Magneto–optical control of bright atomic solitons

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Abstract. In previous work we showed that bright atomic solitons can arise in spinor Bose–Einstein condensates in the form of gap solitons even for repulsive many-body interactions. Here we further explore the properties of atomic gap solitons and show that their internal structure can be used to both excite them and control their centre-of-mass motion using applied laser and magnetic fields. As an illustration we demonstrate a nonlinear atom-optical Mach–Zehnder interferometer based on gap solitons.

1. Introduction

The experimental demonstration of Bose–Einstein condensation in atomic vapours has rapidly led to spectacular new advances in atom optics. In particular, it has enabled its extension from the linear to the nonlinear regime [1], very much like the laser led to the development of nonlinear optics in the 1960s. It is now well established that two-body collisions play for matter waves a role analogous to that of a Kerr nonlinear crystal in optics. The first experimental verification of this analogy was four-wave mixing [2]. In addition, it is also possible to nonlinearly mix optical and matter-waves, as demonstrated recently in matter-wave superradiance [3, 4] and in the first realization of a phase-coherent matter-wave amplifier [5]. In addition, it is known that the nonlinear Schrödinger equation which describes the condensate in the Hartree approximation supports soliton solutions [6]. For the case of repulsive interactions normally encountered in BEC experiments, the simplest solutions are dark solitons, that is, ‘dips’ in the density profile of the condensate [7–10]. These dark solitons have recently been demonstrated in two experiments [11, 12] which appear to be in good agreement with the predictions of the Gross–Pitaevskii equation.

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While very interesting from a fundamental physics point of view, dark solitons would appear to be of limited interest for applications such as atom interferometry, since what would be desirable is to achieve the dispensionless transport of a spatially localized ensemble of atoms, rather than a ‘hole’. In that case, bright solitons are much more interesting. However, the problem is that large condensates are necessarily associated with repulsive interactions, for which bright solitons might appear to be impossible since the nonlinearity cannot compensate for the kinetic energy part (diffraction) in the atomic dynamics. While this is true for atoms in free-space, this is not the case for atoms in suitable potentials, e.g. optical lattices. This is because in that case, it is possible to tailor the dispersion relation of the atoms in such a way that their effective mass becomes negative. For such negative masses, a repulsive interaction is precisely what is required to achieve soliton solutions. This result is known from nonlinear optics, where such soliton solutions, called gap solitons, have been predicted and demonstrated [13, 14].

We showed in previous work [15] that such bright solitons are also possible in matter-wave optics using spinor condensates, but did not discuss explicitly how to excite and control them. The present paper addresses these questions, and shows that a combination of optical and magnetic fields can be used to generate solitons of various velocities, and subsequently, to control them, split and recombine them, etc. This magneto-optical control results from the use of spinor condensates, and can be achieved at minimal cost in terms of atomic loss. In particular, we illustrate how to realize a Mach–Zehnder interferometer for bright atomic solitons. This opens up the way to intriguing new ways to manipulate and transport coherent matter in ways which complement those offered by standard optical tweezers or by atomic wave guides [16, 17].

Among Dan Walls’ many contributions to nonlinear and atom optics he was one of the first to recognize the emerging area of nonlinear atom optics, and he introduced the idea of atomic solitons travelling in laser beams in 1994 along with Zhang and Sanders [18]. The current wave of interest in atomic solitons therefore finds its origin in Dan’s seminal work, and we all greatly miss the originality, insight, and energy he brought to the field.

The paper is organized as follows: section 2 briefly reviews the analysis leading to the predictions of gap solitons [15], while section 3 discusses some of their most important characteristics. Section 4 exploits these properties to develop tools to excite gap solitons and control their dynamical behaviour, leading to the demonstration of an atomic Mach–Zehnder interferometer in section 5. Finally, section 6 is a summary and outlook. While the main text uses a simple model of light–matter interaction for notational clarity, a more realistic coupling scheme, using the full hyperfine structure of the Sodium 3$S_{1/2}$–3$P_{3/2}$ transition, is presented in Appendix A.

2. Physical model

To set the stage for our analysis, we first briefly review the main ingredients of the theory of gap solitons [15]. The system we consider consists of a Bose–Einstein condensate interacting with two counterpropagating, focused Gaussian laser beams of equal frequency $\omega_l$ but opposite circular polarizations (see figure 1). The optical dipole potential associated with the applied laser beams is assumed to
provide tight transverse confinement for the BEC in the \((X, Y)\) plane, thereby forming a cigar shaped condensate of transverse cross-sectional area \(A_T\). In the following, we confine our discussion to the one-dimensional dynamics of the BEC along the \(Z\)-axis for simplicity.

In addition to supplying a transverse optical potential, the laser beams can drive two-photon transitions between different Zeeman sublevels of the atomic ground state. For illustrative purposes we consider the case of Sodium and the two-photon coupling of the Zeeman sublevels \(|-1\rangle = |F_g = 1, M_g = -1\rangle\) and \(|1\rangle = |F_g = 1, M_g = 1\rangle\). For example, starting in the \(|-1\rangle\) state this process involves the absorption of a \(\sigma^+\) photon from the right propagating laser beam followed by emission of a \(\sigma^-\) photon into the left propagating laser beam.

We must of course assume that the excited states involved in the atom-field interaction are far-detuned from the applied laser frequency, a necessary requirement to avoid the detrimental effects of spontaneous emission. This, however raises a serious issue, since for the alkali atoms that we have in mind the two-photon coupling strength vanishes in the limit when the detuning is large compared to the excited-state hyperfine splitting [19, 20]. But this difficulty can be circumvented: we show in the Appendix that by using a four-photon scheme involving an additional \(\pi\)-polarized field incident perpendicular to the \(Z\)-axis, one can achieve an effective two-photon coupling between the states \(|\pm 1\rangle\) which survives in the limit of large detunings. We therefore proceed with our two-photon coupling model and refer the interested reader to the Appendix for details.

By restricting our attention to the coupled states \(|\pm 1\rangle\) the effective single-particle Hamiltonian for our model system can be written as [15, 21]

\[
H_{\text{eff}} = \frac{P_Z^2}{2m} + g\hbar\delta' [|-1\rangle\langle e^{2iK/Z} - 1|1\rangle\langle e^{-2iK/Z}].
\]

where we have omitted constant light-shift terms. Here \(P_Z\) is the atomic centre-of-mass momentum operator along \(Z\), \(m\) the mass of the atom, \(K_l = \omega_l/c\) is the magnitude of the field wave vector along \(Z\), \(g\) is a coupling constant between the ground and excited states characteristic of the atom and transition involved.
\[ \delta' = D^2 \mathcal{E}^2 / \hbar^2 \delta, \] with the detuning \( \delta = \omega_l - \omega_a \), \( \mathcal{E} \) is the laser field amplitude, and \( D \) is the reduced dipole moment for the \(^3S_{1/2} - ^3P_{3/2} \) transition.

The first term of the effective Hamiltonian (1) describes the quantized atomic centre-of-mass motion, and the remaining terms give the effective coupling of the two Zeeman sublevels via the applied laser fields. The exponential terms \( \exp(\pm 2iK_l Z) \) arise from the fact that the two-photon transitions involve the absorption of a photon from one light field and re-emission into the other. Finally, introducing a spinor macroscopic condensate wave function \( \Psi(Z, t) = [\Psi_1(Z, t), \Psi_{-1}(Z, t)]^T \) normalized to the number of atoms \( N \), and including the many-body effects via a mean-field nonlinearity, we obtain the coupled Gross–Pitaevskii equations

\[ i\hbar \frac{\partial \Psi}{\partial t} = H_{\text{eff}} \Psi + U |\Psi|^2 \Psi, \] (2)

where \( U = 4\pi \hbar^2 a_{sc}/m \), \( a_{sc} \) is the s-wave scattering length, \( |\Psi|^2 = |\Psi_1|^2 + |\Psi_{-1}|^2 \), and we have assumed that the magnitude of the self- and cross-nonlinearities are equal for simplicity.

It is convenient to re-express equation (2) in dimensionless form by introducing the scaled variables \( t = t/\tau_c \), \( z = Z/l_c \) and \( \psi_j = \Psi_j/\sqrt{\rho_c} \) where

\[ \tau_c = \frac{1}{g\delta'}, \quad \rho_c = \frac{\hbar \delta'/U}{M^2}. \] (3)

Equations (2) then become

\[ i \frac{\partial}{\partial \tau} \begin{pmatrix} \psi_1 \\ \psi_{-1} \end{pmatrix} = \begin{pmatrix} -M\nabla^2 & e^{2iklz} \\ e^{-2iklz} & -M\nabla^2 \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_{-1} \end{pmatrix} + \text{sgn}(g\delta'/U)|\psi|^2 \begin{pmatrix} \psi_1 \\ \psi_{-1} \end{pmatrix}, \] (4)

where \( M = \hbar \delta'/m/2\hbar K_l^2 \) is a mass-related parameter such that \( k_l = K_l/l_c = 1/2M \). Throughout this paper we use a characteristic laser intensity of \( I = 50 \text{ W cm}^{-2} \) for each of the two counter-propagating laser beams and a wavelength of \( \lambda_l = 985 \text{ nm} \). In addition we use \( g = -\frac{1}{\hbar} \), which is simply the Clebsch–Gordan coefficient for the \( F_g = 1 \leftrightarrow F_e = 1 \) transition. This results in the characteristic scale values \( \tau_c = 685 \text{ \mu s}, \quad l_c = 12.1 \text{ \mu m}, \quad \rho_c = 8.59 \text{ \mu m}^{-3} \) and \( M = 0.0065 \).

3. Gap solitons

The spatially modulated coupling between the optical fields and the condensate induces a single-particle band structure with regions of negative effective mass. As mentioned in the introduction, this leads to the possibility of bright atomic solitons even for repulsive interactions [15]. Their energy lies in forbidden gaps of the linear band structure, hence the name gap solitons.

3.1. Analytical solutions

Approximate analytic expressions for the gap solitons can be obtained by expressing the spinor condensate components as
\[
\psi_{\pm 1}(z, \tau) = e^{\pm i k_l z} e^{-i \gamma \tau / 4M}\phi_{\pm 1}(z, \tau),
\]

where the field envelopes \( \phi_{\pm 1} \) are assumed to be slowly varying in space compared to \( 1/k_l \). Neglecting the second-order spatial derivatives yields the coupled partial differential equations

\[
i \left( \frac{\partial}{\partial \tau} \pm \frac{\partial}{\partial z} \right) \left( \begin{array}{c} \phi_1 \\ \phi_{-1} \end{array} \right) = \left( \begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array} \right) \left( \begin{array}{c} \phi_1 \\ \phi_{-1} \end{array} \right) \pm (|\phi_1|^2 + |\phi_{-1}|^2) \left( \begin{array}{c} \phi_1 \\ \phi_{-1} \end{array} \right),
\]

where the choice \( \pm 1 = \text{sgn} (g \delta'/U) \). For a red-detuned laser and our choice of \( g \) this becomes \( \pm 1 = \text{sgn} (U) \). Aceves and Wabnitz [22] have shown that these dimensionless equations have the explicit two-parameter gap soliton solutions (see also [14])

\[
\begin{align*}
\phi_1 &= \pm \frac{\sin(\eta)}{\beta \gamma^{1/2}} \left( -\frac{e^{2\theta} + e^{i \eta \theta}}{e^{2\theta} + e^{-i \eta \theta}} \right)^{1/2} \text{sech} \left( \theta \mp \frac{i \eta}{2} \right) e^{\pm i \sigma}, \\
\phi_{-1} &= -\frac{\beta \sin(\eta)}{\gamma^{1/2}} \left( -\frac{e^{2\theta} + e^{i \eta \theta}}{e^{2\theta} + e^{-i \eta \theta}} \right)^{1/2} \text{sech} \left( \theta \mp \frac{i \eta}{2} \right) e^{\pm i \sigma},
\end{align*}
\]

with \(-1 < v < 1\) is a parameter which controls the soliton velocity, \( 0 < \eta < \pi \) is a shape parameter, and

\[
\beta = \left( \frac{1 - v}{1 + v} \right)^{1/4}, \quad \gamma = \frac{1}{\sqrt{1 - v^2}},
\]

\[
\theta = -\gamma \sin (\eta) (z - \nu \tau), \quad \sigma = -\gamma \cos (\eta) (\nu z - \tau).
\]

Since we are interested in creating bright solitons in the presence of repulsive interactions we restrict ourselves to \( \text{sgn}(U) = +1 \), corresponding to the choice of the upper sign in the analytic solutions.

The characteristic length scale associated with the solitons is \( l_c \), so that the approximate solitons (7) are valid for \( K l_c = 1/2M \gg 1 \). This inequality is well satisfied for our choice of parameters, which gives \( 1/2M = 76.9 \).

3.2. Characteristic properties

From the dependence of the hyperbolic secant on \( \theta = -\gamma \sin (\eta) (z - \nu \tau) \) in equation (7), we identify the gap soliton parameter \( v = V_g/V_R \) as the group velocity \( V_g \) of the soliton in units of the recoil velocity \( V_R = l_c/\tau_c = \hbar K_l/m \). Since \(-1 < v < 1\), the magnitude of the group velocity is bounded by the recoil velocity, which is \( V_R = 1.77 \text{ cm s}^{-1} \) for the present case of sodium. From equations (7), one can extract further important soliton properties, such as the number \( N_s \) of atoms in the soliton shown in figure 2, and the soliton width \( W_s = \nu_l \) shown in figure 3. Specifically, the number of atoms in a particular gap soliton is given by

\[
N_s = A_T \int \! dZ [ |\Psi_1(Z,t)|^2 + |\Psi_{-1}(Z,t)|^2],
\]

where \( A_T \) is the effective transverse area. Figure 2 illustrates two general soliton properties, namely that increasing the shape parameter \( \eta \) increases the atom
number, but that faster solitons have lower atom numbers. Similarly, figure 3 shows that increasing the shape parameter \( \eta \) decreases the soliton width and that faster solitons have narrower widths.

Given that the analytic gap soliton solutions (7) hold for broad envelopes, we confine our attention to the case \( \eta < 1 \), with atom numbers in the range \( N_s \approx 10^4 - 10^5 \), and soliton widths \( W_s \approx 40 - 100 \mu m \) for the parameters at hand.
We can gain further insight into the structure of the gap solitons by taking the more extreme limit \( \eta \ll 1 \) of equations (7). Using the definitions (5), and returning to dimensional units we have then

\[
\Psi_1(Z, 0) = \frac{\eta}{\beta} \sqrt{\frac{\rho_c}{2}} \operatorname{sech} \left( \frac{Z}{W_0} \right) (-1)^v e^{i(K + K_l)Z},
\]

\[
\Psi_{-1}(Z, 0) = \frac{\beta \eta}{\gamma} \sqrt{\frac{\rho_c}{2}} \operatorname{sech} \left( \frac{Z}{W_0} \right) (-1)^v e^{i(K - K_l)Z + \pi]},
\]

where

\[
W_s = 3.44W_0 = 3.44 \left( \frac{l_c \sqrt{1 - v^2}}{\eta} \right), \quad K = -\frac{\gamma v}{l_c}.
\]

Here \( W_s = 3.44W_0 \) is the soliton width, the factor 3.44 being the numerical conversion from the width of the hyperbolic secant to the \( 1/e^2 \) width of the distribution, and \( K = k/l_c \) a velocity-dependent wave vector shift. This expression agrees well with the features displayed in figure 3, in that the width decreases with increasing \( \eta \) and \( v \). The soliton atom number obtained by combining equations (10) and (11),

\[
N_s = 2(A_T l_c \rho_c \eta) \cdot (1 - v^2),
\]
correctly predicts the scaling properties of figure 2.

An essential point to keep in mind is that the gap solitons are coherent superpositions of the two Zeeman sublevels, and the approximate solutions (11) contain important information on the phase and amplitude relations that need to be created between them to successfully excite and manipulate gap solitons. In particular, they show that there is always a spatially homogeneous \( \pi \) phase difference between the two states. In addition, the two components have the spatial wave vectors

\[
K_{\pm 1} = K \pm K_l,
\]

with \( K = -\gamma v/l_c \). For our parameters, \( K_l = 6.38 \mu m^{-1} \) and \( |K| < 1/l_c = 0.08 \mu m^{-1} \). Despite the fact that it is so small, \( |K| \) is an important factor since it controls the soliton velocity. Finally, it follows from dividing the amplitudes of the two components that

\[
\frac{\left| \Psi_1(Z, t) \right|^2}{\left| \Psi_{-1}(Z, t) \right|^2} = \frac{1 + v}{1 - v},
\]

which shows that their relative occupation depends on the soliton velocity parameter \( v \). For \( v = 0 \) the sublevels are equally populated, but as \( v \to 1 \) the \( |1 \rangle \) sublevel has a larger population, and vice versa for \( v \to -1 \).

The characteristic time scale for the evolution of the gap solitons can be determined from the plane-wave exponential factors in equations (7). Converting to dimensional form the soliton period \( t_s \) is defined as the time to accumulate a \( 2\pi \) phase, or in the limit \( \eta \to 0 \)

\[
t_s = 2\pi t_c \sqrt{1 - v^2}.
\]
Physically, \( t_s \) corresponds to the internal time scale for the gap soliton. In order to observe a soliton-like behaviour, it is therefore necessary to investigate the atomic propagation over several periods.

We conclude this section by noting that Aceves and Wabnitz [22] have shown that the gap soliton solutions (7) are stable solutions of equation (6) in that they remain intact during propagation, even when perturbed away from the exact solutions. However, one should remember equation (6) is only an approximation to the exact system of equation (4) so that in general the gap solitons are solitary wave solutions only. As such, they are not guaranteed to be absolutely stable.

4. Gap soliton control

4.1. State manipulation

Summarizing the previous section, gap solitons require the right population in each Zeeman sublevel, a phase difference of \( \pi \) between these sublevels, and appropriate plane-wave factors \( e^{iK_s Z} \). The shapes of the Hartree wave functions corresponding to the two Zeeman sublevels are hyperbolic secant, which we approximate by a Gaussian in the following sections. They could, for example, be initialized in an optical dipole trap [23]. Manipulating the gap solitons is therefore reduced to the problem of controlling the populations and phases throughout the spinor condensate. This is achieved via a magneto-optical control scheme involving a combination of pulsed coherent optical coupling and of phase-imprinting using spatially inhomogeneous magnetic fields.

Coherent optical coupling can be achieved, for example, by a laser pulse of frequency \( \omega_l \) propagating perpendicularly to the \( Z \)-axis and with linear polarization perpendicular to that axis. For sufficiently short pulses, one can neglect changes in the centre-of-mass motion of the atoms during its duration, leading to a very simple description. We assume for simplicity a plane-wave rectangular pulse of duration \( t_p \) and of spatial extent large compared to the soliton. The Hamiltonian describing the coupling between this pulse and the condensate is then the same as in equation (1) without the linear momentum exchange terms \( \exp(\pm 2iK_l Z) \) and the kinetic energy term, and with \( \delta' \rightarrow \delta_p' \). The state of the system after the pulse is then easily found to be

\[
\begin{pmatrix}
\Psi_1(t_p) \\
\Psi_{-1}(t_p)
\end{pmatrix} =
\begin{pmatrix}
\cos \chi & i \sin \chi \\
-i \sin \chi & \cos \chi
\end{pmatrix}
\begin{pmatrix}
\Psi_1(0) \\
\Psi_{-1}(0)
\end{pmatrix}
\equiv M_L(\chi)
\begin{pmatrix}
\Psi_1(0) \\
\Psi_{-1}(0)
\end{pmatrix},
\tag{17}
\]

where \( \chi = g_\delta' t_p \) is the excitation pulse area and the operator \( M_L \) can be used to control the population transfer by an appropriate choice of \( \chi \).

The required phase relation between the two states can be achieved via Zeeman splitting. Considering for concreteness a spatially inhomogeneous rectangular magnetic field pulse of duration \( t_B \) we have, neglecting again all other effects,

\[
\frac{i\hbar}{\partial t} \Psi_{\pm 1}(Z,t) = \pm \mu_B g_F (B_0 + B' Z) \Psi_{\pm 1}(Z,t),
\tag{18}
\]
where $g_F$ is the Landé g-factor of the hyperfine ground state, $\mu_B$ is the Bohr magneton, $B_0$ the spatially homogeneous component of the magnetic field, and $B'$ its gradient, the direction of the magnetic field being along the $Z$-axis. The application of this field results in the state

$$
\left( \begin{array}{c}
\Psi_1(t_B) \\
\Psi_{-1}(t_B)
\end{array} \right) = \left( \begin{array}{cc}
e^{i(\vartheta+K_BZ)} & 0 \\
0 & e^{-i(\vartheta+K_BZ)}
\end{array} \right) \left( \begin{array}{c}
\Psi_1(0) \\
\Psi_{-1}(0)
\end{array} \right)
$$

$$
\equiv M_B(\vartheta,K_B) \left( \begin{array}{c}
\Psi_1(0) \\
\Psi_{-1}(0)
\end{array} \right),
$$

(19)

where $\vartheta = -(\mu_B g_F/\hbar)B_0 t_B$ and $K_B = -(\mu_B g_F/\hbar)B't_B$ are the imprinted phase shift and phase gradient (or wave vector), respectively. That is, the application of the magnetic pulse results in a phase difference of $2\vartheta$ between the two Zeeman sublevels, and in addition it imparts them wave vectors $\pm K_B$.

We note that although we have treated the pulsed excitations above in an impulsive manner to illustrate their action, our simulations describe the actions of the pulses correctly to the equations of motion (equation 4). The numerics confirm the accuracy of the impulsive approximation for the parameters at hand; a consequence of the fact that we consider pulse durations significantly shorter than the soliton period (equation 16) of $t_s = 4.3$ ms.

4.2. Gap soliton excitation

To illustrate how stationary and moving solitons can be excited using the proposed magneto-optical state scheme, we start from a scalar condensate in the $|1\rangle$ state, $\Psi(Z,0) = [0,\Psi_0(z)]^T$, with spatial mode

$$
\Psi_0(Z) = \frac{N_s}{\sqrt{A_T}} \left( \frac{2}{\pi W_s^2} \right)^{1/4} e^{-Z^2/W_s^2},
$$

(20)

with $W_s$ and $N_s$ the width and atom number of the gap soliton desired (see figures 2 and 3). This Gaussian is chosen to approximate the hyperbolic-secant structure of the analytic gap soliton solution in equation (11). For a stationary solution we need to prepare the Zeeman sublevels with equal population and with a $\pi$ phase difference. We further need to impose wave vectors which are equal in magnitude but opposite in sign $K_{\pm 1} = \pm K_L$. This can be achieved by applying a laser pulse with area $\chi = \pi/4$, followed by a magnetic pulse with $\vartheta = \pi/4$ and $K_B = K_L$. The state then transforms as $(t = t_p + t_B)$

$$
\Psi(Z,t) = M_B(\pi/4,K_L)M_L(\pi/4)\Psi(Z,0)
$$

$$
= \frac{e^{i\vartheta}}{\sqrt{2}} \left( \begin{array}{c}
e^{iK_LZ} \\
e^{-i\pi} e^{-iK_LZ}
\end{array} \right) \Psi_0(Z).
$$

(21)

Using the atomic parameters of Sodium, this situation can be realized for a 10 $\mu$s light pulse with $I = 2.69$ kW cm$^{-2}$, and a 200 $\mu$s magnetic field pulse with $B_0 = 0.89$ mG and $B = 72.5$ G cm$^{-1}$ ($g_F \approx -0.5$ for the $F = 1$ ground state of sodium). Figure 4 shows the resulting stable evolution of the total density $|\Psi(Z,t)|^2$. As a result of the Gaussian approximation to the exact solution there are some slow oscillations imposed on the motion, but the solution remains centred
at $Z = 0$ and stationary over a time $t = 200\,\text{ms}$, much longer than the soliton period $t_s = 4.3\,\text{ms}$.

The excitation of a moving soliton is slightly more complicated since the velocity-dependent wave vector $\mathbf{K} = -\gamma v/l_c$ in equation (11) is no longer zero. To deal with this situation, we first impart the wave vector $\mathbf{K}$ to the initial $\left| -1 \right>$ state in equation (20) using a magnetic pulse with $\theta = 0$, $\mathbf{K}_B = -\mathbf{K}$, which for sodium and a 200 $\mu$s magnetic pulse can be realized using $\mathbf{B}_0 = 0$, $\mathbf{B}' = +0.094$ G cm$^{-1}$. Here we specifically take $v = 0.1$, so that $\mathbf{K} = -830.6$ m$^{-1}$. We further recall that moving solitons must have unequal populations of the two Zeeman sublevels. For $v = 0.1$, we have from equation (15) that the ratio between the $\left| 1 \right>$ and $\left| -1 \right>$ populations should be 1.22. This is achieved by a coherent optical coupling with $\chi = 0.835$, and $I = 2.69$ kW cm$^{-2}$, corresponding to a pulse duration of 10.6 $\mu$s. Finally, we impart the wave vectors $\pm K_I$ and the $\pi$ phase difference between the Zeeman sublevels with a magnetic pulse, as for the stationary soliton above. Summarizing, the full magneto-optical control sequence is described at $(t = 2t_B + t_p)$ by

$$
\Psi(Z,t) = M_B(\pi/4,K_I)M_L(\chi)M_B(0,-K)\Psi(Z,0)
= e^{\frac{\imath}{\hbar}t}
\left(\begin{array}{c}
\sin \chi \, e^{\imath(K+K_I)Z} \\
\cos \chi \, e^{-\imath \pi} \, e^{\imath(K-K_I)Z}
\end{array}\right)\Psi_0(Z).
$$

Figure 5 shows the resulting numerical simulation of a gap soliton with $v = 0.1$ ($V_g = 0.18$ cm s$^{-1}$), illustrating its stable propagation over many soliton periods. We have used the same scheme to launch gap solitons over the full range of velocities.

4.3. Soliton splitting

The next application of magneto-optical control that we consider is soliton splitting. To achieve this goal, we take advantage of the fact that for fast solitons almost all of the population is in one Zeeman sublevel, and the other state can be
viewed as a small perturbation. For example, for \( v = 0.5 \) the ratio of the populations is already 6. This implies that the relative phase between the two states is no longer of importance, so we need only concentrate on getting the plane-wave factors right.

Assume for concreteness that we start from an initial condition with \( N_0 \) atoms in the \( | -1 \rangle \) state, and apply a laser pulse of area \( \pi/4 \) to transfer half the population to the \( | + \rangle \) state. We can then apply a magnetic pulse to impose the wave vectors \( \pm (K_l + K) \), to the \( | \pm 1 \rangle \) states, with \( K = -\gamma v/l_c \) corresponding to a given velocity \( |v| \). Now if we choose \( N_0 = 2N_s \), with \( N_s \) the atom number for that velocity, then for \( |v| \) large enough we may expect to see oppositely moving solitons emerge from the initial state. This is illustrated in figure 6, which clearly demonstrates the emergence of two solitons with opposite velocities. Figure 7, which shows the density profiles for the individual Zeeman sublevels for a time \( t = 82 \) ms, confirms that each gap soliton indeed comprises a dominant Zeeman sublevel plus a small component of the other state. During the early stages of propagation, the solitons rearrange their phase and shape before settling down. This is accompanied by some slowing down and the familiar shedding of ‘radiation’. The emerging solitons are therefore slower than their ‘design velocity’. For the specific example of figure 6, the actual velocity is found to be 1.08 cm s\(^{-1}\).
4.4. Soliton reversal

In addition to offering the possibility of exciting moving gap solitons, magneto-optical control can also reverse their direction of propagation along $Z$. This can again be achieved by making use of the fact that for fast solitons almost all the population is in one Zeeman sublevel, so that we need only concentrate on getting the plane-wave factors right. In the numerical simulation of figure 8 we create a gap soliton with $v = 0.59 \ (V_g = 1 \text{ cm s}^{-1})$ and let it propagate for 100 ms. This soliton consists essentially of a plane-wave factor $e^{i(K+K_i)z}$ multiplying state $|1\rangle$. At $t = 100 \text{ ms}$ we apply a laser pulse of area $\pi/2$ to transfer all the population to state $|–1\rangle$ and change the phase to $e^{-i(K+K_i)z}$ using a magnetic field pulse. This results in the same soliton as before the control sequence, but with opposite velocity. Note, however, the loss of some atomic population to ‘radiation’ in the process.

Figure 7. Two solitons 82.0 ms after the splitting. The plot reveals that each soliton still has a small fraction of the other state bound to it.

Figure 8. The propagation of a fast soliton with velocity $v \approx 1 \text{ cm s}^{-1}$ whose direction is reversed after 100 ms. The process of reversing causes small loss: a packet travels on to the right.

4.4. Soliton reversal

In addition to offering the possibility of exciting moving gap solitons, magneto-optical control can also reverse their direction of propagation along $Z$. This can again be achieved by making use of the fact that for fast solitons almost all the population is in one Zeeman sublevel, so that we need only concentrate on getting the plane-wave factors right. In the numerical simulation of figure 8 we create a gap soliton with $v = 0.59 \ (V_g = 1 \text{ cm s}^{-1})$ and let it propagate for 100 ms. This soliton consists essentially of a plane-wave factor $e^{i(K+K_i)z}$ multiplying state $|1\rangle$. At $t = 100 \text{ ms}$ we apply a laser pulse of area $\pi/2$ to transfer all the population to state $|–1\rangle$ and change the phase to $e^{-i(K+K_i)z}$ using a magnetic field pulse. This results in the same soliton as before the control sequence, but with opposite velocity. Note, however, the loss of some atomic population to ‘radiation’ in the process.
5. Atomic Mach–Zehnder interferometer

As an illustration of the potential use of gap solitons employing magneto-optical control here we consider a nonlinear atomic Mach–Zehnder interferometer. Solitons present some advantages for atom interferometry in that they are many-atom wavepackets which are immune to the effects of spreading, hence allowing longer path lengths, and also reduced signal-to-noise for large atom numbers. Typically many-body effects limit the utility of high-density wavepackets due to spatially varying mean-field phase shifts, but solitons have the cardinal virtue that they have fixed spatial phase variations (for our situation this applies for faster solitons). Thus solitons may provide a key to making maximal use of high density sources for atom interferometry. Indeed they have long been advocated for all-optical switching applications due to these very properties.

Our specific demonstration of a nonlinear atomic interferometer based on solitons involves an initial scalar condensate that is split into two oppositely moving solitons along $Z$, see figure 9 for $t < 60\text{ms}$. At $t = 60\text{ms}$ laser and magnetic pulses are applied which act to reverse the direction of the two solitons. The process of reversing causes some loss of atoms in both solitons as before. The two reversed soliton components come together again at $t = 120\text{ms}$. Since the colliding solitons are predominantly in opposite orthogonal Zeeman sublevels, the interference pattern appearing during the collision is due to the contamination of each soliton by the other state (see figure 7). Figure 10 shows the interference at the soliton collision in the total density (solid line) and also the individual Zeeman sublevels, with a fringe contrast of around 30%. These results demonstrate the potential use of gap solitons for realizing nonlinear atom interferometers with high brightness sources.

6. Summary and outlook

Employing a spinor rather than a scalar condensate gives the opportunity to externally manipulate bright atomic solitons by conceptually simple magneto-optical methods without losing the stable solitonic behaviour. Specifically, the characteristic properties of the gap soliton solutions lead to realistic manipulation schemes that have been demonstrated explicitly in a one-dimensional situation. This scheme has proven successful in exciting solitons with different velocities at a
minimal atom loss rate as well as splitting, and reversing their direction of propagation. The combination of these techniques resulted in the demonstration of an atomic Mach–Zehnder interferometer, which might be of interest in atom optical sensors. We remark that although this was not explicitly discussed here, we have also found numerical evidence for stable three-dimensional gap solitons, when adding an optical potential for confinement in the transverse direction.

It is now becoming amply evident that much of the future of atom optics lies in integrated systems, or ‘atom optics on a chip’ [17]. Optical and magnetic waveguides and beam splitters have recently been demonstrated, but the coupling of a condensate into these guides remains a major experimental challenge. The use of bright solitons, with their potential to transport bright matter waves in a controlled fashion, might offer a solution to this problem, and merits further investigations. Further topics of interest include the manipulation and control of bright solitons using phase imprinting methods that employ the AC–Stark effect, as already realized for dark solitons [10–12].

7. Acknowledgments

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Appendix A: Four-photon coupling

As mentioned in the Introduction, the Raman coupling between the $|F_g = 1, m_g = -1\rangle$ and $|F_g = 1, m_g = 1\rangle$ Zeeman sublevels vanishes in the case of far-off-resonance light, a result of destructive interference with other $3P_{3/2}$ hyperfine excited states [19]. In this limit the system reduces to an effective $|J_g = 1/2\rangle \leftrightarrow |J_e = 3/2\rangle$ transition where no $\Delta m = \pm 2$ transitions are allowed. This difficulty can be overcome by adding a $\pi$-polarized laser beam from the side (e.g. along the X-axis), which will not transfer any momentum along the transverse direction. A proper choice of detunings allows one to use another hyperfine ground state to mediate the $\Delta m = \pm 2$ transition, without populating it significantly.

One disadvantage of this approach is that since this is now a four-photon process, the intensity of the lasers must be increased while still avoiding spontaneous emission. Choosing a frequency difference between the $\pi$-light and the $\sigma$-light a few megahertz larger than the splitting between the hyperfine ground states as shown in figure 11, three time scales govern the dynamics of the system. They are determined by (a) the coupling of the ground state to the excited states, with detunings in the terahertz range; (b) the effective coupling between the hyperfine ground states, with detunings in the gigahertz resp. megahertz range; (c) the desired effective $\Delta m = \pm 2$ coupling. As it turns out, coupling the $|F_g = 2, m_g = -1\rangle$ and $|F_g = 2, m_g = 1\rangle$ levels is a better choice than staying in the $F_g = 1$ manifold, as this minimizes the coupling to the outer states $|F_g = 2, m_g = \pm 2\rangle$. Employing the intermediate $|F_g = 1, m_g = 0\rangle$ state makes the system aware of the hyperfine structure and leads to a non-vanishing coupling.

![Figure 11](image-url)

Figure 11. Four-photon configuration: The $|F_g = 1, m = 0\rangle$ level mediates the coupling between the $|F_g = 2, m = \pm 1\rangle$ levels. The red detuning of the $\pi$-light ensures that all other levels are off resonance. Although the state $|F_g = 1, m = 0\rangle$ is slightly populated, this ensures that we are sensitive to the hyperfine structure.
Separating the time scales leads to an effective coherent evolution between the \( |F_g = 2, m_g = \pm 1 \rangle \) states. It takes the same form as in equation (1), but with

\[
g \to g_{\text{eff}} = \frac{1}{192},
\]

\[
\delta' \to \delta'_{\text{eff}} = \frac{\Delta E^2 \varepsilon^2}{\hbar^2 \delta \Delta},
\]

where \( \varepsilon_\sigma \) is the amplitude for the \( \sigma \)-light resp. \( \varepsilon_\pi \) for the \( \pi \)-light and the detunings \( \delta \) and \( \Delta \) are defined in figure 11. The value of \( g_{\text{eff}} \) involves Clebsch–Gordan and Wigner–6J coefficients for all hyperfine transitions and \( \delta'_{\text{eff}} \) is clearly a four-photon term due to the product of four electric field envelopes. Choosing the wavelength of the \( \sigma \)-light \( \lambda_\sigma = 985 \text{nm} \) as before, \( \Delta = 94.5 \text{MHz} \) and laser intensities \( I_\sigma = 50 \text{kW cm}^{-2} \) and \( I_\pi = 38.8 \text{kW cm}^{-2} \) we obtain an effective coupling \( 1/(g_{\text{eff}} \delta'_{\text{eff}}) = 685.0 \mu\text{s} \), as in section 2.

Numerical simulations of the full-level scheme of sodium is in excellent agreement with the derived coupling in equation (23) and shows that the intermediate level \( |F_g = 1, m_g = 0 \rangle \) contains only around 0.1% of the atomic population. All other levels are negligibly populated, they are indeed far off resonance. Since \( |F_g = 1, m_g = 0 \rangle \) is a ground state, spontaneous emission is not an issue.

The intensities might be reduced if \( \delta \) were chosen smaller, but that would of course alter the properties of the soliton, since they depend on the wavelength of the optical fields. Note that we cannot apply permanent magnetic fields to remove the degeneracy of the hyperfine ground states and thus push non-participating states out of resonance since this would destroy the solitons. The light shifts induced by the lasers are the same for each manifold and will not remove this degeneracy.

References


Magneto-optical control of bright atomic solitons


