V(x,y)$. Now, instead of considering the eigenstates of the 1D potential for each fixed time *t*, we compute eigenstates at each position *y*, and as before combine the lowest energy states to localized

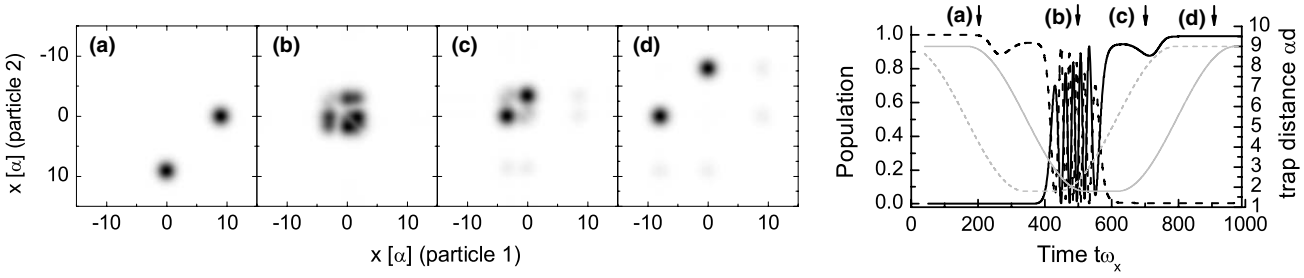

Fig. 6. Robust and coherent transport of a hole from the left trap to the right trap in a system of three traps filled with two bosonic ^{87}Rb atoms with scattering length $a_t = 106a_0$. Shown are plots of the two particle probabilities $|\psi(x_1,x_2)|^2$ at four different times (a–d) as indicated by the arrows in (e). The initial state is $|\psi(t_{\mathrm{init}})\rangle = |0_{\mathrm{M}}\rangle|0_{\mathrm{R}}\rangle + |0_{\mathrm{R}}\rangle|0_{\mathrm{M}}\rangle$, the parameters are $t_r^{\mathrm{LM}} = t_r^{\mathrm{MR}} = 350\omega_x^{-1}$, $t_i^{\mathrm{LM}} = t_i^{\mathrm{MR}} = 100\omega_x^{-1}$, $t_{\mathrm{Delay}} = 180\omega_x^{-1}$, $d_{\mathrm{max}}^{\mathrm{LM}} = d_{\mathrm{max}}^{\mathrm{MR}} = 9\alpha$, and $d_{\mathrm{min}}^{\mathrm{LM}} = d_{\mathrm{min}}^{\mathrm{MR}} = 1.5\alpha$.

wavefunctions $\phi_\alpha(x,y)$, $\alpha \in \{\mathrm{L},\mathrm{M},\mathrm{R}\}$. The full wavefunction can then be decomposed as $\psi(x,y,t) = \sum_\alpha c_\alpha(y,t)\,\phi_\alpha(x,y)$. Inserting this expression into the Schrödinger equation gives the following equation for the evolution of the coefficients $c_\alpha(y,t)$ [20]:

$$i\hbar\frac{\partial c_\alpha}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 c_\alpha}{\partial y^2} + \sum_{\beta=\mathrm{L},\mathrm{M},\mathrm{R}}\left(H_{\alpha\beta} + \frac{\hbar^2}{m}P_{\alpha\beta}\right)c_\beta + \frac{\hbar^2}{2m}\sum_{\beta=\mathrm{L},\mathrm{M},\mathrm{R}} K_{\alpha\beta}\frac{\partial c_\beta}{\partial y}. \quad (5)$$

Here $H_{\alpha\beta}(y) = \int \mathrm{d}x\phi^*_\alpha(x,y)(p_x^2/2m + V(x,y))\phi_\beta(x,y)$ are the Hamiltonian matrix elements for fixed y. $K_{\alpha\beta}(y) = -\int \mathrm{d}x\phi^*_\alpha(x,y)\partial_y\phi_\beta(x,y)$ and $P_{\alpha\beta}(y) = -\int \mathrm{d}x\phi^*_\alpha(x,y)\partial_y^2\phi_\beta(x,y)$ are kinetic and potential couplings, respectively.

For trapped atoms, the STIRAP or CPT sequence was induced by the counterintuitive temporal ordering of the approaching and separation processes. In the case of waveguides, such sequences will be applied in space. To obtain a STIRAP–like transport with the atom initially located in the left arm, first the right guide is approached to the middle one, and, with an appropriate delay, the tunneling is also switched on between the middle and the left guide. Finally, tunneling is turned off in the same order: first between the right and the middle and then between the middle and the left guide. For a CPT-like process to split the atomic wave packet coherently between two waveguides, the approaching sequence has the same order, but the separation is symmetric, see Fig. 7(a). For guided atoms the additional coupling terms in Eq. (5) make the evolution more complex. Expanding c_α into plane waves with momentum $\hbar k$ leads to a diagonal k^2-proportional term which accounts for broadening of the wave packet and to a term proportional to $kK_{\alpha\beta}$ which induces velocity-dependent couplings between the waveguides. Furthermore, a velocity-independent modification of the couplings is introduced through the potential couplings $P_{\alpha\beta}$.

To take again into account further effects beyond this approximation, we have numerically integrated the full 2D Schrödinger equation. We assume to initially have a single atom wave packet located in the left arm, with a gaussian profile in the direction of the waveguide corresponding to mean momentum $\langle k_y\rangle$ and momentum spread $\Delta k_y = k_r$ ($k_r = \sqrt{2m\omega_r/\hbar}$; for the simulations $\omega_r = \omega_x/6$). In the transverse direction the wave packet corresponds to the ground state of the potential. Fig. 7(b)–(d) shows an example of the time evolution in structure (a), generating a splitting of the wave packet through a CPT–like sequence. The wave packet strongly broadens in the direction of propagation, but is still nearly equally split between the left and right outgoing arms with a negligible amount of reflection. Small oscillations occurring in the density of the outgoing dispersive wavepacket are due to the velocity–dependent perturbation of the couplings between the waveguides, i.e., due to the last term in Eq. (5). This splitting is again relatively robust with respect to the parameters describing the potential, provided the symmetry is maintained. Fig. 7(e) shows the change of the atomic fractions in the exit ports of the setup as the minimal distance of the waveguides is varied. The process is not as perfect as its counterpart in traps due to the additional couplings present here. Especially the velocity-dependent couplings modifying the desired CPT–like process play an important role: the larger the mean velocity, the stronger the deviation from the equal splitting, as can be seen from Fig. 7(f).

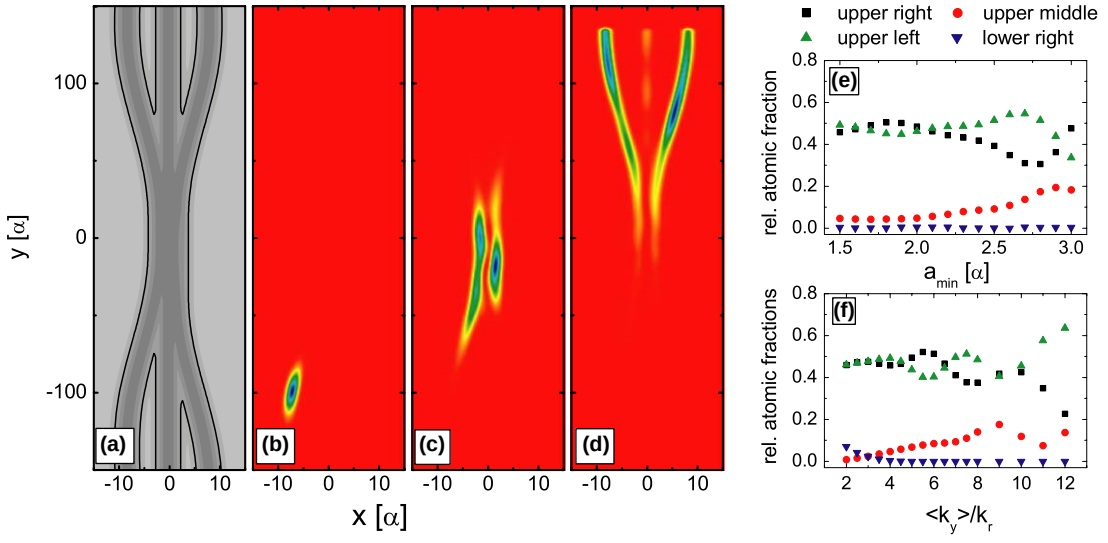

Fig. 7. (a) Plot of the waveguide potential constructed from concatenated harmonic waveguides used to split the wave packet incident in the left arm into a superposition of packets traveling in the left and the right arm using a CPT–like process. The contour line shown corresponds to a distance of 3/2 ground state widths away from the centers of the waveguides; (b)–(d) density plots of $|\psi(x,y)|^2$ at times $t = 20\omega_x^{-1}$, $t = 60\omega_x^{-1}$, $t = 120\omega_x^{-1}$ for a wave packet with mean momentum $\langle k_y\rangle = 3.5k_r$ and initial width $\Delta k_y = k_r$. The minimal distance between waveguides is $d_{\min} = 1.5\alpha$. (e) The relative atomic fractions leaving the setup through the upper and lower exits of the structure as the minimal distance between waveguides is modified. (f) as (e), but as a function of the mean velocity $\langle k_y\rangle$.

4. Conclusions

We have studied the manipulation of the external wavefunction of an atom in a potential consisting of three traps whose coupling can be changed in time. As demonstrated, such a system, if restricted to the lowest eigenstates, constitutes an analogue to the extensively studied system of three internal atomic states coupled via two external laser fields. This allows to apply concepts as STIRAP, CPT, or EIT to coherently and robustly manipulate the external atomic wavefunction. Such processes are of potential interest, e.g., to move around atomic quantum bits or to create superposition states for interferometry. In particular, we have analyzed the robustness of a STIRAP–like process allowing transport of the atom between trapping potentials, and we have also shown that coherent processes are possible for several interacting atoms.

As a different setup, we have studied atomic wave packets propagating in waveguide potentials, where the time dependence of the trap distances is replaced by a spatial variation of the distance between waveguides. Due to additional velocity-dependent couplings, the evolution is more involved and the transport or splitting processes are not as clean as in the case of traps. Still, a stronger robustness as for schemes relying only on Rabi-type tunneling between traps can be achieved, as exemplified through demonstrating the coherent splitting of a wave packet between two arms, a scheme interesting for, e.g., interferometry.

The present work only considers the manipulation of single atom wavepackets or systems of a few atoms. In view of the interesting theoretical and experimental work on tunneling and self-trapping of Bose-Einstein condensates in a double-well potential [24], for future investigations it will be particularly interesting to study how interaction in a many-particle state affects the processes discussed here.

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