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Photon-assisted entanglement creation by minimum-error generalized quantum measurements in the strong-coupling regime

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We explore the possibilities of entangling two distant material qubits with the help of an optical radiation field in the regime of strong quantum electrodynamical couplingwith almost resonant interaction. For this purpose the optimum generalized field measurements are determined which are capable of preparing a two-qubit Bell state by postselection with minimum error. It is demonstrated that in the strong-coupling regime some of the recently found limitations of the nonresonant weak-coupling regime can be circumvented successfully due to characteristic quantum electrodynamical quantum interference effects. In particular, in the absence of photon loss it is possible to postselect two-qubit Bell states with fidelities close to unity by a proper choice of the relevant interaction time. Even in the presence of photon loss this strong-coupling regime offers interesting perspectives for creating spatially well-separated Bell pairs with high fidelities, high success probabilities, and high repetition rates which are relevant for future realizations of quantum repeaters.

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I. INTRODUCTION

Entanglement is a primary resource for quantum technology [1]. For applications in quantum communication, such as quantum key distribution, for example, the generation of well-controlled entanglement between spatially separated quantum systems is of crucial importance. For this purpose quantum repeaters [2] are needed, which counteract the destructive influence of uncontrolled environmental interactions.

Since the early work of Briegel *et al.* [3,4] there have been numerous theoretical proposals suggesting different physical platforms for realizing quantum repeaters [2]. They are based on the main idea of creating entanglement between quantum systems over large distances with the help of a chain of many uncorrelated pairs of quantum systems, each of which is entangled over a significantly shorter distance only. By performing appropriate Bell measurements on each of the two qubits of adjacent entangled pairs, it is possible to swap the already existing short-distance entanglement to the far-separated outermost quantum systems of such a chain.

From the experimental point of view the realization of a quantum repeater still constitutes a major technological challenge. Essential for any such realization are two prerequisites, namely, efficient physical mechanisms for generating highly entangled pairs of quantum systems with high success probabilities and high repetition rates and optimal ways for implementing complete Bell measurements accurately. It has been demonstrated theoretically [2] that the exchange of photons provides a powerful means for entangling material quantum systems at least over distances of moderate lengths, say a few kilometers, thus suggesting practicable solutions for the first prerequisite.

An interesting example in this respect is the recent theoretical proposal of van Loock *et al.* [5–7] of a hybrid quantum repeater based on continuous variables. It suggests the exchange of a single-mode coherent state of an optical radiation field between two cavities for the purpose of entangling spatially separated material qubits. It takes advantage of the facts that experimentally these field states can be controlled well and that they can also be produced with high repetition rates. The main idea of this proposal is to entangle this optical radiation field with two initially uncorrelated material qubit systems quantum electrodynamically and to create entanglement between these two qubits by an appropriate measurement of the quantum state of the radiation field which postselects a Bell state of the two material qubits. In their original proposal van Loock *et al.* [5,6] discuss cases in which the qubit systems couple to the radiation field in a nonresonant way inside two spatially separated cavities connected by an optical fiber so that their quantum electrodynamical interaction is weak and can be described perturbatively. Although offering a transparent theoretical description, this perturbative regime of the electromagnetic coupling also causes major theoretical limitations as far as the achievable degree of entanglement between the material qubits is concerned. They can be traced back to the fact that the relevant field states which have to be distinguished in order to postselect a Bell state of the material qubit pair are not orthogonal. Thus, these field states cannot be distinguished perfectly by any quantum measurement so that the entanglement of the postselected material two-qubit state is never perfect.

The basic ideas of this theoretical proposal offer interesting perspectives for the physical realization of entanglement sources. In view of these developments the natural question arises of whether it is possible to circumvent the theoretical limitations of the weak-coupling regime and to provide a physical mechanism which is capable of producing entangled two-qubit pairs not only with a high repetition rate and high success probability but also with an arbitrarily high degree of entanglement. For any future realization of a quantum repeater such a mechanism for creating at least short- to intermediate-distance entanglement between two qubits is useful as it is expected to increase significantly the final rates of producing long-distance entanglement by subsequent entanglement swapping and quantum state purification. It is a main aim of this paper to address this question.

In the following it is demonstrated that the strong-coupling limit of the quantum electrodynamical interaction offers interesting perspectives for photon-assisted entanglement creation between material quantum systems. By coupling two distant material few-level systems almost resonantly to the quantized radiation field, the performance of entanglement creation processes, such as the one originally proposed by van Loock *et al.* [5], can be improved significantly. In this way it is possible to circumvent previously discussed theoretical limitations which result from the restriction of the quantum electrodynamical interaction to the weak-coupling limit. In contrast, in the strong-coupling limit it is even possible to realize physical situations in which two material quantum systems can be postselected in a perfect Bell state by an appropriate von Neumann measurement of the quantized radiation field. However, for this purpose it is necessary to control the relevant interaction times between the quantized radiation field and the two material few-level systems appropriately. For sufficiently intense radiation fields these interaction times can even be chosen so short that effects of spontaneous emission of photons into other modes of the radiation field can be neglected so that a major decoherence mechanism can be eliminated and all advantages of quantum interferences can be exploited.

In this paper we focus on the exploration of theoretical limits governing entanglement creation between distant material qubits by postselective field measurements in the resonant quantum electrodynamical interaction regime. Experimental realizations of the theoretical scenario discussed require two material qubits each of which is placed inside an optical cavity. The two cavities are connected by a quantum transmission channel, such as an optical fiber, which allows the transmission of the radiation field from one cavity to the other. Although the experimental realization of an efficient coherent transfer of photons between cavities still constitutes a major technological challenge, methods for coping with these challenges have been discussed previously [8–10]. In particular, recently developed sophisticated experimental techniques [11,12] constitute important steps towards achieving almost perfect coherent couplings between a single mode of the radiation field of a Fabry-Pérot cavity and an optical fiber.

This paper is organized as follows. In Sec. II we introduce the quantum electrodynamical model in which two elementary material three-level systems interact with local cavity fields which are coupled by an optical fiber. Furthermore, we develop the general framework for describing the postselection procedure which prepares distant material qubits in a Bell state by an optimal generalized field measurement which introduces minimum errors. Numerical results are presented for characteristic quantities which quantify the success with which a Bell pair is prepared, its fidelity, and the minimum error with which this postselection can be achieved. Whereas in Sec. II we discuss cases in which the propagation of the optical radiation field between the two qubits through an optical fiber is ideal, in Sec. III modifications originating from photon loss during this propagation process are taken into account. In an Appendix we describe the photonic quantum state transfer between two distant optical cavities connected by a long optical fiber, and we determine the conditions for perfect photonic quantum state transfer.

II. PHOTON-ASSISTED ENTANGLEMENT CREATION

In this section we discuss a quantum electrodynamical model in which two spatially well-separated elementary (material) three-level systems are entangled in a Ramsey-type interaction scenario with single-mode photonic quantum states inside cavities connected by a long optical fiber. In particular, we explore the potential of producing high-fidelity material Bell states with the help of postselection by minimum-error field measurements which are capable of distinguishing nonorthogonal photonic states optimally. Thereby, we exploit the fact that perfect photonic quantum state transfer of single-mode quantum states is possible between two optical cavities connected by an optical fiber provided the cavity-fiber couplings are engineered appropriately (compare with the Appendix). Thus, generalizing a recent proposal of van Loock *et al.* [5] to the regime of almost resonant strong quantum electrodynamical coupling, we demonstrate that this dynamical regime combined with optimal postselection by field measurements offers interesting perspectives for producing distant material Bell pairs with high fidelities and with high success probabilities.

A. The quantum electrodynamical model

We consider two spatially separated optical cavities, say A and B, which are connected by a long optical fiber (compare with Fig. 1). We assume that the coupling between these cavities and their connecting optical fiber is engineered in such a way that it is possible to transfer a quantum state prepared in a single mode of cavity A, say with frequency ω, perfectly to a single mode of cavity B with the same frequency. As demonstrated in detail in the Appendix, this is possible if these modes of cavities A and B are coupled resonantly to

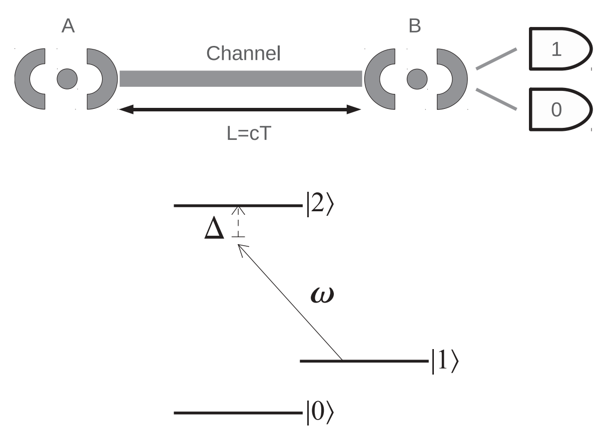

FIG. 1. Schematic representation of photon-assisted entanglement creation: The field state inside cavity A interacts almost resonantly for a short time τ with the material quantum system inside cavity A. The resulting photon state is transferred in a time $T = L/c \gg \tau$ to cavity B by propagation through a connecting optical fiber. (c is the propagation speed in the optical fiber.) By appropriate engineering of the cavity-fiber couplings this quantum-state transfer can be achieved perfectly. After this quantum state transfer an analogous second almost resonant short interaction takes place for a time τ. After this Ramsey-type interaction scenario the photon state of cavity B is measured by a minimum-error two-valued positive operator-valued measure (POVM measurement) with measurement results $m \in \{1,0\}$. The measurement result $m = 1$ prepares both material quantum systems approximately in a Bell state $|\Psi^+\rangle$ with success probability P_{Bell} and with fidelity F_{opt}.

the densely spaced modes of the long connecting optical fiber and if the coupling constants between cavities and optical fiber are engineered in such a way that a time reversal of the decay process from cavity A to the optical fiber is possible. In particular, this implies that the decay rates of the relevant modes of both cavities have to be equal, i.e., $\Gamma_A = \Gamma_B \equiv \Gamma$. The total time T required for such a perfect photonic quantum state transfer between both cavities is determined by the length L of the long optical fiber and by the photonic propagation speed c inside the fiber, i.e., $T = L/c$. Furthermore, this time has to be large in comparison with the decay times of both cavities so that the leakage both out of cavity A into the fiber and from the fiber into cavity B can be completed, i.e., $T \gg 2/\Gamma$. Recent experimental advances constitute highly promising steps towards realizing such links between cavities and an optical fiber [11].

In generalization of the original quantum repeater proposal of van Loock *et al.* [5] we consider two elementary (material) three-level systems, such as trapped atoms or ions, with energy eigenstates $|0\rangle_i$, $|1\rangle_i$, and $|2\rangle_i$ $(i \in \{A,B\})$ and associated energies E_0, E_1, and E_2 (compare with Fig. 1). One of them $(i = A)$ is located in cavity A and the other one $(i = B)$ in cavity B. The lowest-energy eigenstates $|0\rangle_i$ and $|1\rangle_i$ are assumed to be hyperfine-split components of the ground state with long radiative lifetimes so that spontaneous decay of these states by photon emission can be neglected. In the following these two states serve as the qubits which are going to be entangled. The energy eigenstates $|1\rangle_i$ and $|2\rangle_i$ are assumed to be of opposite parity and to be coupled almost resonantly to the single mode of frequency ω of cavity $i \in \{A,B\}$ by an optical dipole transition. The coupling of the far-detuned third level $|0\rangle_i$ to the radiation field is assumed to be negligible. It is the main purpose of our subsequent discussion to demonstrate the creation of entanglement between the two qubits formed by the states $|0\rangle_i$ and $|1\rangle_i$ $(i \in \{A,B\})$ with the help of almost resonant quantum electrodynamical couplings between states $|1\rangle_i$ and $|2\rangle_i$ $(i \in \{A,B\})$ and their respective local cavity fields, which are correlated by photonic quantum state transfer through the connecting optical fiber.

For this purpose we consider a Ramsey-type interaction scenario which starts from an initial state of the total matter-field system of the form

$$|\Psi(t=0)\rangle = \frac{|0\rangle_A + |1\rangle_A}{\sqrt{2}} \otimes \frac{|0\rangle_B + |1\rangle_B}{\sqrt{2}} \otimes |\alpha\rangle_A \otimes |0\rangle. \tag{1}$$

Thus, the two spatially well-separated material qubits are prepared in a particular separable state and the single mode of frequency ω inside cavity A is prepared in a coherent state $|\alpha\rangle$. All other field modes of the radiation field involved are prepared in the vacuum state. After this preparation in a first step the single-mode radiation field inside cavity A interacts with the three-level system A almost resonantly for a time τ which is assumed to be small in comparison with the decay time of this cavity mode, i.e., $\tau \ll 1/\Gamma$. We also assume that this interaction time is so small that spontaneous emission of photons from the excited state $|2\rangle_A$ is negligible. Such a short and almost resonant interaction between the material three-level system and the field mode inside cavity A can be turned on and turned off by Stark switching techniques, for example, by which the dipole transition between levels $|1\rangle_A$ and $|2\rangle_A$ is tuned with the help of an externally applied electric field. After this almost instantaneous interaction (on the time scale of the cavity decay) the resulting photonic quantum state inside cavity A is strongly entangled with the three-level system A and propagates to cavity B through the optical fiber. At time $T = L/c \gg 2/\Gamma \gg \tau$ the photonic quantum state produced by the almost instantaneous interaction inside cavity A has been transferred to the single mode of frequency ω in cavity B. In the second step of the Ramsey-type interaction scenario at time T an analogous second almost resonant interaction of the photon field with the second three-level system is turned on and off for the short time $\tau \ll 2/\Gamma \ll T$ inside cavity B. After these two almost instantaneous matter-field interactions the resulting photon state is measured by photon detectors.

In our subsequent discussion we are interested in cases in which the interaction times τ in cavities A and B are significantly shorter than the decay time of the cavities and the radiative lifetime of the excited states $|2\rangle_i$ $(i \in \{A,B\})$ so that effects of spontaneous decay from these levels during the interaction time can be neglected. In order to ensure that effects of spontaneous decay from these excited states $|2\rangle_i$ $(i \in \{A,B\})$ are negligible also during the long propagation of the photons from cavity A to cavity B, one may transfer the excitations of these levels coherently to radiatively stable hyperfine-split ground-state components, say, $|\tilde{2}\rangle_i$ $(i \in \{A,B\})$, with the help of two (possibly classical) π pulses applied immediately after the interaction of each three-level system with its local single-mode photon field. Thus, by replacing in our subsequent theoretical considerations the excited states $|2\rangle_i$ $(i \in \{A,B\})$ and their energy E_2 by their corresponding radiatively stable states $|\tilde{2}\rangle_i$ $(i \in \{A,B\})$ with corresponding energy $\tilde{E}_2$, effects of spontaneous emission can be neglected during all stages of this interaction scenario.

The dynamics of the short almost resonant interaction between the three-level system $j \in \{A,B\}$ and the occupied local field mode inside cavity j is described by the Hamiltonian

$$\hat{H}_j = \hat{H}_j^{(Q)} + \hbar\omega\hat{a}_j^\dagger\hat{a}_j + \hbar g\hat{a}_j|2\rangle_{jj}\langle 1| + \hbar g^*\hat{a}_j^\dagger|1\rangle_j\langle 2|, \tag{2}$$

with the unperturbed Hamiltonian $\hat{H}_j^{(Q)} = \sum_{k=0,1,2} E_k|k\rangle_{jj}\langle k|$ of the three-level system j. Thereby, the interaction between the local optical field modes and the material systems A and B is described in the dipole and rotating- wave approximation. It is characterized by the coupling parameters $g_j = -{}_j\langle 2|\hat{\mathbf{d}}|1\rangle_j\sqrt{\hbar\omega/(2\epsilon_0)}\mathbf{g}_j(\mathbf{x}_j)/\hbar$ $(j \in \{A,B\})$ which involve the transition dipole moments ${}_j\langle 2|\hat{\mathbf{d}}|1\rangle_j$ and the normalized mode functions of the single-mode radiation field $\mathbf{g}_j(\mathbf{x})$ in their respective cavities. In the following we concentrate on cases with symmetric couplings, i.e., $g = g_A = g_B$. The modulus of this characteristic coupling strength defines the resonant vacuum Rabi frequency $\Omega_{\text{vac}} = |g|$ [13]. The photonic annihilation (creation) operators of the cavity modes A and B are denoted by $\hat{a}_A$ and $\hat{a}_B$ ($\hat{a}_A^\dagger$ and $\hat{a}_B^\dagger$). with the corresponding Fock states $|n\rangle_A$ and $|n\rangle_B$.

It is straightforward to determine the quantum state of the two cavity modes and the two three-level systems at time $t = T$

after the Ramsey-type interaction sequence described above. If we assume that perfect quantum state transfer is achieved between the two cavities coupled by the optical fiber(see the Appendix) the pure state $|\Psi(t)\rangle$ of both material three-level systems and of the field mode inside cavity B is given by

$$
\begin{aligned}
|\Psi(t)\rangle = {} & |g_5(t)\rangle|1\rangle_A|2\rangle_B e^{-i\Phi_{12}} + |g_6(t)\rangle|2\rangle_A|2\rangle_B e^{-i\Phi_{22}} + \tfrac{1}{2}|0\rangle_A|0\rangle_B|\alpha e^{-i\omega t}\rangle e^{-i\Phi_{00}} + |g_1(t)\rangle|\Psi^+\rangle_{AB} e^{-i\Phi_{10}} \\
& + |g_2(t)\rangle(|0\rangle_A|2\rangle_B e^{-i\Phi_{02}} + |2\rangle_A|0\rangle_B e^{-i\Phi_{20}}) + |g_3(t)\rangle|1\rangle_A|1\rangle_B e^{-i\Phi_{11}} + |g_4(t)\rangle|2\rangle_A|1\rangle_B e^{-i\Phi_{21}}
\end{aligned} \tag{3}
$$

with the Bell state $|\Psi^+\rangle_{AB} = (|0\rangle_A|1\rangle_B + |0\rangle_A|1\rangle_B)/\sqrt{2}$ involving the two distant qubits of systems A and B. The time-dependent phases $\Phi_{00} = 2E_0t/\hbar$, $\Phi_{10} = (E_0 + E_1)t/\hbar + \Delta\tau/2$, $\Phi_{20} = (E_0 + E_2)t/\hbar - \Delta\tau/2$, $\Phi_{02} = (E_0 + E_1 + \hbar\omega)t/\hbar + \Delta\tau/2$, $\Phi_{11} = 2E_1t/\hbar + \Delta\tau$, $\Phi_{12} = (2E_1 + \hbar\omega)t/\hbar + \Delta\tau$, $\Phi_{21} = (E_1 + E_2)t/\hbar$, and $\Phi_{22} = (E_1 + E_2 + \hbar\omega)t/\hbar$ characterize the interferences appearing in this Ramsey-type interaction scenario. The detuning from resonance is denoted by $\Delta := (E_2 - E_1)/\hbar - \omega$. The (unnormalized) pure field states $|g_j(t)\rangle$ $(j = 1,\ldots,6)$ and $|\alpha e^{-i\omega t}\rangle$ entering Eq. (3) are defined by

$$
\begin{aligned}
|\alpha e^{-i\omega t}\rangle &= \sum_{n=0}^{\infty} e^{-|\alpha|^2/2}\frac{\alpha^n e^{-in\omega t}}{\sqrt{n!}}|n\rangle_B, \\
|g_1(t)\rangle &= \sum_{n=0}^{\infty} e^{-|\alpha|^2/2}\frac{\alpha^n}{\sqrt{n!}}\frac{1}{\sqrt{2}}\left(\cos[\Omega(n)\tau] + i\frac{\Delta}{2\Omega(n)}\sin[\Omega(n)\tau]\right)e^{-i\omega nt}|n\rangle_B, \\
|g_2(t)\rangle &= -\sum_{n=0}^{\infty} e^{-|\alpha|^2/2}\frac{\alpha^{(n+1)}}{\sqrt{(n+1)!}}\frac{ig\sqrt{n+1}}{\Omega(n+1)}\frac{1}{2}\sin[\Omega(n+1)\tau]e^{-i\omega nt}|n\rangle_B, \\
|g_3(t)\rangle &= \sum_{n=0}^{\infty} e^{-|\alpha|^2/2}\frac{\alpha^n}{\sqrt{n!}}\frac{1}{2}\left(\cos[\Omega(n)\tau] + i\frac{\Delta}{2\Omega(n)}\sin[\Omega(n)\tau]\right)^2 e^{-i\omega nt}|n\rangle_B, \\
|g_4(t)\rangle &= -\sum_{n=0}^{\infty} e^{-|\alpha|^2/2}\frac{\alpha^{(n+1)}}{\sqrt{(n+1)!}}\frac{ig\sqrt{n+1}}{\Omega(n+1)}\sin[\Omega(n+1)\tau]\frac{1}{2}\left(\cos[\Omega(n)\tau] + i\frac{\Delta}{2\Omega(n)}\sin[\Omega(n)\tau]\right)e^{-i\omega nt}|n\rangle_B, \\
|g_5(t)\rangle &= -\sum_{n=0}^{\infty} e^{-|\alpha|^2/2}\frac{\alpha^{(n+1)}}{\sqrt{(n+1)!}}\frac{ig\sqrt{n+1}}{\Omega(n+1)}\frac{\sin[\Omega(n+1)\tau]}{2}\left(\cos[\Omega(n+1)\tau] + i\frac{\Delta}{2\Omega(n+1)}\sin[\Omega(n+1)\tau]\right)e^{-i\omega nt}|n\rangle_B, \\
|g_6(t)\rangle &= -\sum_{n=0}^{\infty} e^{-|\alpha|^2/2}\frac{\alpha^{(n+2)}}{\sqrt{(n+2)!}}\frac{g\sqrt{n+1}}{\Omega(n+1)}\sin[\Omega(n+1)\tau]\frac{g\sqrt{n+2}}{\Omega(n+2)}\frac{1}{2}\sin[\Omega(n+2)\tau]e^{-i\omega nt}|n\rangle_B
\end{aligned} \tag{4}
$$

with the normalized photon-number states $|n\rangle_B$ $(n \in \mathbb{N}_0)$. The parameter $\Omega(n) := \sqrt{\Delta^2/4 + |g|^2 n}$ denotes the effective Rabi frequency associated with the photon number n of the optical radiation field in cavity B.

The quantum state of Eq. (3) yields a complete description of the interaction between the material quantum systems A and B and the optical radiation fields in the idealized case of perfect quantum state transfer between cavities A and B mediated by propagation through the connecting optical fiber. In particular, it clearly exhibits the resulting entanglement between the material systems A and B on the one hand and the radiation field inside cavity B on the other hand. In the weak-coupling limit of large detunings from resonance, this latter entanglement has been used in the proposal by van Loock *et al.* [5] for creating an entangled Bell state $|\Psi^+\rangle_{AB}$ by projecting out the field state $|g_1(t)\rangle$ by a generalized positive-operator-valued quantum measurement [14–16] performed on the optical radiation field. However, in the weak-coupling limit the field states $|g_i(t)\rangle$ $(i = 1,\ldots,6)$ are not orthogonal so that the field state $|g_1(t)\rangle$ cannot be distinguished perfectly from the other field states. This limits the achievable entanglement of the entangled state of the two material qubits significantly. In our subsequent section it will be demonstrated that in the strong-coupling limit of almost resonant quantum electrodynamical coupling these limitations can be circumvented and in certain dynamical regimes even perfect Bell states $|\Psi^+\rangle_{AB}$ can be prepared by suitable quantum measurements performed on the optical field inside cavity B.

B. Postselection of Bell states by minimum-error field measurements

In this section we investigate to what extent the Ramsey-type interaction scenario discussed in the previous section can be optimized in order to prepare a maximally entangled Bell state between the distant material quantum systems A and B by an appropriate minimum-error POVM measurement of the optical field in cavity B. This aspect is of particular interest for future realizations of quantum repeaters which require high-fidelity Bell pairs as a resource. It is demonstrated that in the ideal case of perfect state transfer of the optical radiation field through the fiber from cavity A to system B in the strong-coupling limit of almost resonant interaction between the local cavity fields and the material three-level systems high-fidelity Bell states of the material systems A and B can be prepared provided the relevant interaction times τ are controlled appropriately.

Let us start from the pure quantum state $|\Psi(t)\rangle$ of Eq. (3) which describes the entanglement of the matter-field system after the Ramsey-type interaction scenario with the single occupied field mode inside cavity B. The resulting reduced density operator of the field state $\hat{\rho}_F(t)$ in cavity B is obtained by tracing out the material degrees of freedom, i.e.,

$$\hat{\rho}_F(t) = \mathrm{Tr}_{AB}\{|\Psi(t)\rangle\langle\Psi(t)|\} = p\hat{\rho}_1 + (1-p)\hat{\rho}_2 \tag{5}$$

with the field states

$$\hat{\rho}_1 = \frac{|g_1(t)\rangle\langle g_1(t)|}{p},$$
$$\hat{\rho}_2 = \left(\frac{1}{4}|\alpha e^{-i\omega t}\rangle\langle\alpha e^{-i\omega t}| + 2|g_2(t)\rangle\langle g_2(t)| + \sum_{j=3}^{6}|g_j(t)\rangle\langle g_j(t)|\right)\Big/(1-p), \tag{6}$$

and with $p = \langle g_1(t)|g_1(t)\rangle$ denoting the *a priori* probability of the pure field state $|g_1(t)\rangle$. In general, the quantum states $\hat{\rho}_1$ and $\hat{\rho}_2$ are not orthogonal so that they cannot be distinguished by any quantum measurement perfectly [17–19].

Therefore, in order to optimize the postselection of a perfectly entangled Bell state $|\Psi^+\rangle_{AB}$ it is necessary to perform a POVM measurement on the optical radiation field with two possible measurement results m, say, $m \in \{1,0\}$. The first measurement result $m = 1$ indicates an optimal projection of the field state $\hat{\rho}_F(t)$ onto the pure field state $\hat{\rho}_1$ and the second measurement result $m = 0$ indicates an optimal projection of $\hat{\rho}_F(t)$ onto the mixed field state $\hat{\rho}_2$. Let us denote the positive operators associated with these two measurement results by $\hat{T}_1 \geqslant 0$ and $0 \leqslant \hat{T}_0 := \mathbf{I} - \hat{T}_1$. ($\mathbf{I}$ denotes the unit operator in the Hilbert space of the single-mode radiation field.) The positive operator $\hat{T}_1$ of this POVM $\{\hat{T}_1,\hat{T}_0\}$ has to be determined in such a way that for a given *a priori* probability p the error probability

$$E = p\mathrm{Tr}\{\hat{\rho}_1\hat{T}_0\} + (1-p)\mathrm{Tr}\{\hat{\rho}_2\hat{T}_1\} \tag{7}$$

is as small as possible. Diagonalizing the Hermitian operator $\hat{A} := p\hat{\rho}_1 - (1-p)\hat{\rho}_2$ according to $\hat{A} = \sum_\lambda \lambda|\lambda\rangle\langle\lambda|$ the solution of this optimization problem is given by the projection operator [14–16]

$$\hat{T}_1 = \sum_{\lambda\geqslant 0}|\lambda\rangle\langle\lambda| = \mathbf{I} - \hat{T}_0, \tag{8}$$

which projects onto the non-negative spectral components of the operator $\hat{A}$. The corresponding minimal error probability $E_{\min}$ of the optimal POVM measurement defined by Eq. (8) is determined by the trace norm distance between the two (unnormalized) components $p\hat{\rho}_1$ and $(1-p)\hat{\rho}_2$ of the quantum state $\hat{\rho}_F(t)$, i.e. [14,16],

$$E_{\min} = \tfrac{1}{2}[1 - ||p\hat{\rho}_1 - (1-p)\hat{\rho}_2||_1]. \tag{9}$$

The probability P_{Bell} with which this optimal POVM measurement of the optical radiation field prepares the distant material quantum systems A and B in the Bell state $|\Psi^+\rangle$ is thus given by

$$P_{\mathrm{Bell}} = p\mathrm{Tr}_{\mathrm{field}}\{\hat{\rho}_1\hat{T}_1\}. \tag{10}$$

From Eqs. (9) and (10) it is apparent that if the quantum states $\hat{\rho}_1$ and $\hat{\rho}_2$ were orthogonal the positive operator $\hat{T}_1$ of the POVM measurement would be a projection operator onto the support of the state $\hat{\rho}_1$ and the success probability P_{Bell} would be given by p. In addition, the minimal error probability $E_{\min}$ would vanish. However, the typical nonorthogonality of the field states $\hat{\rho}_1$ and $\hat{\rho}_2$ complicates matters and causes unavoidable errors even if minimum-error POVM measurements are performed.

With the optimal POVM measurement $\{\hat{T}_1,\hat{T}_0\}$ as defined by Eq. (8) we can associate a deterministic quantum operation [20] with the Kraus operators $\{\hat{U}_1\sqrt{\hat{T}_1},\hat{U}_0\sqrt{\mathbf{I}-\hat{T}_1}\}$ which allows us to determine the quantum state of the matter-field system associated with the two measurement results $m \in \{1,0\}$ of the optimal POVM. Thereby, the linear operators $\hat{U}_1$ and $\hat{U}_0$ are partial isometries between the ranges of $\sqrt{\hat{T}_1}$ and $\sqrt{\hat{T}_0}$ and the Hilbert space of the optical radiation field. After a successful POVM measurement the state of both material quantum systems A and B is given by

$$\hat{\rho}_{AB}(t) = \frac{\mathrm{Tr}_{\mathrm{fields}}\{|\Psi(t)\rangle\langle\Psi(t)|\hat{T}_1\}}{\mathrm{Tr}_{A,B,\mathrm{fields}}\{|\Psi\rangle\langle\Psi|\hat{T}_1\}}. \tag{11}$$

Note that this quantum state of systems A and B is independent of the transformation $\hat{U}_1$. Thus, the fidelity F_{opt} of an optimally prepared Bell pair which is postselected by a measurement result with value $m = 1$ (corresponding to the POVM operator $\hat{T}_1$) is given by

$$F_{\mathrm{opt}} = \sqrt{\langle\Psi^+|\hat{\rho}_{AB}(t)|\Psi^+\rangle}. \tag{12}$$

In order to obtain some insight into the dynamical parameter regimes in which this postselection procedure may yield high-fidelity Bell pairs, let us concentrate on the practically important case of large mean photon numbers, i.e., $\overline{n} = |\alpha|^2 \gg 1$, and on intermediate values of the interaction times τ of the optical radiation field with the quantum systems A and B so that in Eq. (3) the photon-number-dependent Rabi frequencies $\Omega(n)$ can be linearized around the mean photon number $\overline{n}$. This linearization is valid if the condition

$$\frac{1}{2}\left|\frac{d^2\Omega}{dn^2}(n)\right|_{n=\overline{n}}\tau(\Delta n)^2 = \frac{1}{8\overline{n}}\frac{|\overline{\Omega}_0|^4\tau}{|\overline{\Omega}|^3} \ll \pi \tag{13}$$

is fulfilled so that revival phenomena [13] can be neglected. Thereby, $\Delta n = |\alpha| = \sqrt{\overline{n}}$ denotes the photon-number uncertainty of the coherent state $|\alpha\rangle$ and $\overline{\Omega} = \sqrt{\Delta^2/4 + |g|^2\overline{n}}$ and $\overline{\Omega}_0 = |g|\sqrt{\overline{n}}$ are the effective Rabi frequency and the resonant Rabi frequency associated with the mean photon number $\overline{n}$. In this linearization the field states of Eq. (3) can be approximated by

$$|g_1\rangle = |\alpha e^{-i\omega t}e^{i\theta}\rangle e^{i\omega_c\tau}\frac{1+\Delta/(2\overline{\Omega})}{2\sqrt{2}} + |\alpha e^{-i\omega t}e^{-i\theta}\rangle e^{-i\omega_c\tau}\frac{1-\Delta/(2\overline{\Omega})}{2\sqrt{2}},$$
$$|g_2\rangle = -e^{i\varphi}\frac{\overline{\Omega}_0}{4\overline{\Omega}}(|\alpha e^{-i\omega t}e^{i\theta}\rangle e^{i\omega_c\tau} - |\alpha e^{-i\omega t}e^{-i\theta}\rangle e^{-i\omega_c\tau}),$$
$$|g_3\rangle = |\alpha e^{-i\omega t}e^{2i\theta}\rangle e^{2i\omega_c\tau}\frac{[1+\Delta/(2\overline{\Omega})]^2}{8} + |\alpha e^{-i\omega t}e^{-2i\theta}\rangle e^{-2i\omega_c\tau}\frac{[1-\Delta/(2\overline{\Omega})]^2}{8} + |\alpha e^{-i\omega t}\rangle\frac{1-[\Delta/(2\overline{\Omega})]^2}{4}, \tag{14}$$
$$|g_4\rangle = |g_5\rangle = -e^{i\varphi}\frac{\overline{\Omega}_0}{8\overline{\Omega}}\left[|\alpha e^{-i\omega t}e^{2i\theta}\rangle e^{2i\omega_c\tau}\left(1+\frac{\Delta}{2\overline{\Omega}}\right) - |\alpha e^{-i\omega t}e^{-2i\theta}\rangle e^{-2i\omega_c\tau}\left(1-\frac{\Delta}{2\overline{\Omega}}\right) - |\alpha e^{-i\omega t}\rangle\frac{\Delta}{\overline{\Omega}}\right],$$
$$|g_6\rangle = e^{2i\varphi}\frac{\overline{\Omega}_0^2}{8\overline{\Omega}^2}\left(|\alpha e^{-i\omega t}e^{2i\theta}\rangle e^{2i\omega_c\tau} + |\alpha e^{-i\omega t}e^{-2i\theta}\rangle e^{-2i\omega_c\tau} - 2|\alpha e^{-i\omega t}\rangle\right)$$

with $g = e^{i\varphi}|g|, \theta = \overline{\Omega}_0^2\tau/(2\overline{\Omega}\overline{n}) \ll 1$, and with the modified effective Rabi frequency $\omega_c = \overline{\Omega}[1 - \overline{\Omega}_0^2/(2\overline{\Omega}^2)]$. This linearization implies that the *a priori* probability p entering Eq. (5) can be approximated by

$$p = \frac{\Delta^2}{8\overline{\Omega}^2} + \left[\frac{1}{4} - \left(\frac{\Delta}{4\overline{\Omega}}\right)^2\right]\left\{1 + \cos(2\overline{\Omega}\tau)\exp\left\{-\left[\overline{\Omega}_0^2\tau/(\overline{\Omega}\sqrt{\overline{n}})\right]^2/2\right\}\right\}. \tag{15}$$

Furthermore, the overlaps between the coherent state $|g_1(t)\rangle$ and the states $|\alpha e^{i\omega t}\rangle$ and $|g_3(t)\rangle$ constituting significant parts of the quantum state $\hat{\rho}_2$ reduce to

$$\langle\alpha e^{-i\omega t}|g_1(t)\rangle = \frac{1}{\sqrt{2}}\exp\left\{-\left[\overline{\Omega}_0^2\tau/(\overline{\Omega}\sqrt{\overline{n}})\right]^2/2\right\}\left(\cos(\overline{\Omega}\tau) + i\frac{\Delta}{2\overline{\Omega}}\sin(\overline{\Omega}\tau)\right),$$
$$\langle g_3(t)|g_1(t)\rangle = e^{-[\overline{\Omega}_0^2\tau/(2\overline{\Omega}\sqrt{\overline{n}})]^2/2}\left(e^{-i\overline{\Omega}\tau}\frac{[1+\Delta/(2\overline{\Omega})]^3}{16\sqrt{2}} + e^{i\overline{\Omega}\tau}\frac{[1-\Delta/(2\overline{\Omega})]^3}{16\sqrt{2}}\right)$$
$$+ e^{-9[\overline{\Omega}_0^2\tau/(2\overline{\Omega}\sqrt{\overline{n}})]^2/2}\left(e^{-3i\overline{\Omega}\tau}\frac{[1+\Delta/(2\overline{\Omega})]^2[1-\Delta/(2\overline{\Omega})]}{16\sqrt{2}} + e^{3i\overline{\Omega}\tau}\frac{[1+\Delta/(2\overline{\Omega})][1-\Delta/(2\overline{\Omega})]^2}{16\sqrt{2}}\right)$$
$$+ e^{-[\overline{\Omega}_0^2\tau/(2\overline{\Omega}\sqrt{\overline{n}})]^2/2}\left(e^{i\overline{\Omega}\tau}\frac{[1+\Delta/(2\overline{\Omega})]\{1-[\Delta/(2\overline{\Omega})]^2\}}{8\sqrt{2}} + e^{-i\overline{\Omega}\tau}\frac{[1-\Delta/(2\overline{\Omega})]\{1-[\Delta/(2\overline{\Omega})]^2\}}{8\sqrt{2}}\right). \tag{16}$$

From Eq. (16) it is apparent that in the case of resonant excitation, i.e., $\Delta = 0$, these overlaps vanish at interaction times $\tau = (\pi + 2k\pi)/(2\overline{\Omega}_0)$ ($k \in \mathbb{N}_0$). Furthermore, at these particular interaction times also all other overlaps $\langle g_j(t)|g_1(t)\rangle$ with $j = 2,4,5,6$ vanish. Thus, in this linearization approximation the quantum states $\hat{\rho}_1$ and $\hat{\rho}_2$ are orthogonal at these interaction times so that they can be distinguished perfectly by a von Neumann measurement described by the projection operators $\{\hat{T}_1 = \hat{\rho}_1, \hat{T}_0 = \mathbf{I} - \hat{T}_1\}$. In this case the success probability reduces to $p = \{1 - \exp[-(\overline{\Omega}_0\tau/\sqrt{\overline{n}})^2/2]\}/4$ and approaches the value of $p = 1/4$ in the limit of sufficiently large interaction times of the order of the inverse vacuum Rabi frequency, i.e., $\tau \gg \sqrt{\overline{n}}/\overline{\Omega}_0 = 1/\Omega_{\rm vac}$. In addition, the error probability $E_{\min}$ of this optimal von Neumann measurement vanishes, and the fidelity $F_{\rm opt}$ of the prepared Bell state $|\Psi^+\rangle$ equals unity.

This resonant behavior at these particular interaction times is in marked contrast to the behavior at large detunings $|\Delta/2| \gg |\overline{\Omega}_0|$ at which we obtain $\overline{\Omega} \to |\Delta/2|$ so that the above-mentioned overlaps tend to the nonvanishing values $|\langle\alpha e^{-i\omega t}|g_1\rangle| = \exp\{-[\overline{\Omega}_0^2\tau/(\overline{\Omega}\sqrt{\overline{n}})]^2/2\}/\sqrt{2}$ and $|\langle g_3|g_1\rangle| = \exp\{-[\overline{\Omega}_0^2\tau/(2\overline{\Omega}\sqrt{\overline{n}})]^2/2\}/(8\sqrt{2})$. As a result, in this weak-coupling limit, it is only for extremely large interaction times, i.e., $\tau \geqslant |\Delta/\overline{\Omega}_0|/\Omega_{\rm vac} \gg 1/\Omega_{\rm vac}$, that these overlaps become small. However, at these interaction times, which are significantly larger than the inverse vacuum Rabi frequency, typically additional effects originating from spontaneous emission of photons also have to be taken into account which have been neglected in our analysis. Thus, apart from these extremely large interaction times, in the weak-coupling limit it is never possible to distinguish the relevant field states $\hat{\rho}_1$ and $\hat{\rho}_2$ perfectly so that the preparation of perfect Bell states $|\Psi^+\rangle$ is impossible.

Physically speaking, these marked differences between the strong- and the weak-coupling cases are due to the characteristic dephasing phenomena which are also responsible for the well-known collapse phenomena in the Jaynes-Cummings-Paul model [13]. For the case of interaction times τ characterized by the inequalities (13) these interference phenomena are captured quantitatively by the various slightly shifted coherent states entering Eq. (16). Although these coherent states of the form $|\alpha e^{-i\omega t}e^{i\theta k}\rangle$ with $\pm k = 0,1,2$ are shifted in their phases only slightly by multiples of the small amount $\theta = \overline{\Omega}_0^2\tau/(2\overline{\Omega}\overline{n}) \ll 1$, their overlaps are given by

$$|\langle\alpha e^{-i\omega t}e^{-ik\theta}|\alpha e^{-i\omega t}e^{-ik'\theta}\rangle| = \exp\left\{-\left[(k'-k)\overline{\Omega}_0^2\tau/(\overline{\Omega}\sqrt{\overline{n}})\right]^2/2\right\}. \tag{17}$$

This implies that for very small interaction times τ or large mean photon numbers $\overline{n}$, i.e., $\overline{\Omega}_0^2\tau/(\overline{\Omega}\sqrt{\overline{n}})/2 \ll 1$, the coherent states entering Eq. (16) are almost identical. Thus, in this limit the dynamics is well approximated by the semiclassical limit, i.e., $\overline{n} \to \infty$, in which the influence of the single-mode radiation field can be approximated well by a classical field and the influence of photon fluctuations is negligible. However, for intermediate interaction times of the order of $\overline{\Omega}_0^2\tau/(\overline{\Omega}\sqrt{\overline{n}})/2 \geqslant 1$, the overlaps of the coherent states entering Eq. (16) become small so that their interferences are suppressed significantly. This suppression of interference due to dephasing of these coherent states is also the reason for the appearance of the well-known collapse phenomena in the Jaynes-Cummings-Paul model. In particular, in the case of resonant excitation it implies that even for arbitrary interaction times of the order of $\tau \geqslant \sqrt{\overline{n}}/\overline{\Omega}_0 := 1/\Omega_{\text{vac}}$ the field states $\hat{\rho}_1$ and $\hat{\rho}_2$ are almost orthogonal so that they can be distinguished almost perfectly by an appropriate quantum measurement. Effects of spontaneous emissions of photons from the excited state $|2\rangle$ with a spontaneous decay rate Γ can still be neglected as long as the matter-field coupling as characterized by the vacuum Rabi frequency is sufficiently large so that $\Omega_{\text{vac}} \gg \Gamma$. Meeting these requirements is within reach of current experimental possibilities. The recent experiment by McKeever *et al.* [21], for example, was performed in the optical frequency regime and was characterized by the parameters $\Omega_{\text{vac}}/(2\pi) = 16$ MHz and $\Gamma/(2\pi) = 2.6$ MHz. More recent experiments have even reported vacuum Rabi frequencies $\Omega_{\text{vac}}/(2\pi)$ exceeding 20 GHz [12]. However, in cases of large detunings, i.e., $|\Delta/2| \gg \overline{\Omega}_0$, according to Eq. (16) strong dephasing requires significantly larger interaction times of the order of $\tau \geqslant |\Delta/\overline{\Omega}_0|/\Omega_{\text{cav}} \gg 1/\Omega_{\text{vac}}$, for which typically the influence of spontaneous decay processes can no longer be neglected.

In Figs. 2, 3, and 4 numerical results are presented for characteristic quantitative measures which exhibit to what extent postselection by a minimum-error POVM measurement on the optical radiation field is capable of preparing a Bell state $|\Psi^+\rangle_{AB}$. These numerical results are based on the exact quantum state of Eq. (3) from which the minimum-error POVM measurement is determined according to Eq. (8). This optimum minimum-error POVM depends on the characteristic electrodynamical interaction parameters involved, namely, the interaction time τ, the mean photon number $\overline{n}$, the detuning from resonance Δ, and the strength of the quantum electrodynamical coupling as measured by the resonant mean Rabi frequency $\overline{\Omega}_0$. In all these figures the mean photon number of the initially prepared coherent field state $|\alpha\rangle$ is given by $\overline{n} = |\alpha|^2 = 100$, so that typical quantum electrodynamical effects originating from photon-number fluctuations as measured by $\Delta n = \sqrt{\overline{n}} = 10$ are still apparent. In particular, this implies that deviations from the previously discussed analytical predictions of the linearization approximation are still observable.

Characteristic features of the strong-coupling regime, i.e., $\Delta = 0$, are depicted in Fig. 2 as a function of the dimensionless parameter $\overline{\Omega}\tau/(2\pi)$ which involves the interaction time τ and the effective mean Rabi frequency $\overline{\Omega}$. Consistent with the approximate analytical expression for the *a priori* probability p as given by Eq. (15) for shorter interaction times τ the success probability P_{Bell} of Eq. (10) exhibits maxima at integer multiples of the interaction time $\tau = \pi/\overline{\Omega}$ (Fig. 2, top). These maxima correspond to multiples of Rabi cycles at which the material three-level systems are found again in their initially prepared states $|0\rangle$ and $|1\rangle$ which constitute the qubits to be entangled. Correspondingly. also minima appear at odd integer multiples of the interaction time $\tau = (\pi/2)/\overline{\Omega}$ at which the three-level systems involved populate

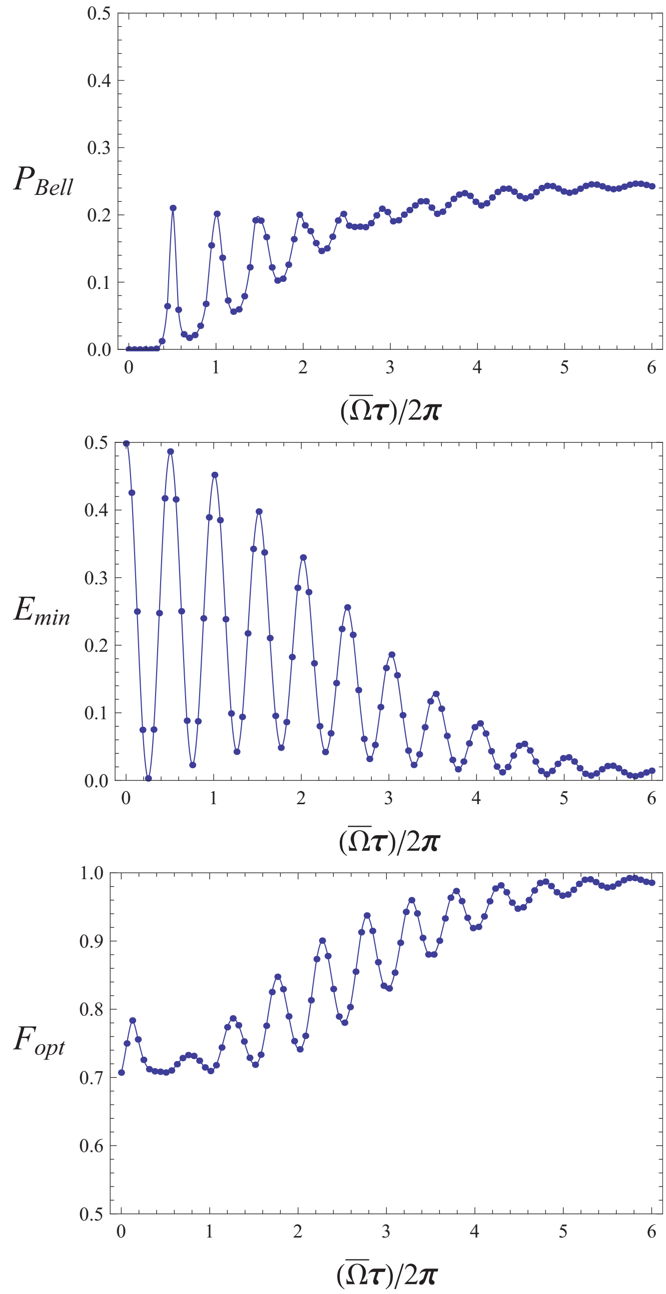

FIG. 2. (Color online) Dependence of characteristic quantities on the interaction time τ in the strong-coupling limit $\Delta = 0$: Top: The success probability for postselecting a Bell state $|\Psi^+\rangle_{AB}$ of Eq. (10); middle: the minimum-error probability of the POVM measurement of the field of Eq. (9); bottom: the fidelity of the postselected Bell state $|\Psi^+\rangle_{AB}$ of Eq. (12).

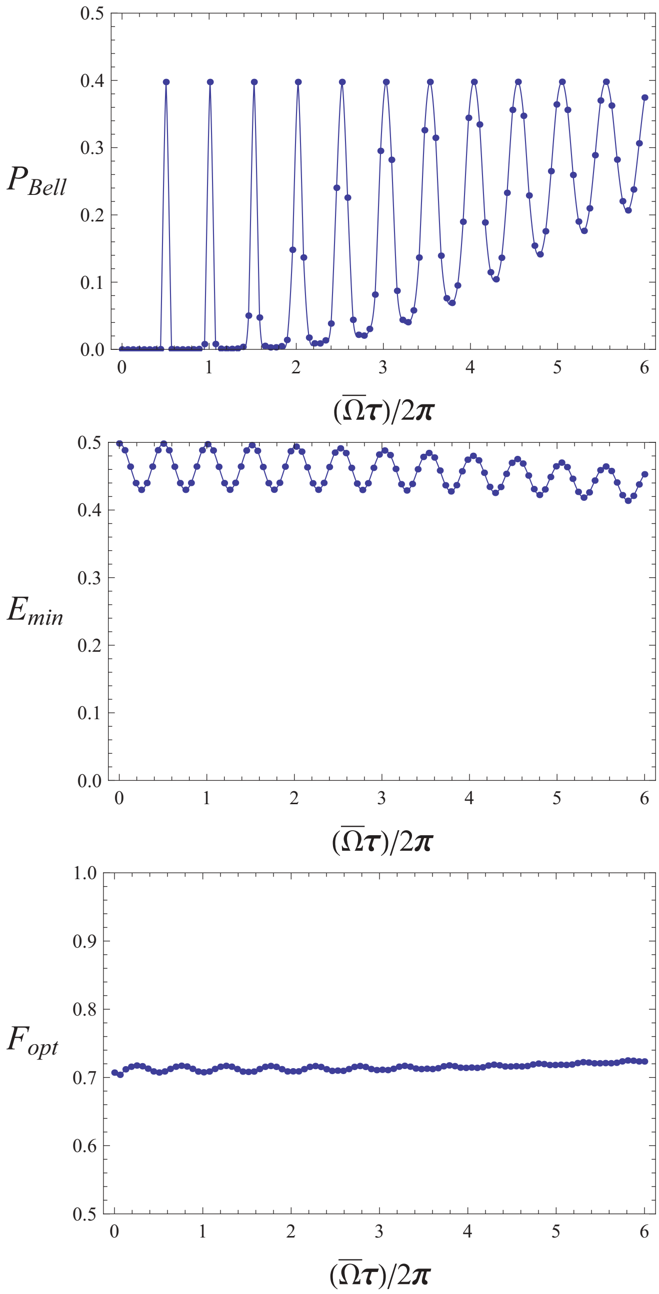

FIG. 3. (Color online) Dependence of characteristic quantities on the interaction time τ in the weak-coupling limit $\Delta = 5\overline{\Omega}_0$: Top: The success probability for postselecting a Bell state $|\Psi^+\rangle_{AB}$ of Eq. (10); middle: the minimum-error probability of the POVM measurement of the field of Eq. (9); bottom: the fidelity of the postselected Bell state $|\Psi^+\rangle_{AB}$ of Eq. (12).

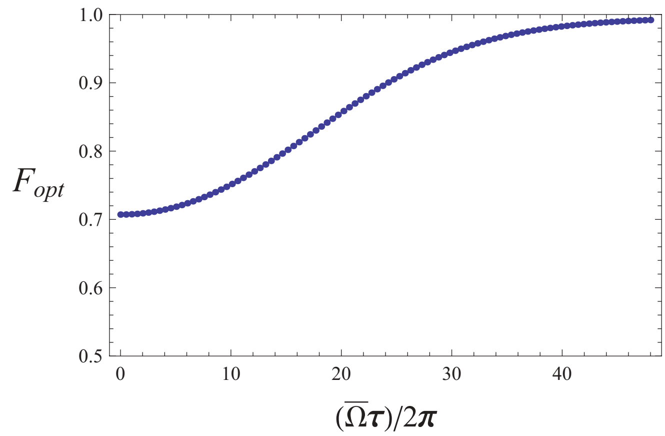

FIG. 4. (Color online) Fidelity of the postselected Bell state $|\Psi^+\rangle_{AB}$ of Eq. (12) in the weak-coupling limit $\Delta = 5\overline{\Omega}_0$ for long interaction times: The effects of dephasing lead to an asymptotic increase to unity.

the excited state $|2\rangle$. However, for larger interaction times these maxima and minima become less pronounced. This reflects the influence of the dephasing originating from the dependence of the Rabi frequencies $\Omega(n)$ on the photon number n in Eq. (3), which is also responsible for the collapse phenomena in the Jaynes-Cummings-Paul model [13]. In the limit of large interaction times, i.e., $\overline{\Omega}\tau \geqslant \sqrt{\overline{n}}$ the success probability approaches the value $1/4$ consistent with Eq. (15). The minimum-error probability of Eq. (9) always exhibits maxima at completed Rabi cycles, i.e., at integer multiples of the interaction time $\tau = \pi/\overline{\Omega}$ (Fig. 2, middle). Thus, at these interaction times it is difficult to distinguish the field states $\hat{\rho}_1$ and $\hat{\rho}_2$ even by a minimum-error POVM measurement. This is due to the fact that in view of the periodic Rabi oscillations at these interaction times the atom-field state is similar to the initially prepared quantum state of Eq. (1) which is characterized by the property that the field states $\hat{\rho}_1$ and $\hat{\rho}_2$ are identical and thus indistinguishable. This is also the reason for the vanishing success probability at $\tau = 0$. However, due to dephasing the difference between minima and maxima of the minimum-error probability eventually vanishes in the limit of sufficiently large interaction times. Similarly, also the fidelity of a Bell pair which is postselected by such a minimum-error POVM field measurement exhibits periodic oscillations with the Rabi frequency $\overline{\Omega}$ (Fig. 2, bottom). The maxima of these oscillations appear at odd integer multiples of the interaction time $\tau = (\pi/2)/\overline{\Omega}$. At these interaction times the three-level systems are likely to be found in the excited states $|2\rangle$ so that we expect small success probabilities at these interaction times. However, due to dephasing and the related collapse phenomena at sufficiently long interaction times, the differences between minima and maxima of the fidelity eventually tend to zero and the achievable fidelity approaches its maximum possible value of unity. These sufficiently long interaction times of the order of $\tau \geqslant \sqrt{\overline{n}}/\overline{\Omega}_0 = 1/\Omega_{\text{vac}}$ are therefore most favorable for preparing a high-fidelity material Bell pair in the Bell state $|\Psi^+\rangle_{AB}$ provided spontaneous emission of photons into other modes of the radiation field is negligible. Recent quantum electrodynamical experiments performed in the strong-coupling regime [21] demonstrate that such large vacuum Rabi frequencies are within current experimental possibilities.

This strong-coupling behavior is in marked contrast to the dependence of these characteristic quantities on the interaction time in the weak-coupling limit in which the detuning is large, i.e., $|\Delta| \gg \overline{\Omega}_0$. This case is depicted in Figs. 3 and 4. From the interaction times shown in Fig. 3 characteristic oscillations of these quantities with the mean Rabi frequency $\overline{\Omega} \to |\Delta/2|$ are apparent. They originate from the instantaneous turning on and off of the interactions between the optical radiation field and the quantum systems A and B. Although at their maxima the success probabilities $P_{\rm Bell}$ are slightly larger than in the resonant case of Fig. 2, the fidelity of the postselected Bell pairs is significantly smaller and oscillates slightly around the value of $F_{\rm opt} = 1/\sqrt{2}$. In view of the large detuning considered, the effects of dephasing are negligible for the interaction times depicted in Fig. 3. The effects of dephasing become important only in the limit of extremely long interaction times of the order of $\tau \geqslant |\Delta/\overline{\Omega}_0|/\Omega_{\rm cav} \gg 1/\Omega_{\rm vac}$. The resulting collapse phenomena cause an increase of the achievable fidelities so that asymptotically they approach the maximum possible value of unity (compare with Fig. 4). However, typically at these extremely long interaction times the influence of spontaneous emission of photons into other modes of the radiation field is no longer negligible, and our theoretical model is no longer adequate for describing such cases. Therefore, for the preparation of high-fidelity Bell states by postselection the weak-coupling regime exhibits clear limitations even if the postselection is performed by minimum-error POVM measurements.

III. EFFECTS OF PHOTON LOSS

In this section we investigate additional effects of photon loss during the propagation of the optical radiation field through the optical fiber by modeling the propagation-induced photon loss by the dynamics of damped harmonic oscillators with equal decay rates. This may describe a physical situation, for example, in which only a single transverse but many longitudinal modes of the radiation field are excited in the optical fiber during the photonic propagation process and in which the photon loss is due to leakage of this transversal mode out of the optical fiber. It is demonstrated that in the strong-coupling regime of resonant interactions, besides an overall decay of success probabilities and achievable fidelities of the postselected Bell pairs, characteristic interference oscillations also appear which originate from dephasing.

Let us consider again the quantum electrodynamical model of Sec. II A with the only difference that during the propagation of the optical radiation field through the fiber the dynamics is described by damped harmonic oscillators. Thus, during the time interval $T - 2\tau$ (with $\tau \ll 2/\Gamma \ll T$) of the propagation through the optical fiber the free dynamics of the optical radiation field previously described by the Hamiltonian $\sum_{i\in L} \hbar\omega_i \hat{a}_i^\dagger \hat{a}_i$ (compare with the Appendix) is replaced by the Lindbladian master equation

$$\frac{d\hat{\rho}}{dt} = -i\sum_{i\in L}\omega_i[\hat{a}_i^\dagger\hat{a}_i,\hat{\rho}] + \sum_{i\in L}([\hat{L}_i,\hat{\rho}\hat{L}_i^\dagger] + [\hat{L}_i\hat{\rho},\hat{L}_i^\dagger]) = \mathcal{L}\hat{\rho} \qquad (18)$$

for the field state $\hat{\rho}(t)$ with the Lindblad operators $\hat{L}_i = \sqrt{\gamma/2}\hat{a}_i$. Thereby, the damping rate γ characterizes the photon loss in the optical fiber.

Expanding the density operator of a particular mode, say i, into photon-number states, i.e.,

$$\hat{\rho}^{(i)}(t) = \sum_{n,m=0}^{\infty} \rho_{n,m}^{(i)}(t)\exp[-i\omega_i(n-m)t]|n\rangle_{ii}\langle m|, \qquad (19)$$

and taking matrix elements of the density operator equation (18) we obtain the result

$$\dot{\rho}_{n,m}^{(i)} = -\frac{\gamma}{2}(m+n)\rho_{n,m}^{(i)} + \gamma\sqrt{(n+1)(m+1)}\rho_{n+1,m+1}^{(i)}. \qquad (20)$$

A general solution of this equation can be obtained with the help of the Laplace transformation

$$\hat{\rho}^{(i)}(z) := \int_0^\infty \hat{\rho}^{(i)}(t)e^{-zt}\,dt \qquad (21)$$

which transforms the master equation with the initial condition $\hat{\rho}^{(i)}(t=0)$ into an algebraic equation for $\hat{\rho}^{(i)}(z)$. Inverting its solution with the help of the inverse relation

$$\hat{\rho}^{(i)}(t) = \frac{1}{2\pi i}\int_{\mathcal{C}} e^{zt}\hat{\rho}^{(i)}(z)\,dz, \qquad (22)$$

we obtain the corresponding solution $\hat{\rho}^{(i)}(t)$ for the field state of mode i at time t. Thereby, the path of integration $\mathcal{C}$ has to be chosen in such a way that all poles of $\hat{\rho}^{(i)}(z)$ are included.

In the photon-number-state representation the time-dependent solution of the master equation (18) can be determined by induction, thus yielding the result

$$\rho_{n,m}^{(i)}(t) = e^{-\gamma(m+n)t/2}\sum_{j=0}^{\infty}\rho_{n+j,m+j}^{(i)}(t=0) \times \frac{\sqrt{(n+j)!}\sqrt{(m+j)!}}{\sqrt{n!}\sqrt{m!}}\frac{(1-e^{-\gamma t})^j}{j!} \qquad (23)$$

for mode i. With the help of this solution it is straightforward to propagate field coherences of the form $|n\rangle_{ii}\langle m|$ of any mode i from an initial time immediately after the excitation by the optical cavity field A to the time after completion of the propagation through the optical fiber. Thus, an initial coherence between coherent states, such as $|\beta\rangle_{ii}\langle\alpha|$, for example, evolves to a coherence of the form

$$e^{\mathcal{L}T}|\beta\rangle_{ii}\langle\alpha| = e^{-[1-\exp(-\gamma T)](\frac{|\alpha|^2+|\beta|^2}{2}-\beta\alpha^*)}|\beta e^{-\gamma T/2}\rangle_{ii}\langle\alpha e^{-\gamma T/2}|. \qquad (24)$$

In Fig. 5 numerical results are presented which demonstrate characteristic properties of the postselection of a Bell state $|\Psi^+\rangle_{AB}$ by a minimum-error POVM measurement of the optical radiation field in the presence of photon loss during propagation through the optical fiber. Thereby, the quantum state resulting from all interactions between the optical field and the quantum systems A and B has been evaluated numerically with the help of relation (23). Apart from the photon loss during the propagation through the optical fiber and the choice of a fixed interaction time the parameters are the same as in Fig. 2. The interaction time $\tau = (23/4)2\pi/\overline{\Omega}_0$ has been chosen in such a way that in the absence of photon loss the fidelity of creating a Bell state has a maximum

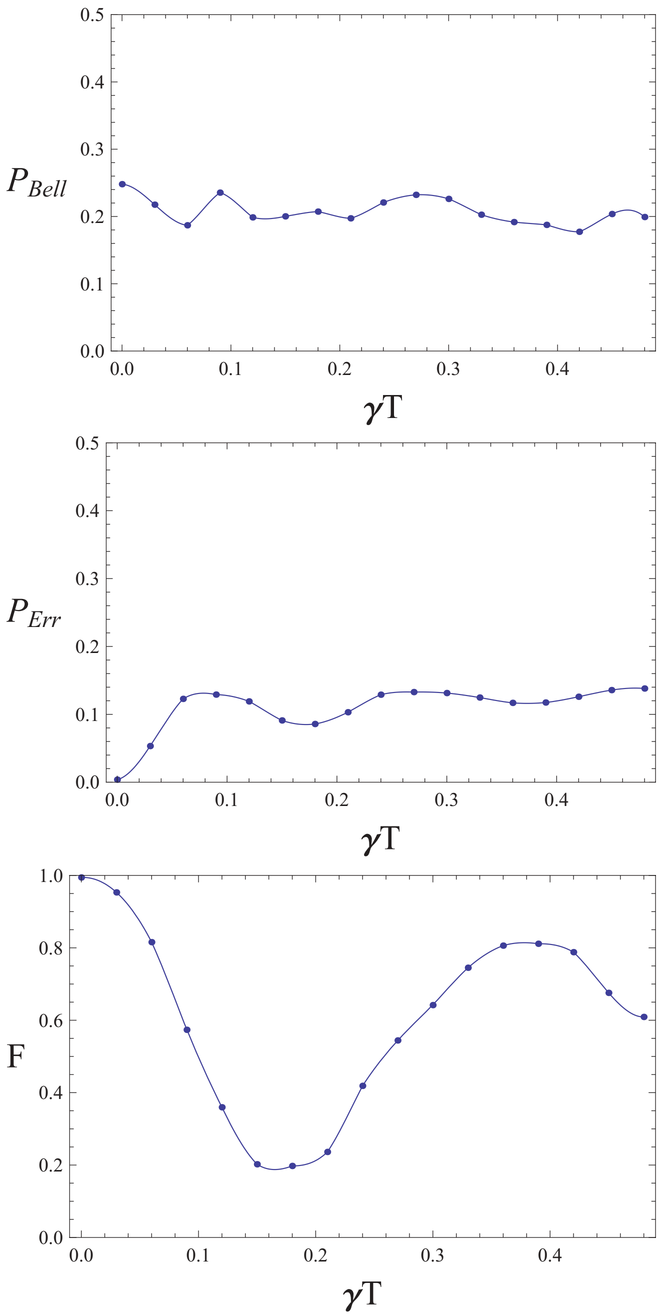

FIG. 5. (Color online) Dependence of characteristic quantities on the propagation time T (in units of $1/\gamma$) through a lossy optical fiber in the strong-coupling limit $\Delta = 0$ for an interaction time $\tau = (23/4)2\pi/\overline{\Omega}_0$: Top: The success probability for postselecting a Bell state $|\Psi^+\rangle_{AB}$ of Eq. (10); middle: the minimum-error probability of the POVM measurement of the field of Eq. (9); bottom: the fidelity of the postselected Bell state $|\Psi^+\rangle_{AB}$ of Eq. (12).

and that its value is close to unity (compare with Fig. 2). Figure 5 depicts the dependence of the characteristic quantities $P_{\rm Bell}$, $E_{\rm min}$, and $F_{\rm opt}$ of this minimum-error postselection procedure on the time T of propagation through the fiber. It is apparent that photon loss tends to decrease the success probability $P_{\rm Bell}$ and the fidelity $F_{\rm opt}$ of the postselected Bell pair. At the same time it also increases the minimum error $E_{\rm min}$. For an optical fiber with an intensity loss of D dB/m propagation durations T can be translated into lengths L of the optical fiber by the relation $L = (\gamma T)20/(D \ln 10)$. Thus, at a wavelength of 1550 nm with a photon loss of 0.2dB/km, for example, the maximum value of $\gamma T = 0.3$ depicted in Fig. 5 corresponds to a fiber length of $L = 13\,029$ m. It is interesting to note that all the characteristic quantities depicted in Fig. 5 exhibit also an oscillatory behavior. It can be traced back to the fact that according to Eq. (24) for small photon loss, i.e., $\gamma T \ll 1$, all relevant field coherences between coherent states also involve a characteristic frequency $\tilde{\omega} = \gamma \mathrm{Im}(\alpha^* \beta)$. According to Eq. (14) due to dephasing typical relevant coherent states are of the form $|\alpha e^{ik\theta}\rangle$ with $k = \pm 1, \pm 2, \ldots$. Therefore, an estimate of these characteristic frequencies can be obtained by assuming that $\alpha = \sqrt{\overline{n}} e^{i\theta}$ and $\beta = \sqrt{\overline{n}}$, for example, which yields an oscillation frequency of the order of $\tilde{\omega} = \gamma \overline{n}[\overline{\Omega}_0 \tau/(2\overline{n})] = \overline{\Omega}_0 \gamma \tau/2$ for resonant coupling, i.e., $\Delta = 0$.

IV. SUMMARY AND CONCLUSION

In the context of the hybrid quantum repeater model, we have studied optimal possibilities of preparing high-fidelity Bell pairs of two material qubits by postselection with the help of two single-mode cavities and an optical fiber connecting them. Whereas the original proposal of van Loock *et al.* [5] concentrated on the dynamical regime of weak coupling in which the interaction between the optical radiation field and the two material quantum systems involved can be described perturbatively, our discussion concentrated on the strong-coupling regime of almost resonant interaction. We also discussed the problem of photonic quantum state transfer through an optical fiber and determined by what choices of the coupling parameters between cavities and optical fiber perfect photonic quantum state transfer is possible. We determined the optimum POVM measurements which have to be performed on the single-mode optical radiation field of cavity B in order to postselect a Bell pair with minimum error. On the basis of this analysis we demonstrated that some of the limitations of the nonresonant weak-coupling limit can be circumvented successfully in the strong-coupling regime. In particular, provided the propagation of the optical radiation field through the fiber is ideal it is possible to create Bell pairs of fidelities arbitrarily close to unity provided the interaction time between the material quantum systems and the optical field is chosen properly. This is due to the fact that at these particular interaction times the quantum states of the optical radiation field which are entangled with the desired two-qubit Bell state and with the other material quantum states involved are almost orthogonal so that they can be distinguished almost perfectly by a von Neumann measurement. According to Eq. (14) this von Neumann measurement involves an approximate projection onto a cat-state-like superposition of two coherent states. This is in marked contrast to the weak-coupling regime of nonresonant interaction where the relevant field states are always nonorthogonal, so that they cannot be distinguished perfectly by any quantum measurement and the resulting postselection is never perfect. Furthermore, due to photon-induced dephasing effects which are characteristic for the collapse phenomena of the Jaynes-Cummings-Paul model at these interaction times, the success probabilities of the strong-coupling regime tend to the limiting value of 1/4 and are not significantly smaller than the corresponding values (of the order of 0.4) achievable in the weak-coupling regime. We have also explored effects originating from photon loss taking place during the propagation of the optical radiation field through the optical fiber. In addition to an overall decrease of achievable fidelities and success probabilities, in the strong-coupling regime also an oscillatory behavior is apparent. Thus, the strong-coupling regime of the hybrid quantum repeater model offers interesting perspectives for entangling material quantum systems over not too long distances and thus for providing high-fidelity Bell pairs at high repetition rates for future realizations of quantum repeaters.

ACKNOWLEDGMENTS

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APPENDIX: PERFECT QUANTUM STATE TRANSFER BETWEEN TWO SINGLE-MODE CAVITIES COUPLED BY AN OPTICAL FIBER

In this Appendix it is demonstrated that within the validity of the rotating-wave and pole approximations perfect quantum state transfer between two single modes of two spatially well-separated cavities connected by a long optical fiber is possible provided the coupling between the cavities and the optical fiber is engineered appropriately. Explicit expressions for these optimal couplings are determined.

In the following we consider a quantum state transfer scenario in which a single mode of cavity A with frequency ω is coupled almost resonantly to a dense set of modes of an optical fiber. Initially, a photonic quantum state is prepared in this single mode of cavity A and all other modes of the fiber and of the second distant cavity B are prepared in their vacuum states. After having left cavity A the photons propagate through the fiber so that cavity A is left in its vacuum state (apart from exponentially small terms). After this decay process a photonic wave packet propagates through the optical fiber whose spatial extension is of the order of c/Γ_A, with c denoting the speed of photonic propagation in the fiber and with Γ_A denoting the resonant decay rate of cavity A into the fiber. In particular, we assume that the length L of the optical fiber is large, i.e., $L \gg c/\Gamma_A$. Thus, during the propagation of the photon wave packet through the optical fiber the coupling of the fiber modes to the second field mode of frequency ω which is localized in the second distant cavity B is turned on adiabatically. In the following it is shown that the photonic wave packet can leak from the optical fiber into cavity B almost perfectly provided the couplings between both cavities and the optical fiber are engineered appropriately.

In the rotating-wave approximation the almost resonant coupling of the single mode of frequency ω in cavity A to the optical fiber modes is described by the Hamiltonian

$$\hat{H} = \hbar\omega\hat{a}_A^\dagger\hat{a}_A + \sum_{i\in L}\hbar\omega_i\hat{a}_i^\dagger\hat{a}_i + \sum_{i\in L}(\kappa_i\hat{a}_i^\dagger\hat{a}_A + \kappa_i^*\hat{a}_A^\dagger\hat{a}_i)$$
$$= \sum_{k,j\in I}\hat{a}_k^\dagger H_{kj}\hat{a}_j \quad \text{(A1)}$$

with L denoting the set of modes of the optical fiber and with $I = L \cup \{A\}$. As we want to transfer a photonic quantum state through a long optical fiber we assume in our subsequent discussion that only one transverse mode but a large number of longitudinal modes of the optical fiber couple to the optical cavity A almost resonantly in the frequency band $(-\Delta + \omega, \omega + \Delta)$. The rotating-wave approximation is valid if $\Delta \ll \omega$. In a simple approximation one may assume that in this frequency band the frequencies of the almost resonantly coupled modes of the optical fiber can be described by the relation $\omega_n = 2\pi cn/l$ with integer values of n and with l denoting the length of the optical fiber. (c is the speed of light inside the optical fiber.) Accordingly, in terms of the bare modes the matrix representation of the Hamiltonian of Eq. (A1) is given by

$$\hat{H} \longrightarrow \begin{pmatrix} \hbar\omega & \kappa_1^* & \kappa_2^* & . & . \\ \kappa_1 & \hbar\omega_1 & 0 & . & . \\ \kappa_2 & 0 & \hbar\omega_2 & . & . \\ . & . & . & . & . \\ . & . & . & . & . \end{pmatrix}. \quad \text{(A2)}$$

This Hamiltonian can be diagonalized by a unitary transformation $\hat{U}$ which describes the transformation to normal coordinates or to dressed modes, i.e.,

$$\hat{H} = \hat{U}\Lambda\hat{U}^\dagger, \quad \Lambda \longrightarrow \begin{pmatrix} \hbar\lambda_0 & 0 & 0 & . & . \\ 0 & \hbar\lambda_1 & 0 & . & . \\ 0 & 0 & \hbar\lambda_2 & . & . \\ . & . & . & . & . \\ . & . & . & . & . \end{pmatrix} \quad \text{(A3)}$$

with the dressed eigenfrequencies $\{\lambda_j\}$ $(j \in \mathbb{N}_0)$ and the destruction and creation operators of the dressed eigenmodes $\hat{a}'_j = \sum_{i\in I}\hat{U}^\dagger_{ji}\hat{a}_i$ and $\hat{a}'^\dagger_j = \sum_{i\in I}U_{ji}\hat{a}_i^\dagger$, respectively. Thus, in these dressed modes the Hamiltonian is diagonal, i.e.,

$$\hat{H} = \sum_{j\in\mathbb{N}_0}\hbar\lambda_j\hat{a}'^\dagger_j\hat{a}'_j. \quad \text{(A4)}$$

For $i \in L$ and $j \in \mathbb{N}_0$ the matrix elements of the diagonalizing unitary transformation $\hat{U}$ are given by

$$U_{ij} = -\frac{\kappa_i}{\hbar\omega_i - \hbar\lambda_j}U_{Aj} := U_i(\lambda_j),$$
$$U_{Aj} = \frac{1}{\sqrt{1 + \sum_{n\in L}\frac{|\kappa_n|^2}{(\hbar\omega_n - \hbar\lambda_j)^2}}} := U_A(\lambda_j) \quad \text{(A5)}$$

with the dressed frequencies $\{\lambda_j\}$ being determined by the zeros of the quantization function $f(\lambda)$, i.e.,

$$f(\lambda_j) := \hbar\omega - \hbar\lambda_j - \sum_{n\in L}\frac{|\kappa_n|^2}{\hbar\omega_n - \hbar\lambda_j} = 0. \quad \text{(A6)}$$

With the help of this transformation to normal coordinates the time evolution of any coherent initial state $|\Psi(t=0)\rangle = |\{\alpha_i\}\rangle = |\alpha_0\rangle_A \otimes |\alpha_1\rangle_1 \otimes |\alpha_2\rangle_2 \cdots$. of the bare modes with $\hat{a}_i|\beta\rangle_i = \beta|\beta\rangle_i$ and $i \in I$ can be determined easily because for $t \geqslant 0$

$$e^{-it\hat{H}/\hbar}|\{\alpha_i\}\rangle = e^{-\sum_{i\in I}|\alpha_i|^2/2}e^{\sum_{i\in I}\hat{a}_i^\dagger\alpha_i(t)}|0\rangle \equiv |\{\alpha_i(t)\}\rangle \quad \text{(A7)}$$

with the vacuum state of all modes $|0\rangle$ and with $(i \in I)$

$$\alpha_i(t) = \sum_{k\in\mathbb{N}_0, j\in I} U_{ik} e^{-i\lambda_k t} U^*_{jk}\alpha_j. \tag{A8}$$

Thereby, we have taken into account the identity

$$\sum_{j\in I} |\alpha_j(t)|^2 = \sum_{j\in I} |\alpha_j|^2, \tag{A9}$$

which is a consequence of the unitary time evolution. The time evolution of $\alpha_i(t)$ $(i \in I)$ as given by Eq. (A7) can also be determined by complex integration, i.e.,

$$\alpha_i(t) = \frac{-1}{2\pi i}\int_{-\infty+i0}^{\infty+i0} d\lambda U_i(\lambda)\frac{df/d\lambda(\lambda)}{f(\lambda)}\sum_{m\in I}\alpha_m U^*_m(\lambda)e^{-i\lambda t}. \tag{A10}$$

Evaluating Eq. (A10) with the help of the calculus of residues one arrives at Eq. (A7).

In the continuum limit of a very long optical fiber, in which the bare mode frequencies $\{\omega_i,\ i \in I\}$ are so densely spaced that they can be described by a continuum, the quantization function of Eq. (A6) can be approximated by

$$f(\lambda + i0) = \hbar(\omega - \delta\omega) - \hbar\lambda - i\frac{\hbar\Gamma_A}{2} \tag{A11}$$

with

$$\Gamma_A = \frac{2\pi}{\hbar}|\kappa_n|^2\frac{dn}{d\hbar\omega_n}\bigg|_{\omega_n=\omega}. \tag{A12}$$

This approximation is an immediate consequence of the relation

$$\sum_{n\in L}\frac{|\kappa_n|^2}{\hbar\omega_n - \hbar\lambda - i0} = \hbar\delta\omega + i\frac{\hbar\Gamma_A}{2} \tag{A13}$$

and of the Weisskopf-Wigner or pole approximation [22]. This latter approximation assumes that the interaction-induced frequency shift $\delta\omega$ as well as the decay rate Γ_A are slowly varying functions of λ so that they can be replaced by their values at $\lambda = \omega$ and can be considered as being λ independent. For its validity it is necessary that both the frequency shift $\delta\omega$ and the decay rate Γ_A are small in comparison with the cavity frequency ω, i.e., $|\delta\omega|,\ \Gamma_A \ll \Delta \ll \omega$. In the following we absorb the frequency shift $\delta\omega$ in a renormalized cavity frequency, i.e., $\omega - \delta\omega \to \omega$. The λ-independent decay rate Γ_A describes the loss of photons from cavity A due to the coupling of the cavity mode with frequency ω to the optical fiber.

From Eqs. (A10) and (A11) the solution of the Schrödinger equation with Hamiltonian (A1) and with initial condition

$$|\psi(t=0)\rangle = |\alpha\rangle_A \prod_{i\in L}|0\rangle_i \tag{A14}$$

is given by $|\psi(t)\rangle = |\alpha(t)\rangle_A \prod_{i\in L}|\alpha_i(t)\rangle_i$ with

$$\begin{aligned}\alpha(t) &= \alpha e^{-i\omega t - \Gamma_A t/2},\\ \alpha_i(t) &= \frac{\alpha\kappa_i}{\hbar\omega_i - \hbar\omega + i\hbar\Gamma_A/2}(e^{-i\omega_i t} - e^{-i\omega t - \Gamma_A t/2}).\end{aligned} \tag{A15}$$

Thus, for a sufficiently long interaction time, i.e., $T_1 \gg 1/\Gamma_A$, apart from exponentially small terms of the order of $\exp(-\Gamma_A T_1/2)$ the depletion of the cavity mode is perfect and $|\psi(t)\rangle$ describes a pure quantum state in which the cavity mode A is in its vacuum state and a photonic wave packet propagates through the optical fiber.

Let us now consider a time T_1 with $T_1 \gg 1/\Gamma_A$ at which the occupied modes of the radiation field are described by the quantum state $\prod_{i\in L}|\alpha_i(T_1)\rangle_i$ and at which cavity A is approximately in its vacuum state. This quantum state describes a photonic wave packet in the optical fiber with a spatial extension of the order of c/Γ_A. If the optical fiber is sufficiently large, i.e., $l \gg c/\Gamma_A$, at this time T_1 this photonic wave packet is localized well inside the optical fiber and propagates towards the second cavity B, which is assumed to be prepared in its vacuum state at time T_1. In the course of this propagation process the coupling between the optical fiber and cavity B which is negligible at time T_1 is turned on adiabatically and eventually leads to the leakage of this photon wave packet into cavity B. Thus, in analogy with Eq. (A1) for times $t \geqslant T_1$ the interaction Hamiltonian between the optical fiber and this second cavity B is given by

$$\hat{H}' = \hbar\omega\hat{a}^\dagger_B\hat{a}_B + \sum_{i\in L}\hbar\omega_i\hat{a}^\dagger_i\hat{a}_i + \sum_{i\in L}(\kappa'_i\hat{a}^\dagger_i\hat{a}_B + \kappa'^*_i\hat{a}^\dagger_B\hat{a}_i) \tag{A16}$$

with complex-valued coupling coefficients κ'_i. In general they may differ from the coupling coefficients κ_i for cavity A. Again, in the continuum limit and in the pole approximation one can characterize the coupling between the optical fiber and cavity B by a decay rate

$$\Gamma_B = \frac{2\pi}{\hbar}|\kappa'_n|^2\frac{dn}{d\hbar\omega_n}\bigg|_{\omega_n=\omega}. \tag{A17}$$

Evaluating the quantum state of the optical fiber and of cavity mode $B|\psi(T_1+t)\rangle$ for $t \gg 1/\Gamma_B$ with the help of Eq. (A10), we obtain the result

$$|\psi(T_1+t)\rangle = |\alpha(T_1+t)\rangle_B \prod_{i\in L}|\alpha_i(T_1+t)\rangle_i \tag{A18}$$

with

$$\begin{aligned}\alpha(T_1+t) &= \sum_{i\in L}\frac{\kappa'^*_i}{\hbar\omega_i - \hbar\omega + i\hbar\Gamma_B/2}\frac{\alpha\kappa_i e^{-i\omega_i(T_1+t)}}{\hbar\omega_i - \hbar\omega + i\hbar\Gamma_A/2},\\ \alpha_i(T_1+t) &= \frac{1}{2\pi i}\int_{-\infty+i0}^{\infty+i0} d\lambda\frac{\kappa'_i}{\hbar\omega_i - \hbar\lambda}\frac{e^{-i\lambda t}}{\omega - \lambda - i\Gamma_B/2}\\ &\quad\times\sum_{j\in L}\frac{\alpha\kappa_j e^{-i\omega_j T_1}}{\hbar\omega_j - \hbar\omega + i\hbar\Gamma_A/2}\frac{\kappa'^*_j}{\hbar\omega_j - \hbar\lambda},\end{aligned} \tag{A19}$$

where in the first equation we already neglected terms which are exponentially small in the parameter $\exp(-\Gamma_B t/2)$. A comparison of Eq. (A19) with the initial condition of Eq. (A14) shows that for interaction times $T_1 + t$ with $t \gg 1/\Gamma_B$ the necessary and sufficient condition for perfect quantum state transfer between cavities A and B is $\alpha = \alpha(T_1 + t)$. A sufficient condition for satisfying this condition is to choose the coupling constants between the optical fiber and the second cavity B in such a way that

$$\kappa'_i = \kappa^*_i = |\ \kappa_i\ |\ e^{-i\varphi_i}, \quad e^{2i\varphi_i} = \frac{\hbar\omega_i - \hbar\omega + i\hbar\Gamma_A/2}{\hbar\omega_i - \hbar\omega - i\hbar\Gamma_A/2}, \tag{A20}$$

and to choose the interaction time $t = T_2$ with $T_1, T_2 \gg 1/\Gamma_A, 1$ so that $\omega_n(T_1 + T_2)$ is an integer multiple of 2π. If the relevant modes of the optical fiber fulfill the condition $\omega_n = 2\pi cn/l$ with integer values of n, for example, this latter condition can be fulfilled by the choice $c(T_1 + T_2) = l$. This condition describes the fact that the total interaction time $T = T_1 + T_2$ has to allow for a propagation of photons through the optical fiber of length l. In addition, this interaction time has to be large enough so that the leaking out of cavity A and the leaking into cavity B can be completed, i.e., $T_1, T_2 \gg 1/\Gamma_A$. The complex conjugation involved in Eq. (A20) reflects the fact that for perfect quantum state transfer a time-reversal process is necessary. The phase modulation of the coupling constants κ_i required by Eq. (A20) is characteristic for the scattering phase shift of a Breit-Wigner resonance with a Lorentzian frequency distribution.

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