

Hybrid quantum repeater based on resonant qubit-field interactions

József Zsolt Bernád*

*Institut für Angewandte Physik, Technische Universität Darmstadt, D-64289, Germany;
 Institute for Solid State Physics and Optics, Wigner Research Centre, Hungarian Academy of Sciences,
 P.O. Box 49, H-1525 Budapest, Hungary;
 and Department of Physics, University of Malta, Msida MSD 2080, Malta*

(Received 6 July 2017; revised manuscript received 13 September 2017; published 22 November 2017)

We propose a hybrid quantum repeater based on ancillary coherent field states and material qubits coupled to optical cavities. For this purpose, resonant qubit-field interactions and postselective field measurements are determined which are capable of realizing all necessary two-qubit operations for the actuation of the quantum repeater. We explore both theoretical and experimental possibilities of generating near-maximally-entangled qubit pairs ($F > 0.999$) over long distances. It is shown that our scheme displays moderately low repeater rates, between 5×10^{-4} and 23 pairs per second, over distances up to 900 km, and it relies completely on current technology of cavity quantum electrodynamics.

DOI: [10.1103/PhysRevA.96.052329](https://doi.org/10.1103/PhysRevA.96.052329)**I. INTRODUCTION**

Distribution of well-controlled entanglement over long distances is essential for quantum communication [1,2]. In practice, quantum channels connecting spatially separated quantum systems are subject to information loss. For example, direct transmission of photons via optical fibers, quantum channels, is limited, at best, to a few hundred km [3,4]. In the case of free space channels, there are new developments in satellite-Earth-based entanglement distribution [5], which go beyond 1000 km, though terrestrial free space quantum communication has a limitation of a few hundred km due to the curvature of Earth [6]. As straightforward amplification is not an option in quantum communication due to the no-cloning theorem [7,8], one possibility is to use the quantum repeater protocol [9,10], which improves the low success rates. Here, the total distance between the quantum systems is divided into smaller distances, i.e., elementary links, with repeater nodes in between. There are already various proposals for quantum repeaters and impressive experimental efforts are being made for their implementations [11].

An interesting proposal of van Loock *et al.* [12–14], a hybrid quantum repeater, uses coherent states to distribute entanglement between the nodes of the quantum repeater. This scheme has the advantage that a high repetition rate can be achieved, mainly due to almost unit efficiencies of homodyne photodetection of coherent states, in contrast to low efficiencies of single-photon detectors. Furthermore, for this type of quantum repeater every logical two-qubit gate is realized with the help of qubit-field interactions within cavity quantum electrodynamics (QED) scenarios [15,16]. These proposals are based on far-off-resonant qubit-field interactions, which impose limitations on the orthogonality of the field states involved in postselective homodyne measurements, thus affecting the fidelity of the entangled states. In order to have unit fidelities in these approaches, long interaction times or high numbers of mean photons are required. If the interaction times are much longer than the characteristic times of dipole transitions, then

the qubit-field interaction model has to be extended beyond the dipole approximation. High numbers of mean photons usually are not an experimental issue; however, to obtain better fidelities one has to increase the mean photon number. In this case, the justification of the far-off-resonant model requires significantly increased difference between the frequency of the qubit transition and the frequency of the single-mode field, which means that the cavities in use have to have adjustable frequencies. It has been demonstrated by us that this hurdle can be overcome by resonant qubit-field interactions [17]. As a continuation of this work, we have proposed building blocks for a hybrid quantum repeater, which is completely based on cavity QED and resonant qubit-field interactions [18–20].

In this paper, we go one step further and propose an entire quantum repeater scheme by assembling all three building blocks. These building blocks are based on two-level atomic ensembles, single-mode cavities, postselective field measurements, and input coherent states of the radiation field; hence, they are compatible with each other. Besides assembling a quantum repeater, we also extend our analysis to two building blocks. In our previous article [17], we have studied entanglement generation between two spatially separated material qubits, and now we augment this approach with the reflection of photons from the surface of the cavities, a source of decoherence, and we replace minimum-error field measurements with balanced homodyne photodetection. We also review the entanglement swapping protocol, because in our previously proposed scheme the success probability is found to be less than one [20]. In this paper, we present a new set of field measurements, which postselect deterministically the four Bell states. The purification protocol of Ref. [19] is applied straightforwardly to output states of the entanglement generation block.

Another aim of this paper is to calculate the average rates of near-maximally-entangled pairs per second between the end points of the repeater chain. We focus on a standard quantum repeater scheme [9] and avoid new generation of schemes, for now [21]. As a result, the performance of the presented quantum repeater it is expected to be moderate. The rate analysis will be done in the context of some current experimental setups with the following assumptions: Qubits do not decay and detectors have unit efficiency.

*zsolt.bernad@um.edu.mt

This paper is organized as follows. In Sec. II, we present the scheme of the hybrid quantum repeater. In Subsec. II A, we introduce the theoretical model for the building block of entanglement generation and determine the form of the entangled state generated between the repeater nodes. This result is employed in Subsec. II B, where the entanglement purification protocol and its theoretical model are discussed. In Subsec. II C, we present the entanglement swapping protocol and its theoretical model. In Sec. III, we collect some experimental setups and data, which may play a role in the future implementation of the proposal. Based on these experimental setups, we determine the repeater rates of near-maximally-entangled pairs separated by a distance of 100 km.

II. PROTOCOL

In this section, we discuss a quantum repeater, which is based on models of cavity quantum electrodynamics. We follow the seminal idea of Ref. [9], where entanglement over distance L is created by dividing the distance into n elementary links and inserting nodes at their intersection. Thus, each node is connected by elementary links with length $L_0 = L/n$ either to neighboring nodes or to the endpoints of repeater chain. The quantum repeater presented here consists of three building blocks: Entanglement is generated between neighboring nodes, then with the help of an entanglement purification one can purify the effects of any kind of decoherence, and finally entanglement swapping is applied to increase the distance of shared entanglement. The physical resources of this quantum repeater are atoms, optical conveyor belts, optical or microwave cavities with initially prepared coherent states, state-selective detectors for the qubits, and postselective field measurements implemented by balanced homodyne photodetection. The main physical mechanism for the realization of two-qubit operations is the resonant qubit-field interaction in dipole and rotating-wave approximation. In this context, we employ the Jaynes-Cummings-Paul and Tavis-Cummings models with an interaction time in the region of the so-called collapse phenomena [22]. Furthermore, we consider that the initial coherent states have large mean photon numbers $\bar{n} \geq 100$.

In Fig. 1, we display the sketch of the setup. The status of some current experimental settings, which are strongly related to our proposed setup, is thoroughly discussed in Sec. III. In the first step, the qubits (implemented by atoms) are loaded from a magneto-optical trap (MOT) into a dipole trap, which can be set into motion, thus realizing an optical conveyor belt. These qubits interact with single-mode electromagnetic fields in cavities denoted by C_1 , which are coupled by optical fibers to neighboring nodes. The emerging fields propagate to neighboring nodes, where they interact with the local qubits and field measurements (D_1 detectors) are performed in order to postselect entangled states between qubits separated by an elementary link. Afterward, entanglement purification is implemented with the help of qubit-field interactions in cavities denoted by C_2 , postselective field measurements (D_1 detectors), and qubit measurements (D_2 detectors). Finally, two cavities (denoted by C_2), where simultaneously two qubits can interact with the single-mode of the radiation field, and two postselective field measurements are used to generate

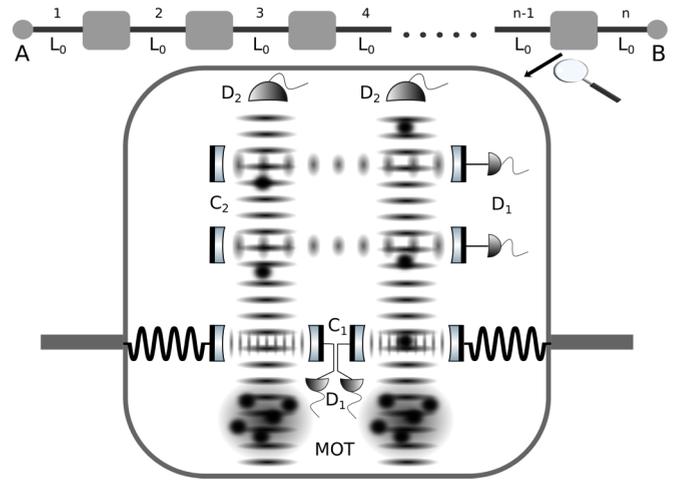


FIG. 1. Schematic representation of a cavity QED-based quantum repeater protocol. The total distance between the end points A and B is divided in n elementary links with length L_0 . At each node there are four cavities: For the entanglement generation protocol there are two C_1 cavities, in which single qubits can interact with single-mode electromagnetic fields; and for the implementation of unambiguous Bell measurements there are two C_2 cavities, in which two qubits can interact symmetrically and simultaneously with the single-mode electromagnetic fields. The entanglement purification protocol is implemented with cavities C_2 , such that there is only one qubit present in the cavity during the qubit-field interaction. The atoms (black dots) implementing the qubits are collected from a magneto-optical trap (MOT) and transferred into an optical conveyor belt. The conveyor belt can move the qubits in and out of the cavities. There are also two types of detectors: Four D_1 detectors realize postselective field measurements for all three building blocks of the quantum repeater, and two D_2 detectors measure the states of the qubits for both the purification protocol and the Bell measurement.

unambiguous and noninvasive Bell measurements. These Bell measurements realize entanglement swapping between the nodes, and after applying them in every node we are able to generate near-maximally-entangled pairs between the endpoints of the repeater chain.

In the subsequent subsections, we discuss in more detail the quantum electrodynamical models of all three building blocks. The main aim is to explore the possibilities of generating near-maximally-entangled pairs with high success probabilities.

A. Entanglement generation between neighboring nodes

An elementary link between two neighboring nodes is modeled by two spatially separated cavities A , B and a long optical fiber connecting them. The qubits in the nodes are two atoms in conveyor belts with ground states $|0\rangle_i$ and excited states $|1\rangle_i$ ($i \in \{A, B\}$). In the first step, the single-mode radiation field of cavity A interacts with qubit A . This is followed by the leakage of the single-mode field into the optical fiber, the propagation of the radiation field inside the fiber from system A to system B , and a leakage of the fiber's radiation field into cavity B . Finally, the single-mode radiation field in cavity B , which is generated by the leakage of the fiber into the cavity, interacts with qubit B . A postselective measurement on the emerged radiation field in cavity B

generates entanglement between the two remote qubits. In the subsequent argumentation, we present a minimal model, which is capable of effectively describing this physical setup, and we analyze its efficiency with respect to the model's parameters in order to generate highly entangled qubit states with high success probabilities.

We consider a model where both qubits and cavities are similar. There are three main time scales in this system: the qubit-field interaction time τ , the decay time of the cavities τ_c , and the propagation time T in the fiber. We assume that $\tau \ll \tau_c \ll T$, which also encodes our intention that the distance between the two qubits is large. In order to avoid spontaneous decay of qubit A during the long propagation time, one can coherently transfer the population of the states involved in the interaction to radioactively stable electronic levels, which are assumed to not decay during the whole process. Furthermore, we also consider that the frequency of the a single-mode radiation field is in resonance with the qubits' transition frequency. In the dipole and rotating-wave approximation, the Hamiltonian for the qubit-field interaction is ($\hbar = 1$) [23,24]

$$\hat{H}_1 = \begin{cases} \omega_c \hat{\sigma}_z^A / 2 + g \hat{a} \hat{\sigma}_+^A + g \hat{a}^\dagger \hat{\sigma}_-^A, & t \in [0, \tau], \\ \omega_c \hat{\sigma}_z^B / 2 + g \hat{b} \hat{\sigma}_+^B + g \hat{b}^\dagger \hat{\sigma}_-^B, & t \in [T', T' + \tau], \end{cases}$$

$$T' = \tau + T + 2\tau_c,$$

where $\sigma_z = |1\rangle_i \langle 1| - |0\rangle_i \langle 0|$, $\hat{\sigma}_+^i = |1\rangle_i \langle 0|$ and $\hat{\sigma}_-^i = |0\rangle_i \langle 1|$ ($i \in \{A, B\}$). The coupling constant g characterizes the strength of the dipole interaction of the qubits with the single-mode field, and thus $2g$ is the vacuum Rabi splitting. ω_c is the frequency of the single-mode fields in both cavities and also the transition frequency of the qubits' state. \hat{a} and \hat{b} (\hat{a}^\dagger and \hat{b}^\dagger) are the annihilation (creation) operators of the field mode in cavities A and B .

The optical fiber is considered to be a single-mode fiber with frequency ω and its modes which can couple to the cavities form a frequency band ($\omega - \delta\omega, \omega + \delta\omega$). This usually means that there is only one wave function considered as the solution of the Helmholtz equation in the cross section of the fiber and many along the length of the fiber [25]. In the rotating-wave approximation, i.e., $\delta\omega \ll \omega$, the interaction Hamiltonian between the single-mode cavities and the fiber modes is

$$\hat{H}_2 = \begin{cases} \sum_i \kappa_{i,A} \hat{a}_i^\dagger \hat{a} + \kappa_{i,A}^* \hat{a}^\dagger \hat{a}_i, & t \in [\tau, \tau + \tau_c], \\ \sum_i \kappa_{i,B} \hat{a}_i^\dagger \hat{b} + \kappa_{i,B}^* \hat{b}^\dagger \hat{a}_i, & t \in [\tau + T, \tau + T + \tau_c], \end{cases}$$

where $\kappa_{i,A}$ ($\kappa_{i,B}$) describes the coupling between the single-mode of cavity A (B) and the i th mode of the fiber. \hat{a}_i (\hat{a}_i^\dagger) is the annihilation (creation) operator of the i th mode of the fiber.

Another important phenomena is the photon loss during the propagation of the radiation field through the optical fiber. We consider a model, where each mode of the fiber is described by a damped harmonic oscillator and the decay rates are equal. The decaying mechanism is given by the master equation

$$\frac{d\hat{\rho}}{dt} = -\frac{\gamma}{2} \sum_i (\hat{a}_i^\dagger \hat{a}_i \hat{\rho} - 2\hat{a}_i \hat{\rho} \hat{a}_i^\dagger + \hat{\rho} \hat{a}_i^\dagger \hat{a}_i) = \mathcal{L}\hat{\rho},$$

for propagation times $t \in [\tau + \tau_c, \tau + T + \tau_c]$. γ is the damping rate, which characterizes the photon loss in the fiber.

The free Hamiltonian of the complete radiation field is

$$\hat{H}_0 = \omega_c (\hat{a}^\dagger \hat{a} + \hat{b}^\dagger \hat{b}) + \sum_i \omega_i \hat{a}_i^\dagger \hat{a}_i,$$

with $\omega_i \in (\omega - \delta\omega, \omega + \delta\omega)$ being the frequency of the i th fiber mode.

The first purpose of this subsection is to investigate the time evolution of the whole setup described by

$$\frac{d\hat{\rho}}{dt} = -i[\hat{H}_0 + \hat{H}_1 + \hat{H}_2, \hat{\rho}] + \mathcal{L}\hat{\rho}. \quad (1)$$

Our main strategy is to split the above evolution into parts and the output state of one part is considered as input state for the subsequent one. As we have already stated that $\tau_c \ll T$, it is reasonable to take the whole time evolution equal to $2\tau + T$ and thus considering the leakages as almost instantaneously occurring effects in regard to the propagation time of the radiation field inside the fiber. Therefore, the time evolution can be split into three parts: the qubit-field interaction in cavity A , the leakages and the photon loss during the propagation, and the qubit-field interaction in cavity B .

First, we investigate the time evolution in cavity A with an initial state

$$|\Psi_0\rangle = |0\rangle_A |\alpha\rangle_A, \quad (2)$$

where the single-mode field is in the coherent state

$$|\alpha\rangle = \sum_{n=0}^{\infty} e^{-\frac{|\alpha|^2}{2}} \frac{\alpha^n}{\sqrt{n!}} |n\rangle, \quad \alpha = \sqrt{\bar{n}} e^{i\phi}. \quad (3)$$

The other modes (fiber and cavity B) of the radiation field are considered to be in the ground state. The initial state of the qubit B is not taken into account yet, because it will be prepared after the elapsed time $\tau + T$ and thus qubit B will interact with the emerging field in cavity B right after its preparation procedure. Time evolution of Eq. (1) for times $0 \leq t \leq \tau$ and initial condition (2) is based on the solutions of the resonant Jaynes-Cummings-Paul model:

$$\begin{aligned} |\Psi(t)\rangle &= e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \left[\cos(g\sqrt{n}t) \frac{\alpha^n}{\sqrt{n!}} |0\rangle_A e^{i\omega_c t/2} \right. \\ &\quad \left. - i \sin(g\sqrt{n+1}t) \frac{\alpha^{n+1}}{\sqrt{(n+1)!}} |1\rangle_A e^{-i\omega_c t/2} \right] e^{-i\omega_c n t} |n\rangle_A. \end{aligned}$$

In the following discussion, we focus on large mean photon number $\bar{n} \gg 1$ and interaction times τ such that the Rabi frequency $g\sqrt{\bar{n}}$ can be linearized around \bar{n} . Thus the obtained joint state of qubit and single-mode field can be approximated by

$$\begin{aligned} |\Psi(\tau)\rangle &\approx \frac{|0\rangle_A e^{i\omega_c \tau/2} + |1\rangle_A e^{-i\omega_c \tau/2} e^{i\phi}}{2} e^{-ig\sqrt{\bar{n}}\tau/2} |\alpha(\tau) e^{-i\phi}\rangle_A \\ &\quad + \frac{|0\rangle_A e^{i\omega_c \tau/2} - |1\rangle_A e^{-i\omega_c \tau/2} e^{i\phi}}{2} e^{ig\sqrt{\bar{n}}\tau/2} |\alpha(\tau) e^{i\phi}\rangle_A, \\ \varphi &= \frac{g\tau}{2\sqrt{\bar{n}}}, \quad \alpha(\tau) = \alpha e^{-i\omega_c \tau}, \end{aligned} \quad (4)$$

provided that the interaction time τ fulfills the condition $g\tau/\sqrt{\bar{n}} \ll 16\pi$. This is a time scale below the well-known revival phenomena of the population inversion in the Jaynes-Cummings-Paul model [26].

In the next step, qubit A moves out of cavity A and the single-mode radiation field starts its leakage into the optical fiber. In order to deal with the dynamics of the second part, leakage out from cavity A , propagation in the fiber, and the leakage out from the fiber into cavity B , we recall and make full advantage of the results derived in Ref. [17]. The initial condition for the leakage is given by (4), which we rewrite in a more convenient form and we add the ground state of the fiber:

$$\begin{aligned} |\psi(t = \tau)\rangle &= (|\Psi_1\rangle_A |\alpha^-\rangle_A + |\Psi_2\rangle_A |\alpha^+\rangle_A) \prod_i |0\rangle_i, \\ |\Psi_1\rangle_A &= \frac{|0\rangle_A e^{i\omega_c\tau/2} + |1\rangle_A e^{-i\omega_c\tau/2} e^{i\phi}}{2} e^{-ig\sqrt{\bar{n}}\tau/2}, \\ |\Psi_2\rangle_A &= \frac{|0\rangle_A e^{i\omega_c\tau/2} - |1\rangle_A e^{-i\omega_c\tau/2} e^{i\phi}}{2} e^{ig\sqrt{\bar{n}}\tau/2}, \\ \alpha^- &= \alpha e^{-i\omega_c\tau - i\phi}, \quad \alpha^+ = \alpha e^{-i\omega_c\tau + i\phi}. \end{aligned}$$

The solution to (1) for times $\tau \leq t \leq \tau + \tau_c$ is given by

$$\begin{aligned} |\psi(t)\rangle &= |\Psi_1\rangle_A |\alpha^-(t)\rangle_A \prod_i |\alpha_i^-(t)\rangle_i \\ &+ |\Psi_2\rangle_A |\alpha^+(t)\rangle_A \prod_i |\alpha_i^+(t)\rangle_i \end{aligned}$$

with

$$\begin{aligned} \alpha^\pm(t) &= \alpha e^{\pm i\phi - i\omega_c t} e^{-i\omega_c t - \kappa_A t/2}, \\ \alpha_i^\pm(t) &= \frac{\alpha e^{\pm i\phi - i\omega_c \tau} \kappa_{i,A}}{\omega_i - \omega_c + i\kappa_A/2} (e^{-i\omega_i t} - e^{-i\omega_c t - \kappa_A t/2}), \end{aligned}$$

and κ_A being the cavity A 's decay constant (see the appendix in Ref. [17] for a detailed derivation). Provided that the leakage time τ_c is sufficiently long, i.e., $\kappa_A \tau_c \gg 1$, and neglecting the small exponential terms, we have that the depletion of the cavity mode is perfect.

In the following step, the propagation of the radiation field from cavity A to cavity B is discussed. The initial condition for Eq. (1) with respect to the propagation is

$$\begin{aligned} |\psi(\tau + \tau_c)\rangle &= |\Psi_1\rangle_A \prod_i |\alpha_i^-\rangle_i + |\Psi_2\rangle_A \prod_i |\alpha_i^+\rangle_i, \\ \alpha_i^\pm &= \frac{\alpha e^{\pm i\phi - i\omega_c \tau - i\omega_i \tau_c} \kappa_{i,A}}{\omega_i - \omega_c + i\kappa_A/2}, \end{aligned} \quad (5)$$

which also means that we neglect to follow the evolution of the empty cavity A . In order to calculate $e^{\mathcal{L}t}(|\psi(\tau + \tau_c)\rangle\langle\psi(\tau + \tau_c)|)$ for times $\tau + \tau_c \leq t \leq \tau + \tau_c + T$, we recall the results of Sec. III in Ref. [17]. Coherent states and coherences between coherent states evolve as

$$\begin{aligned} e^{\mathcal{L}t} |\alpha_i^l\rangle_i \langle\alpha_i^k| &= e^{-f_i(t)_{l,k}} |\alpha_i^l e^{-\gamma t/2 - i\omega_i t}\rangle_i \langle\alpha_i^k e^{-\gamma t/2 - i\omega_i t}|, \\ f_i(t)_{l,k} &= (1 - e^{-\gamma t}) \left[\frac{|\alpha_i^l|^2 + |\alpha_i^k|^2}{2} - \alpha_i^l (\alpha_i^k)^* \right], \end{aligned}$$

with $l, k \in \{+, -\}$. We observe that $f_i(t)_{+,+} = f_i(t)_{-,-} = 0$ for all i . For the other two terms, we have

$$\begin{aligned} f_i(t)_{+,-} &= (1 - e^{-\gamma t}) |\alpha_i^+|^2 (1 - e^{2i\phi}), \\ f_i(t)_{-,+} &= (1 - e^{-\gamma t}) |\alpha_i^+|^2 (1 - e^{-2i\phi}), \end{aligned}$$

where we used the relation $|\alpha_i^+| = |\alpha_i^-|$. We can now conclude that initial to the leakage out from the fiber into cavity B the joint state of fiber modes and qubit A has the following form:

$$\begin{aligned} &|\Psi_1\rangle_A \langle\Psi_1| \prod_i |\beta_i^-\rangle_i \langle\beta_i^-| + |\Psi_2\rangle_A \langle\Psi_2| \prod_i |\beta_i^+\rangle_i \langle\beta_i^+| \\ &+ |\Psi_1\rangle_A \langle\Psi_2| \prod_i e^{-f_i(T)_{-+}} |\beta_i^-\rangle_i \langle\beta_i^+| \\ &+ |\Psi_2\rangle_A \langle\Psi_1| \prod_i e^{-f_i^*(T)_{-+}} |\beta_i^+\rangle_i \langle\beta_i^-|, \end{aligned} \quad (6)$$

where

$$\beta_i^\pm = \frac{\alpha e^{\pm i\phi} \kappa_{i,A}}{\omega_i - \omega_c + i\kappa_A/2} e^{-i\omega_i T - i\omega_c \tau - \gamma T/2} \quad (7)$$

and due to the relation $T \gg \tau_c$, we have also considered that $T + \tau_c \approx T$.

Equation (6) can be considered as an initial condition for (1) and we introduce the decay constant κ_B for cavity B . We assume that $\kappa_B \tau_c \gg 1$ and as described in Ref. [17] we have the following conditions for perfect leakage into cavity B : Choose the coupling constants between the fiber and cavity B in such a way that

$$\begin{aligned} \kappa_{i,B} &= \kappa_{i,A}^* = |\kappa_i| e^{-i\phi_i}, \\ e^{2i\phi_i} &= \frac{\omega_i - \omega_c + i\kappa_A/2}{\omega_i - \omega_c - i\kappa_A/2}, \end{aligned}$$

which also yields that $\kappa_A = \kappa_B$, and $\omega_i T$ is an integer multiple of 2π . If these conditions are not fulfilled, then many photons are reflected from the surface of the mirror, which forms cavity B and connects it with the optical fiber.

In general, these conditions are hard to realize in current experimental setups and therefore we consider a simple yet detailed enough model, such that it is able to describe effects of photon reflection from the surface of the mirror. We set $\kappa_{i,A} = \kappa_{i,B}$, i.e., $\kappa_A = \kappa_B$ and $\omega_i T$ is an integer multiple of 2π for all i . The last condition can be fulfilled if the relevant modes of the fiber have approximately a frequency spacing of c/L_0 , where L_0 is the length of the fiber and c is the speed of the light in the fiber. Thus, we can introduce (see Eq. A(19) in Ref. [17])

$$\sqrt{\eta} = \sum_i \frac{|\kappa_{i,A}|^2}{(\omega_i - \omega_c + i\kappa_A/2)^2}, \quad (8)$$

the transmittance of the mirror and η quantifies the fraction of photons which are not reflected back from the surface of cavity B .

After τ_c time, a part of the propagating radiation field is able to leak into cavity B and the reflected field we consider as a lost information; hence we trace out the state of the fiber after the reflection. By using the relation

$$\text{Tr}_{\text{fiber}}\{|\{\alpha_i\}\rangle\langle\{\beta_i\}|\} = e^{-\sum_i \frac{|\alpha_i|^2 + |\beta_i|^2 - 2\alpha_i \beta_i^*}{2}}$$

with $|\{\alpha_i\}\rangle = |\alpha_1\rangle_1 \otimes |\alpha_2\rangle_2 \otimes \dots$, we obtain the following joint state of qubit A and the single-mode of field in cavity B :

$$\begin{aligned} \hat{\rho} = & |\Psi_1\rangle_A \langle \Psi_1| |\alpha'^-\rangle_B \langle \alpha'^-| + |\Psi_2\rangle_A \langle \Psi_2| |\alpha'^+\rangle_B \langle \alpha'^+| \\ & + F(T, \eta, \varphi) |\Psi_1\rangle_A \langle \Psi_2| |\alpha'^-\rangle_B \langle \alpha'^+| \\ & + F^*(T, \eta, \varphi) |\Psi_2\rangle_A \langle \Psi_1| |\alpha'^+\rangle_B \langle \alpha'^-|, \end{aligned} \quad (9)$$

with

$$\alpha'^{\pm} = \sqrt{\eta} e^{-\gamma T/2} \alpha e^{-i\omega_c(\tau+T) \pm i\varphi},$$

$$\begin{aligned} F(T, \eta, \varphi) = & \exp \left\{ - \sum_i f_i(T)_{-,+} + |\beta_i^R|^2 (1 - e^{-2i\varphi}) \right\} \\ = & \exp \left\{ -|\alpha|^2 (1 - e^{-2i\varphi}) (1 - \eta e^{-\gamma T}) \right\}, \end{aligned}$$

where we have used Eq. (5) for the relation $\sum_i |\alpha_i^{\pm}|^2 = |\alpha|^2$ and Eq. (7) to approximate the reflected average photon number $\sum_i |\beta_i^R|^2$ as $(1 - \eta) \sum_i |\alpha_i^{\pm}|^2 = (1 - \eta) |\alpha|^2 e^{-\gamma T}$.

Finally, we consider the last part of the dynamics, where we include the state of qubit B , which moves through cavity B after the leakage from the fiber is considered to reach its maximum. The initial condition for (1) is

$$\hat{\rho} \otimes |1\rangle_B \langle 1|,$$

where $\hat{\rho}$ is defined in Eq. (9). In order to obtain the solution for the density matrix of the two qubits and the single-mode field, we separate the initial condition into four parts. We make use of the resonant Jaynes-Cummings-Paul model for the initial condition $|1\rangle_B |\alpha'^-\rangle_B$ and obtain in the coherent state approximation

$$\begin{aligned} & \frac{|1\rangle_B e^{-i\omega_c \tau/2} + |0\rangle_B e^{i\omega_c \tau/2} e^{-i\phi}}{2} e^{-ig\sqrt{\bar{n}}\tau/2} |\alpha'^- e^{-i\phi}\rangle_B \\ & + \frac{|1\rangle_B e^{-i\omega_c \tau/2} - |0\rangle_B e^{i\omega_c \tau/2} e^{-i\phi}}{2} e^{ig\sqrt{\bar{n}}\tau/2} |\alpha'^- e^{i\phi}\rangle_B, \end{aligned}$$

where we have considered that $\phi + \varphi \approx \phi$. In a similar way, for the initial condition $|1\rangle_B |\alpha'^+\rangle_B$ we get

$$\begin{aligned} & \frac{|1\rangle_B e^{-i\omega_c \tau/2} + |0\rangle_B e^{i\omega_c \tau/2} e^{-i\phi}}{2} e^{-ig\sqrt{\bar{n}}\tau/2} |\alpha'^+ e^{-i\phi}\rangle_B \\ & + \frac{|1\rangle_B e^{-i\omega_c \tau/2} - |0\rangle_B e^{i\omega_c \tau/2} e^{-i\phi}}{2} e^{ig\sqrt{\bar{n}}\tau/2} |\alpha'^+ e^{i\phi}\rangle_B. \end{aligned}$$

In order to present a clear picture of the obtained density matrix of qubits and single-mode field in cavity B , we transform out the phases acquired during the qubit-field interactions and the propagation phase through the optical fiber

$$\begin{aligned} \hat{\rho}(2\tau + T) = & \hat{U}(2\tau + T) \hat{\rho}' \hat{U}^\dagger(2\tau + T), \\ \hat{U}(2\tau + T) = & e^{-i\omega_c \hat{\sigma}_z^A \tau/2 - i\omega_c \hat{\sigma}_z^B \tau/2 - i\omega_c \hat{b}^\dagger \hat{b}(2\tau + T)}. \end{aligned} \quad (10)$$

We introduce the Bell states

$$\begin{aligned} |\Psi^\pm\rangle = & \frac{1}{\sqrt{2}} (|0\rangle_A |1\rangle_B \pm |1\rangle_A |0\rangle_B), \\ |\Phi_\phi^\pm\rangle = & \frac{1}{\sqrt{2}} (e^{-i\phi} |0\rangle_A |0\rangle_B \pm e^{i\phi} |1\rangle_A |1\rangle_B), \end{aligned} \quad (11)$$

and the unnormalized states

$$\begin{aligned} |\Phi_1\rangle = & \frac{|\Psi^+\rangle + |\Phi_\phi^+\rangle}{2\sqrt{2}} e^{-ig\sqrt{\bar{n}}\tau} |\alpha_F e^{-2i\varphi}\rangle_B, \\ |\Phi_2\rangle = & \frac{|\Psi^-\rangle - |\Phi_\phi^-\rangle}{2\sqrt{2}} |\alpha_F\rangle_B, \quad |\Phi_3\rangle = \frac{|\Psi^-\rangle + |\Phi_\phi^-\rangle}{2\sqrt{2}} |\alpha_F\rangle_B, \\ |\Phi_4\rangle = & \frac{|\Psi^+\rangle - |\Phi_\phi^+\rangle}{2\sqrt{2}} e^{ig\sqrt{\bar{n}}\tau} |\alpha_F e^{2i\varphi}\rangle_B, \end{aligned}$$

where $\alpha_F = \sqrt{\eta} e^{-\gamma T/2} \alpha$. The state of qubits and single-mode field in the interaction picture defined in (10) takes the form

$$\begin{aligned} \hat{\rho}' = & \sum_{i,j=1}^2 |\Phi_i\rangle \langle \Phi_j| + \sum_{i,j=3}^4 |\Phi_i\rangle \langle \Phi_j| \\ & + F(T, \eta, \varphi) (|\Phi_1\rangle + |\Phi_2\rangle) (|\Phi_3\rangle + |\Phi_4\rangle) + \text{H.c.} \end{aligned}$$

In the next step, we briefly investigate the possibility of a field measurement which is capable of realizing conditionally an entangled two-qubit state. First, we consider the overlaps

$$\begin{aligned} F_* = & |\langle \alpha_F | \alpha_F e^{-2i\varphi} \rangle_B| = |\langle \alpha_F e^{2i\varphi} | \alpha_F \rangle_B^*| \\ = & |\exp \{ -\eta e^{-\gamma T} |\alpha|^2 (1 - e^{-2i\varphi}) \}| \approx e^{-\eta e^{-\gamma T} \frac{g^2 \tau^2}{2}}, \end{aligned} \quad (12)$$

where we have used the relation $\varphi = \frac{g\tau}{2\sqrt{\bar{n}}}$. The approximation holds for $g\tau \ll \sqrt{\bar{n}}$ and shows that the overlap nearly vanishes for interaction times $\tau \geq 4\sqrt{e^{\gamma T}/\eta}/g$. In order to ensure that (12) is almost zero, i.e., $|\alpha_F\rangle_B$ is orthogonal to $|\alpha_F e^{2i\varphi}\rangle_B$ and $|\alpha_F e^{-2i\varphi}\rangle_B$, and the coherent state approximation is still valid, the interaction times have to fulfill the following condition:

$$4\sqrt{\frac{e^{\gamma T}}{\eta}} \leq g\tau \ll 50\sqrt{\bar{n}}. \quad (13)$$

This condition shows clearly the destructive effects of the photon loss during propagation and the photon reflection from the surface of cavity B ; i.e., the left-hand side of Eq. (13) is the smallest when $\gamma T = 0$ and $\eta = 1$, which correspond to lossless propagation or no propagation and perfect leakage into cavity B . Thus, for interaction times, which fulfill the conditions in (13), there is a postselective field measurement of $|\alpha_F\rangle$ by means of balanced homodyne photodetection [27] (see also our discussion in Ref. [18]), which is able to prepare the two-qubit state

$$\begin{aligned} \hat{\rho}_{AB} = & \frac{(1+x)|\Psi^-\rangle \langle \Psi^-| + (1-x)|\Phi_\phi^-\rangle \langle \Phi_\phi^-|}{2}, \\ x = & \exp\{-\bar{n}[1 - \cos(2\varphi)](1 - \eta e^{-\gamma T})\} \\ & \times \cos[\bar{n} \sin(2\varphi)(1 - \eta e^{-\gamma T})] \end{aligned} \quad (14)$$

with success probability

$$P_{\text{Gen}} = 0.5. \quad (15)$$

In the case when $x = 0$, $\hat{\rho}_{AB}$ is not an entangled state and hence not purifiable. Furthermore, the introduced parameter x is a good entanglement measure, because the concurrence of the state in Eq. (14) is $|x|$ [28].

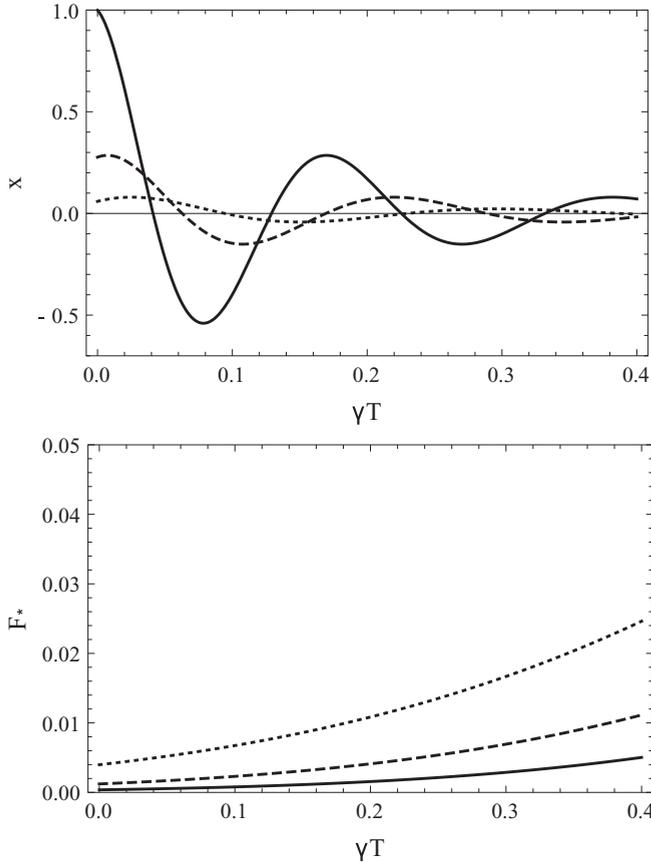


FIG. 2. Top panel: Concurrence x of the state in Eq. (14) as a function of γT . Bottom panel: The overlap of the field states, F_* in Eq. (12), as a function of γT . The interaction time was set to $\tau = 4/g$ with mean photon number $\bar{n} = 100$. Three curves are presented for different values of η : 1.0 (full line), 0.85 (dashed line), and 0.7 (dotted line).

The last part of this section is devoted to the analysis of x as a function of η and γT . Figure 2 shows the limitations of entanglement generation between the two remote qubits. We require in general that the absolute value of x in Eq. (14) to be close to unity and on the other hand that the overlap between components of the field state are zero, which is a necessary condition for successful postselection of (14). These two requirements are hard to hold for cases when the photon loss in the fiber is high and there is a big reflection rate from the surface of cavity B . The separation angle φ between the coherent state during the qubit-field interactions cannot be too large ($\approx \pi/4$), because then $|x|$ is too small, but when φ is too small then the overlap between the field states is not vanishing. This is an optimization problem with competing objectives and the ideal φ , which is determined by the interaction time [see Eq. (4)], has to be found for every experimental scenario independently.

We conclude at the end of this subsection that entanglement generation between two nodes connected by a optical fiber is obtained. The resulting two-qubit states are not perfect Bell states, but they are entangled unless the parameter x is equal to zero. In the following subsection, we investigate these states as input states for the entanglement purification protocol.

B. Entanglement purification

In this subsection, we present an entanglement purification protocol, which is capable of increasing the degree of entanglement of the state obtained in Sec. II A [see Eq. (14)]. The protocol we have in mind is a recurrence protocol [29,30], which works in a recursive way; i.e., it uses two copies of the same state for the next purification step. Thus, it is assumed that the entanglement generation procedure have resulted in a presumably large ensemble of similarly entangled states between the repeater nodes. In our previous work [19], we have discussed an implementation and here we briefly recapitulate it.

We consider two qubits A_1 and A_2 in one node with ground states $|0\rangle_i$ and excited states $|1\rangle_i$ ($i \in \{A_1, A_2\}$). These qubits move sequentially through a cavity and interact resonantly with single-mode field prepared initially in a coherent state $|\alpha\rangle$ [see Eq. (3)]. We take a general initial state with no correlations between the field and the qubits

$$|\Psi_0\rangle = (c_{00}|00\rangle + c_{01}|01\rangle + c_{10}|10\rangle + c_{11}|11\rangle)|\alpha\rangle,$$

with the basis $|ij\rangle = |i\rangle_{A_1}|j\rangle_{A_2}$ ($i, j \in \{0, 1\}$). The Hamiltonian in the dipole and rotating-wave approximation reads

$$\hat{H} = \begin{cases} \omega_c \hat{\sigma}_z^{A_1}/2 + \omega_c \hat{a}^\dagger \hat{a} + g \hat{a} \hat{\sigma}_+^{A_1} + g \hat{a}^\dagger \hat{\sigma}_-^{A_1}, & t \in [0, \tau], \\ \omega_c \hat{\sigma}_z^{A_2}/2 + \omega_c \hat{a}^\dagger \hat{a} + g \hat{a} \hat{\sigma}_+^{A_2} + g \hat{a}^\dagger \hat{\sigma}_-^{A_2}, & t \in [\tau, 2\tau], \end{cases}$$

where $\sigma_z^i = |1\rangle_i\langle 1| - |0\rangle_i\langle 0|$, $\hat{\sigma}_+^i = |1\rangle_i\langle 0|$ and $\hat{\sigma}_-^i = |0\rangle_i\langle 1|$ ($i \in \{A_1, A_2\}$). $2g$ is the vacuum Rabi splitting and ω_c is the frequency of the single-mode field in the cavity and also the transition frequency of the qubits' state. \hat{a} (\hat{a}^\dagger) is the annihilation (creation) operator of the field mode in the cavity.

By solving the Jaynes-Cummings-Paul model in sequence for interaction times characterizing the collapse phenomena [26] and projecting onto the field state contribution $|\alpha\rangle$ in the full solution by means of balanced homodyne photodetection [19], one is able to generate the probabilistic two-qubit quantum operation at each node,

$$\hat{M}_{A_1, A_2} = |\Psi^-\rangle\langle\Psi^-| + |\Phi_\phi^-\rangle\langle\Phi_\phi^-|, \quad (16)$$

where these Bell states are defined by Eq. (11) with the indices $A_1 = A$ and $A_2 = B$. This probabilistic quantum operation takes over the role of the controlled-NOT gate employed in the seminal protocols of Refs. [29,30].

We have demonstrated in Ref. [19] that for large mean photon number $\bar{n} = 500$, cavity damping, and spontaneous emission of the qubits (parameter values based on Ref. [31]), the following protocol is very robust:

(I) The quantum operation M is applied locally at each node A and B to the initial state

$$\hat{\rho} = \hat{\rho}_{A_1, B_1} \otimes \hat{\rho}_{A_2, B_2},$$

where both $\hat{\rho}_{A_1, B_1}$ and $\hat{\rho}_{A_2, B_2}$ have the form of Eq. (14). After successful applications of the quantum operations at each node A and B , we get the following four-qubit state:

$$\hat{\rho}^{(1)} = \frac{\hat{M} \hat{\rho} \hat{M}^\dagger}{\text{Tr}\{\hat{M}^\dagger \hat{M} \hat{\rho}\}}, \quad \hat{M} = \hat{M}_{A_1, A_2} \hat{M}_{B_1, B_2}.$$

(II) One of the pairs is measured, where the choice of the measured pair is unimportant. There are four possible states

in which one can find, for example, the pair (A_2, B_2) . The measurement of one of the states $|ij\rangle_{A_2, B_2}$ with $i, j \in \{0, 1\}$ results in the two-qubit state

$$\hat{\rho}_{A_1, B_1}^{i, j} = \text{Tr}_{A_2, B_2} \{ |ij\rangle_{A_2, B_2} \langle ij| \hat{\rho}^{(1)} \}. \quad (17)$$

(III) In the next step, we apply the unitary operator $\hat{U}_{A_1}^i \hat{U}_{B_1}^{j+1}$ at each node to the state $\rho_{A_1, B_1}^{i, j}$, where

$$\hat{U}^i = (|1\rangle\langle 1| + i|0\rangle\langle 0|)(|1\rangle\langle 0| + |0\rangle\langle 1|)^j.$$

The final two-qubit state is obtained after a measurement-dependent (A_2 and B_2 qubits found in the states $|ij\rangle_{A_2, B_2}$) unitary transformation

$$\hat{\rho}_{A_1, B_1}^{(1)} = (\hat{U}_{A_1}^i \hat{U}_{B_1}^{j+1}) \hat{\rho}_{A_1, B_1}^{i, j} (\hat{U}_{A_1}^i \hat{U}_{B_1}^{j+1})^\dagger.$$

Now, we recall the result of Sec. II A on the state generated between two spatially separated qubits. For simplicity, we set the phase ϕ in (14) equal to zero and we apply a local unitary transformation at both nodes A and B such that $|\Psi^-\rangle$ picks up a global phase and $|\Phi^-\rangle \rightarrow |\Phi^+\rangle$. Substituting this state into the purification protocol, we get

$$\hat{\rho}_{A_1, B_1}^{(1)} = \frac{(1+x)^2}{2+2x^2} |\Psi^-\rangle\langle\Psi^-| + \frac{(1-x)^2}{2+2x^2} |\Psi^+\rangle\langle\Psi^+|,$$

with a success probability $(1+x^2)/4$. After repeating the protocol for N times, i.e., 2^N qubit pairs were used to get a single two-qubit state, we obtain

$$\hat{\rho}^{(N)} = f_{(N)}(x) |\Psi^-\rangle\langle\Psi^-| + g_{(N)}(x) |\Psi^+\rangle\langle\Psi^+|, \quad (18)$$

with success probability

$$P_{\text{Pur}} = [P_{(0)}]^{2^{N-1}} [P_{(1)}]^{2^{N-2}} \dots [P_{(N-2)}]^2 P_{(N-1)},$$

$$P_{(k)} = \frac{f_{(k)}^2(x) + g_{(k)}^2(x)}{2}, \quad (19)$$

where

$$f_{(k+1)}(x) = \frac{f_{(k)}^2(x)}{f_{(k)}^2(x) + g_{(k)}^2(x)}, \quad f_{(0)}(x) = \frac{1+x}{2},$$

$$g_{(k+1)}(x) = \frac{g_{(k)}^2(x)}{f_{(k)}^2(x) + g_{(k)}^2(x)}, \quad g_{(0)}(x) = \frac{1-x}{2}.$$

Equation (19) shows the overall success probability of N purification rounds, provided that in the first iteration 2^{N-1} qubits, in the second iteration 2^{N-2} qubits, and so on, are successfully purified.

It is worth noticing that we purify in the case of $x > 0$ toward the Bell state $|\Psi^-\rangle$ and for $x < 0$ toward the other Bell state $|\Psi^+\rangle$. In Fig. 3, we show the tradeoff between the entanglement of the state and the overall success probability after several iterations of the purification protocol. The figures demonstrate that the purification protocol is successful only at the expense of the qubit resources due to the low probabilities involved [32]. For example, $N = 4$ iterations with 16 qubit pairs can purify a wide range of badly entangled states toward a Bell state; however, the overall success probability is approximately 10^{-8} to 10^{-9} . For lower number of iterations, we can cover a smaller range of badly entangled states, but with higher overall success probability and better resource

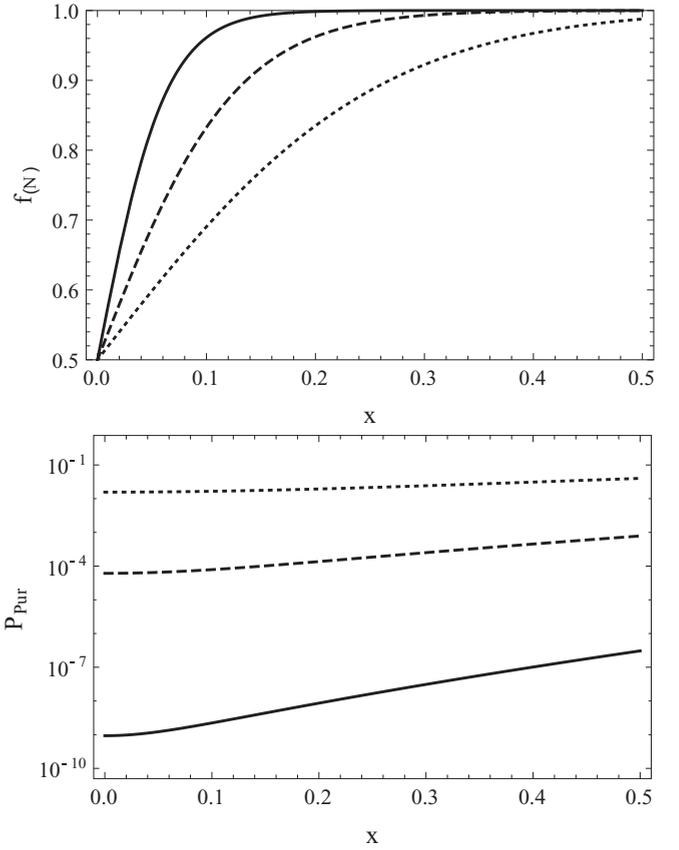


FIG. 3. Top panel: The coefficient $f_{(N)}$ in Eq. (18) as a function of x after N iterations of the purification protocol. Bottom panel: Semilogarithmic plot of the overall success probability P_{Pur} [see Eq. (19)] as a function of x . The value of x , concurrence of the state in Eq. (14), is considered to be not larger than 0.5 due to the findings in Fig. 2. Three curves are presented for different values of iterations N : 4 (full line), 3 (dashed line), and 2 (dotted line).

management. We presented a figure only for the coefficient $f_{(N)}$, because $g_{(N)}$ is nothing else than the reflection of $f_{(N)}$ about the y axis. The absolute value of x , i.e., concurrence of (14), is limited by a value of 0.5 due to our findings in the Sec. II A.

C. Entanglement swapping

In an entanglement swapping, the goal is to increase the distance of the shared entanglement. In other words, take three repeater nodes, labeled by the letters A , B , and C , where A - B and B - C are neighboring nodes which share a Bell pair. Thus, B has two qubits, and by performing a projective Bell measurement on these two qubits and communicating the results to A and C , one can generate a Bell pair between A and C . Applying the swapping protocol to all intermediate nodes results in a Bell pair between the endpoints of the repeater chain.

The most important ingredient here is the realization of Bell measurements. We have already introduced and discussed such a scenario, where we have analyzed the realization of a noninvasive unambiguous Bell measurements [18,20]. A noninvasive Bell measurement means that the qubits are

projected on a Bell state without destroying them. Therefore, measuring later these two-qubit states one can demonstrate that they are indeed in a Bell state. Our proposal is compatible with the other two protocols presented in Secs. II A and II B, because it is based on the Tavis-Cummings model [33], i.e., two material qubits interact simultaneously with the single-mode radiation field inside a cavity, and postselective field measurements. However, the proposed scheme in Ref. [20] requires special conditions on the mean photon number of the single-mode fields. In the subsequent discussion, we briefly present the scheme proposed in Refs. [18,20] with a different postselective field measurement scheme, based also on balanced homodyne photodetection.

Let us consider that two qubits A and B simultaneously move through a cavity and resonantly interact with the single-mode radiation field, where the path of the qubits is designed such a way that the dipole couplings g are equal. Thus, the Hamiltonian in the dipole and rotating-wave approximation reads

$$\hat{H} = \omega_c \hat{a}^\dagger \hat{a} + g \hat{a} (\hat{\sigma}_+^A + \hat{\sigma}_+^B) + g \hat{a}^\dagger (\hat{\sigma}_-^A + \hat{\sigma}_-^B) + \omega_c (\hat{\sigma}_z^A/2 + \hat{\sigma}_z^B/2),$$

where $\sigma_z^i = |1\rangle_i \langle 1| - |0\rangle_i \langle 0|$, $\hat{\sigma}_+^i = |1\rangle_i \langle 0|$ and $\hat{\sigma}_-^i = |0\rangle_i \langle 1|$ ($i \in \{A, B\}$). ω_c is the frequency of the single-mode field in the cavity and also the transition frequency for qubits A and B . \hat{a} (\hat{a}^\dagger) is the annihilation (creation) operator of the single-mode field.

The field is prepared initially in a coherent state and after the interaction it is postselected by balanced homodyne photodetection. In the next step, the two qubits move through the second cavity and interact with the single-mode radiation field, prepared also in a coherent state. The emerged state of the field is again postselected. If each of the two postselections has two outputs, then there are four possible two-qubit states which are generated in the protocol. The main task is to find those conditions which allow that these four postselected qubit states are the Bell states.

The initial condition before the first interaction is

$$|\Psi_0\rangle = (a_- |\Psi^- \rangle + a_+ |\Psi^+ \rangle + b_- |\Phi^- \rangle + b_+ |\Phi^+ \rangle) |\alpha\rangle, \quad (20)$$

where $\alpha = \sqrt{\bar{n}}$, and the Bell states are defined in Eq. (11) with the following adopted notation:

$$|\Phi^\pm\rangle = |\Phi_{\phi=0}^\pm\rangle.$$

By solving the resonant Tavis-Cummings model for interaction times characterizing the collapse phenomena [26] and approximating the field contributions one order beyond the coherent state approximation, one obtains (see the appendix in Ref. [20])

$$\begin{aligned} |\Psi(\tau)\rangle \approx & (a_- |\Psi^- \rangle + b_- |\Phi^- \rangle) |\alpha\rangle \\ & + \frac{a_+ - b_+}{2} (|\Psi^+ \rangle - |\Phi_{2\pi\tau}^+ \rangle) |\alpha_+\rangle \\ & + \frac{a_+ + b_+}{2} (|\Psi^+ \rangle + |\Phi_{-2\pi\tau}^+ \rangle) |\alpha_-\rangle, \quad (21) \end{aligned}$$

where τ is a dimensionless parameter of the interaction time equal to $\frac{gt}{\pi\sqrt{4\bar{n}+2}}$ and we have introduced the field states

$$|\alpha_\pm\rangle = \sum_{n=0}^{\infty} \frac{\alpha^n e^{-\frac{|\alpha|^2}{2}}}{\sqrt{n!}} e^{\pm i 2\pi\tau[\bar{n}+1+n-\frac{(n-\bar{n})^2}{4\bar{n}+2}]} |n\rangle. \quad (22)$$

The collapse phenomena occurs when $1/4 \leq \tau \leq 3/4$. If $\tau = 1/2$, then the field states $|\alpha_\pm\rangle$ have made half a rotation in phase space and lie on the opposite site to the initial coherent state $|\alpha\rangle$, i.e., $\langle \alpha | \alpha_\pm \rangle \approx 0$. This is a special case because the qubit states has the following relation:

$$|\Phi_\pi^+\rangle = |\Phi_{-\pi}^+\rangle = -|\Phi^+\rangle. \quad (23)$$

Furthermore, $|\alpha_-\rangle$ ($|\alpha_+\rangle$) rotates clockwise (counterclockwise) during the interaction time on the circle with radius $\sqrt{\bar{n}}$ and at $\tau = 1/2$ $\langle \alpha_- | \alpha_+ \rangle \neq 0$.

In the next step, a postselective measurement on the field state $\hat{\rho} = \text{Tr}\{|\Psi(\tau)\rangle\langle\Psi(\tau)|\}$ is performed with the help of balanced homodyne photodetection. Here, we briefly recapitulate the basics of this measurement, because the arguments presented in this subsection differ from our former study in Ref. [20]. So, the quantum state of the field, which we want to measure, interferes with an intense coherent state $|\alpha_L\rangle e^{i\phi_L}$ of a local oscillator on a 50:50 beam splitter. The two modes emerging from the beam splitter are directed to two photodetectors, which generate an electric current proportional to the photon number. The two photocurrents are subtracted and thus by the difference of photon numbers n_- is measured. Provided that the local oscillator state is intense, i.e., $|\alpha_L| \gg 1$, and the photodetectors have unit efficiency, the measurement is equivalent to a projective von Neumann measurement [27]. If a and a^\dagger are the annihilation and creation operator of the mode to be measured, then a quadrature state $|x_{\phi_L}\rangle$ is defined by the relation

$$\frac{1}{\sqrt{2}} (\hat{a} e^{-i\phi_L} + \hat{a}^\dagger e^{i\phi_L}) |x_{\phi_L}\rangle = x_{\phi_L} |x_{\phi_L}\rangle. \quad (24)$$

A balanced homodyne measurement projects onto the quadrature eigenstate $|x_{\phi_L}\rangle$ according to the probability distribution

$$P_{\phi_L}(x_{\phi_L}) = \text{Tr}\{\hat{\rho} |x_{\phi_L}\rangle\langle x_{\phi_L}|\}, \quad (25)$$

where $x_{\phi_L} = n_- / \sqrt{2|\alpha_L|^2}$. For the case $\tau = 1/2$, we consider the following projective measurement,

$$\hat{\mathcal{P}}_1 = \int_0^\infty |x_{\pi/2}\rangle\langle x_{\pi/2}| dx_{\pi/2}, \quad (26)$$

with properties

$$\hat{\mathcal{P}}_1 |\alpha\rangle = |\alpha\rangle, \quad \hat{\mathcal{P}}_1 |\alpha_\pm\rangle = 0. \quad (27)$$

The projector $\hat{\mathcal{P}}_1$ represents a postselective balanced homodyne measurement where the right side of the phase space is measured only.

In the case when the detectors are signaling, we postselect from the joint state of field and qubits (21) the state

$$a_- |\Psi^- \rangle + b_- |\Phi^- \rangle = a_- |\Psi^- \rangle - b_- |\Phi_{\pi/2}^+ \rangle. \quad (28)$$

Hence, the initial state before the second interaction reads

$$|\Psi_0\rangle = (a_- |\Psi^- \rangle - b_- |\Phi_{\pi/2}^+ \rangle) |\alpha e^{i\pi/2}\rangle. \quad (29)$$

After $\tau = 1/2$ interaction time, we get the state

$$\begin{aligned} & a_- |\Psi^-\rangle |\alpha'\rangle + b_- |\Phi_{-\pi/2}^+\rangle (|\alpha'_+\rangle + |\alpha'_-\rangle) \\ & + b_- |\Psi^+\rangle (|\alpha'_-\rangle - |\alpha'_+\rangle), \end{aligned} \quad (30)$$

where $\alpha' = \alpha e^{i\pi/2}$ and we have used the relation $|\Phi_{-\pi/2}^+\rangle = |\Phi_{3\pi/2}^+\rangle$. The field of the second cavity is measured with the help of the following projector,

$$\hat{P}_2 = \int_0^\infty |x_0\rangle \langle x_0| dx_0, \quad (31)$$

with properties

$$\hat{P}_2 |\alpha'\rangle = |\alpha'\rangle, \quad \hat{P}_2 |\alpha'_\pm\rangle = 0. \quad (32)$$

The projector \hat{P}_2 represents a postselective balanced homodyne measurement where the upper side of the phase space is measured only. If the detector signals, then we postselect the two qubit state $|\Psi^-\rangle$ with success probability $|a_-|^2$ [compare with the initial condition in (20)]. Otherwise, we have

$$\begin{aligned} & (\hat{I} - \hat{P}_2) |\alpha'\rangle = 0, \\ & (\hat{I} - \hat{P}_2) (|\alpha'_-\rangle - |\alpha'_+\rangle) = 0, \\ & (\hat{I} - \hat{P}_2) (|\alpha'_+\rangle + |\alpha'_-\rangle) = (|\alpha'_+\rangle + |\alpha'_-\rangle), \end{aligned} \quad (33)$$

where \hat{I} is the identity operator. Thus, in the case of no signaling we postselect the two-qubit state $|\Phi_{-\pi/2}^+\rangle$ with success probability $|b_-|^2$.

Let us turn back to the case when the detectors which postselect the field of the first cavity do not signal. In this case, we obtain the following two-qubit state:

$$a_+ |\Psi^+\rangle + b_+ |\Phi_{\pi/2}^+\rangle = a_+ |\Psi^+\rangle - b_+ |\Phi_{\pi/2}^-\rangle. \quad (34)$$

Thus, before the second interaction, the initial state reads

$$|\Psi_0\rangle = (a_+ |\Psi^+\rangle - b_+ |\Phi_{\pi/2}^-\rangle) |\alpha e^{i\pi/2}\rangle, \quad (35)$$

and after $\tau = 1/2$ interaction time, we have the following joint state of field and qubits:

$$\begin{aligned} & -b_+ |\Phi_{\pi/2}^-\rangle |\alpha'\rangle + a_+ |\Phi_{-\pi/2}^+\rangle (|\alpha'_-\rangle - |\alpha'_+\rangle) \\ & + a_+ |\Psi^+\rangle (|\alpha'_+\rangle + |\alpha'_-\rangle). \end{aligned} \quad (36)$$

In case of successful signaling when \hat{P}_2 is applied, the protocol postselects the state $|\Phi_{\pi/2}^-\rangle$ with success probability $|b_+|^2$; otherwise, i.e., $\hat{I} - \hat{P}_2$ is applied, we get $|\Psi^+\rangle$ with $|a_+|^2$ success probability.

Let us apply these results to the three-node scheme (A, B, and C) explained at the beginning of this subsection. The initial state we consider is

$$\begin{aligned} |\Psi_0\rangle & = |\Psi^-\rangle_{AB_1} \otimes |\Psi^-\rangle_{B_2C} \\ & = -\frac{1}{2} |\Psi^-\rangle_{AC} |\Psi^-\rangle_{B_1B_2} - \frac{1}{2} |\Phi^+\rangle_{AC} |\Phi^+\rangle_{B_1B_2} \\ & \quad + \frac{1}{2} |\Psi^+\rangle_{AC} |\Psi^+\rangle_{B_1B_2} + \frac{1}{2} |\Phi^-\rangle_{AC} |\Phi^-\rangle_{B_1B_2}. \end{aligned} \quad (37)$$

It is immediate that each of the Bell measurements occurs with 25% probability and toward A and C the following classical

communication protocol is applied:

$$\begin{aligned} & \{\hat{P}_1, \hat{P}_2\} \rightarrow -|\Psi^-\rangle_{AC}, \\ & \{\hat{P}_1, \hat{I} - \hat{P}_2\} \rightarrow e^{i\pi/2} |\Phi^-\rangle_{AC}, \\ & \{\hat{I} - \hat{P}_1, \hat{P}_2\} \rightarrow e^{-i\pi/2} |\Phi^+\rangle_{AC}, \\ & \{\hat{I} - \hat{P}_1, \hat{I} - \hat{P}_2\} \rightarrow |\Psi^+\rangle_{AC}. \end{aligned}$$

III. PERFORMANCE AND LIMITATIONS

In the subsequent discussion, we compare the prerequisite of our proposal with the status of current developments in experimental physics. In view of these experimental setups and their parameters, we give the performance of our proposed quantum repeater.

A. Experimental considerations

In our proposal, each repeater node requires four cavities as explained in Sec. II, where two of the cavities are coupled one sided to fibers. In order that the cavities couple efficiently to the fiber links, they may be built with asymmetric mirror transmissions. If mirrors with high transmission rates are coupled to the single-mode optical fiber, then this leads to a highly directional single-mode output [34]. In this configuration together with ^{87}Rb atoms, the whole setup operates in the intermediate-coupling regime of cavity QED $\{g, \kappa, \Gamma\} \approx 2\pi \times \{5, 3, 3\}$ MHz. The other requirement is that these cavities are also coupled to outer lasers which prepare the coherent states inside them and to balanced homodyne measurement setups. A possible solution is to pierce a small hole in the center of the mirror with low transmission rate, thus allowing a good in and out coupling. This experimental technique has already been reported for microwave cavities [35]. The other two cavities, which are used in the entanglement purification protocol and in the generation of complete Bell measurements, have to be able to couple strongly and symmetrically the supported single-mode radiation field to two atoms. This implementation of the two-atom Tavis-Cummings model has been experimentally reported for neutral Cs atoms with $\{g, \kappa, \Gamma\} \approx 2\pi \times \{18.0, 0.4, 5.2\}$ MHz [36], for $^{40}\text{Ca}^+$ ions with $\{g, \kappa, \Gamma\} \approx 2\pi \times \{1.0, 0.05, 11.5\}$ MHz [37], and for ^{87}Rb atoms with $\{g, \kappa, \Gamma\} \approx 2\pi \times \{7.6, 2.8, 3.0\}$ MHz [38]. Furthermore, these cavities have to have a good in and out coupling with external radiation fields, such that the preparation of coherent state and the postselective field measurements via balanced homodyne photodetection can effectively be carried out.

Another critical step in our proposal is that the atoms, implementing the qubits, can be transported through the cavities. The transport has to be highly controllable and a possible solution is the use of optical conveyor belts. Cs atoms can be captured from a vapor, cooled down, and trapped in a high-gradient magneto-optical trap [39]. A dipole trap is formed with help of two counterpropagating laser beams and the atoms are transferred without loss from the magneto-optical trap into the dipole trap [40]. By detuning the frequencies of the laser beams, one can set into motion the standing wave which acts as a optical conveyor belt and transports the atoms with high position precision into the

cavities. Furthermore, the speed of the conveyor belts defines the interaction time between the atoms and the radiation field inside the cavities. Because of the purification protocol, we also require that the number of atoms loaded in the optical conveyor belt is high as possible (2^N with N iterations) and a decade ago it was reported that it is possible to load 19 atoms efficiently into the dipole trap [41]. The atoms in the dipole trap can be subject to coherent manipulations as imposed in the purification protocol, in which measurement-dependent transformations have to be carried out (see Sec. II B). After the qubit-field interactions, information on the internal atomic state have to be extracted. This can be done by applying push-out lasers on the conveyor belt and depending on the internal atomic state the Cs atom either remain in the trap or get pushed out [42]. In experiments without conveyor belts, one may use state-selective field ionization detectors for rubidium atoms [31].

Postselective field measurements are at the core of our proposed quantum repeater. These measurements are carried out via balanced homodyne photodetection. Here, the purpose is not to perform a complete state tomography on the radiation field emerging from the qubit-field interactions, but instead to measure a specific field state component and by thus conditionally postselect qubit states. All three protocols presented in Sec. II depend on the realization of such a measurement, which is capable of discriminating a coherent state from the rest of the field states, where all states are well separated from each other in the phase space. There are already investigations for such situations. For example, Ref. [43] has shown that the error probability of a scheme, where $|-α\rangle$ and $|\alpha\rangle$ are to be discriminated, is small also for the small mean number of photons involved, i.e., $\bar{n} \approx 0.4\text{--}1.4$. We have considered in our scheme $\bar{n} \approx 100$, so we believe that the implementation of our proposal with postselective field measurements is in the range of current experimental technologies and furthermore due to the large number of mean photons involved detector inefficiencies can also be overcome. The duration time of performing a quadrature measurement depends on the setup. Here, we estimate it to be equal to 5.5 ns [43–45].

Optical fibers are the key elements defining the distance between the nodes of a quantum repeater. The critical parameter defining this distance is the attenuation length, which is maximal at telecom wavelengths around $1.5\ \mu\text{m}$ [4]. The only issue here is that most of the atomic transitions of typical cavity QED atoms, which couple resonantly to the single-mode radiation field, are not at telecom wavelength and therefore the field state is not suitable for long-distance transmission over optical fibers due to high losses. There are two possible approaches: realizing cascade transitions [46,47] or using wavelength conversion [48]. These experiments are subject to the generation or conversion of single photons. We remind the reader that our scheme is based on multiphoton field states and therefore further experimental developments are required by our proposal in order to use optical fibers with telecom wavelength. Otherwise, resonant photons with atomic transitions will suffer high attenuation in optical fibers with frequencies equal to the atomic transitions. Provided that we are able to use optical fibers with telecom wavelength, then the photon loss in our theoretical proposal can be considered

to be approximately 0.2 dB/km. Thus, the propagation time T and the damping rate γ can be translated into a length L_0 , which characterizes the distance between two repeater nodes, by the relation $L_0 = 20(\gamma T)/(0.2\ \text{log } 10)$.

B. Rate analysis

In this subsection, we compute the rates at which near-maximally-entangled pairs are generated between the endpoints of the repeater chain. An important parameter is the time T_{link} which is required to purify a near-maximally-entangled state between two neighboring repeater nodes and the overall success probability P of this process. We are going to estimate these parameters depending on the number of elementary links n , the number of iterations N required in purification protocol, and the success probabilities obtained in Sec. II.

First, we analyze the repeaterless entanglement generation between two points separated by a distance L_0 . The total time T_1 attempting to generate an entangled qubit between these two points has the following parts: the time required for the two qubit-field interactions $2 \times 1/(2g)$ (characteristic time of the collapse phenomena in the Jaynes-Cummings model); the leak-in and leak-out processes $2 \times 1/\kappa$; the propagation time L_0/c where $c \approx 2 \times 10^8$ m/s is the speed of light in a telecom optical fiber; the time required for the balanced homodyne measurement T_{det} ; and the time L_0/c of classical communication in order to confirm or deny the success of the procedure. We consider a case where the reinitialization of the cavities and the detectors is done during the classical communication. Thus, we obtain

$$T_1 = \frac{1}{g} + \frac{2}{\kappa} + \frac{L_0}{c} + T_{\text{det}} + \frac{L_0}{c}. \quad (38)$$

Provided that at least two entangled qubit pairs are generated, the purification protocol may start and the total time T_2 attempting to purify a state out of these two states has the following parts: time required for the two qubit-field interactions $2 \times 1/(2g)$; the time T_{det} required for the balanced homodyne measurements and the qubit measurements; and the time of classical communication. Here, the classical communication is required both to confirm the success of the protocol and to postprocess the obtained state depending on the results of the qubit measurements. These considerations yield

$$T_2 = \frac{1}{g} + T_{\text{det}} + \frac{L_0}{c}. \quad (39)$$

Now, taking into account the experimental parameters discussed in the previous section, we realize that for distances by means of $L_0 > 2$ km both T_1 and T_2 have L_0/c as the dominant time, because T_{det} is mainly determined by the leaking out of the fields from the cavity into the measurement setups, whereas the quadrature measurements are very fast. We consider that qubit measurements are also fast. These considerations yield that $T_1 \approx 2L_0/c$ and $T_2 \approx L_0/c$.

If the distance L_0 is below 2 km then $1/\kappa$ is the dominant term in the duration of the process. In this case, the reinitialization of the cavities and the leaking of the fields toward the detectors must be taken into consideration and we estimate the whole time to be at least $2/\kappa$. Thus, for small distances both

T_1 and T_2 are considered to be approximately equal to $10 \mu\text{s}$ ($\kappa = 2\pi \times 0.05 \text{ MHz}$ from Ref. [37]).

Thus, for a distance L_0 (later an elementary link in the repeater chain), the time required to generate 2^N entangled pairs and obtain a highly entangled pair by N purification rounds is given by

$$T_{\text{link}} = 2^N T_1 + (2^N - 1)T_2, \quad (40)$$

where $2^N - 1$ is the number of the purification protocols applied. During this time, the overall success probability is

$$P = [P_{\text{Gen}}]^{2^N} P_{\text{Pur}}. \quad (41)$$

According to the purification protocol in Sec. II B, we purify toward $|\Psi^-\rangle$ or $|\Psi^+\rangle$ depending on the sign of x in Eq. (14). Therefore, we define the fidelity

$$F = \max\{f_{(N)}, g_{(N)}\}, \quad (42)$$

where $f_{(N)}$ and $g_{(N)}$ are given in Eq. (18), i.e., the general form of a purified state after N purification rounds. Applying our protocols of entanglement generation and purification to a point-to-point situation, one may talk about the rate of entangled pairs generated across a distance L_0 only if the achievable fidelities are also displayed as well. In this case, we are able to compare the performance of our scheme to a recent results of Ref. [49], where the ultimate rate of repeaterless quantum communication is given by $-\log_2(1 - \chi)$. This formula gives the rate of Bell pairs per channel use, which results in the rate of Bell pairs per second by multiplying it with the repetition rate $c/(2L_0)$; i.e., entanglement generation over the quantum channel is assisted by a classical communication. The parameter χ quantifies the fraction of photons surviving the channel, i.e., the transmissivity of the channel, which in our case yields $\chi = \eta \exp\{-\gamma T\}$. η and γ have been introduced in Sec. II A to characterize the fraction of photons not reflected from the surface of the second cavity and the decay rate of the optical fiber.

In Fig. 4, average rates of generated entangled pairs over a distance L_0 are analyzed for different numbers $N \in \{1, 2, 3, 4\}$ of purification rounds and $\eta \in \{0.8, 1\}$. These figures clearly demonstrate that there is a tradeoff between average rates and the fidelity of the pairs with respect to a Bell state ($|\Psi^-\rangle$ or $|\Psi^+\rangle$). If $\eta = 1$, then near-maximally-entangled pairs are either generated on very short distances $L_0 \approx 500 \text{ m}$ with an average rate ≈ 2625 pairs per second or larger distances $L_0 \approx 4 \text{ km}$ with an average rate $\approx 10^{-2}$ pairs per second. If $\eta = 0.8$, then we require at least $N = 4$ purification rounds and for a distance of $L_0 \approx 2.5 \text{ km}$ we obtain a very low average rate $\approx 10^{-11}$ pairs per second. These distances and average rates define also the possible elementary links of the repeater chain, because applying entanglement swapping procedure to low-fidelity pairs reduces the fidelity of the output pairs even further. Now, if we compare our results with the ultimate rate of Ref. [49], it becomes clear that our protocol has a low performance unless $\eta \approx 1$ and $L_0 \leq 0.5 \text{ km}$. However, the result in Ref. [49] is an upper bound for rates assuming arbitrary local operations and unlimited classical communication, thus being a benchmark rate for quantum repeater proposals.

In the next step, we are going to discuss several scenarios where the quantum repeater protocol is in use. Let us denote by $L = nL_0$ the length of the repeater chain with L_0 being the length of an elementary link and consequently n being the number of the links. We calculate the average number of attempts of preparing one near-maximally-entangled pair between all the repeater nodes. The best strategy here is to use memories and implement n parallel processes. As soon as one near-maximally-entangled pair has been generated along one elementary link, its state is saved in a quantum memory, while between the other nodes the process is repeated until we succeed. Provided that we are successful along all the links, the average number of attempt is [50]

$$A_n = \sum_{i=1}^n \binom{n}{i} \frac{(-1)^{i+1}}{1 - (1 - P)^i} \quad (43)$$

with P given in Eq. (41).

Another characteristic time of a quantum repeater, which is T_{swap} , stands for the overall time required to entangle the two endpoints of the repeater chain with the help of entanglement swapping procedures. Based on our proposal in Sec. II C, the swapping procedure is deterministic and the characteristic time of a single swap contains the following processes: the time required for the two subsequent qubit-field interactions $2 \times 1/(\sqrt{2}g)$ (characteristic time of the collapse phenomena in the two-qubit Tavis-Cummings model); the time T_{det} required for the two balanced homodyne measurements; and the time of classical communication between the nodes in order to inform the parties about which state has been swapped. In order to speed up the whole swapping process, parallel entanglement swappings are carried out for intermediate nodes. For example, $n = 100$ elementary links are reduced in the first round to $n = 50$ links, in the second round to $n = 26$ links, and so on, until we have $n = 1$ link, which means that we reached the endpoints of the repeater chain. Hence,

$$T_{\text{swap}} = \lceil \log_2 n \rceil \left(\frac{\sqrt{2}}{g} + 2T_{\text{det}} + \frac{L_0}{c} \right), \quad (44)$$

where $\lceil \cdot \rceil$ is the ceiling function. The time T_{swap} has the dominant term L_0/c unless $L_0 < 2 \text{ km}$, when we consider $T_{\text{swap}} = 10 \mu\text{s}$, where $g = 2\pi \times 1.0 \text{ MHz}$ and $T_{\text{det}} \approx 1/\kappa$ with $\kappa = 2\pi \times 0.05 \text{ MHz}$ (see Ref. [37]).

An interesting feature arises when we apply the swapping procedure to $\hat{\rho}_{A, B_1} \otimes \hat{\rho}_{B_2, C}$ with Bell measurements on qubit systems B_1 and B_2 and the shared state between the nodes being the output state of the purification protocol [see Eq. (18)]

$$\hat{\rho} = p|\Psi^-\rangle\langle\Psi^-| + (1 - p)|\Psi^+\rangle\langle\Psi^+|.$$

We have iterated the entanglement purification until either p or $1 - p$ is larger than 0.999; however, the equality $p = 1$ or $p = 0$ in principle cannot be reached in finite numbers of purification rounds. During the applications of the swapping procedure, this is an issue, because the fidelity F introduced in Eq. (42) reduces after k rounds of parallel swapping procedures as

$$\begin{aligned} F^{(k)} &= f(F^{(k-1)}), \quad F^{(0)} = F, \\ f(x) &= 1 - 2x + 2x^2. \end{aligned} \quad (45)$$

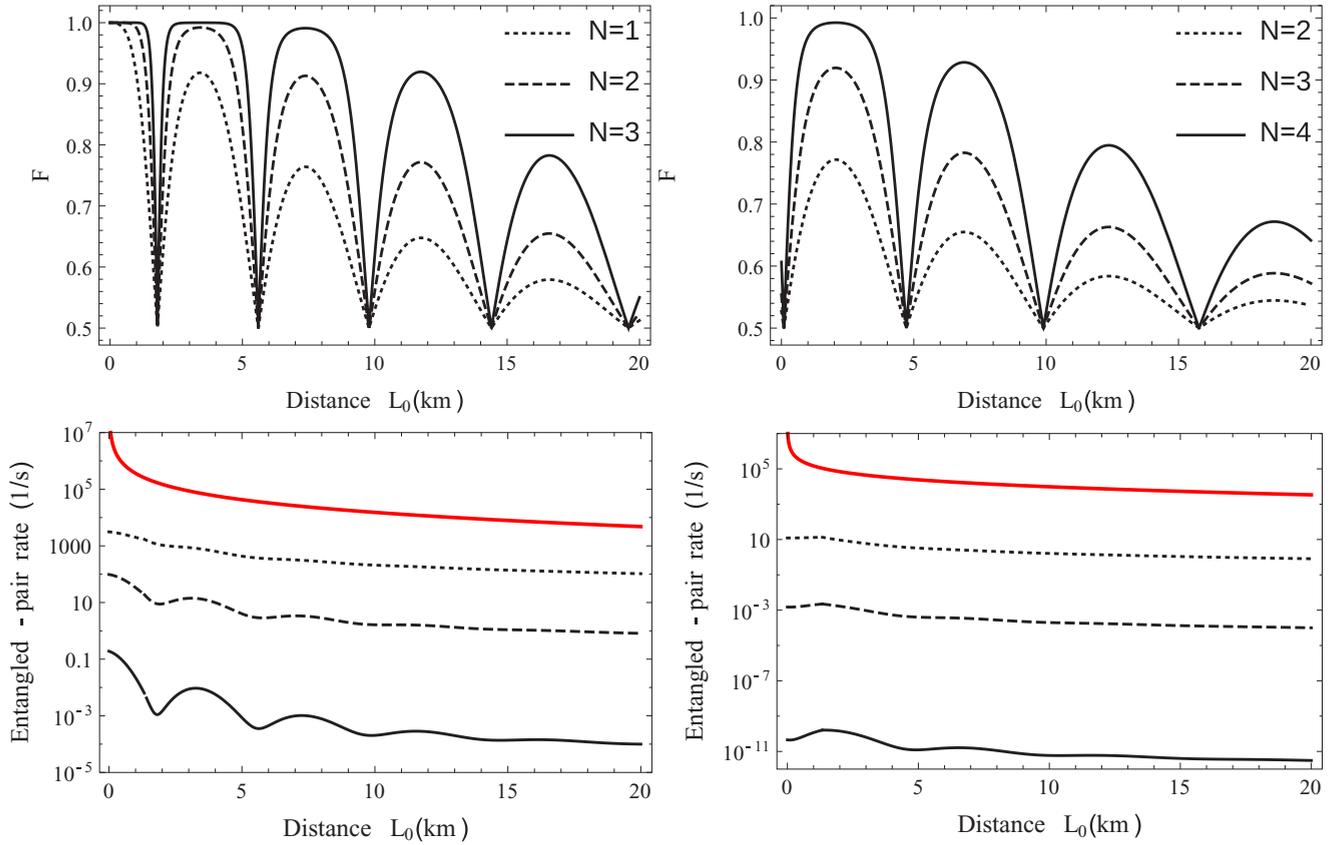


FIG. 4. Entanglement generation in repeaterless scenarios. Top panels: Achievable fidelities F [see (42)] of entangled pairs generated between two points separated by the distance L_0 . N is the number of purification rounds. Bottom panels: Semilogarithmic plot of average rates of entangled pairs with the same purification rounds as in the same (right or left) top figure. Left panels: The rate $1 - \eta$ of reflected number of photons from the surface of the second cavity (see Sec. II A) is taken to be zero. Right panels: $1 - \eta = 0.2$. The red (gray) curves are the ultimate rates of Bell pairs per second in repeaterless quantum communication [49], where the transmissivity of the channel with length L_0 is defined by pure fiber loss in the left panel and fiber loss and η in the right panel. Note that not all rate values are accompanied by high fidelities in the respective top figure and for isolated cases when $F = 0.5$ the pairs are not even entangled. The interaction time has been set to $\tau = 4/g$ with mean photon number $\bar{n} = 100$.

As an example, consider a repeater chain with $n = 60$ elementary links, which means that the number of parallel swapping procedures $k = \lceil \log_2 60 \rceil = 6$. If the fidelity of the pairs between the nodes is $F = 0.999$, then the fidelity of the final pair between the end points is $F^{(6)} = 0.939$. Therefore, in order that we can talk about near-maximally-entangled pairs ($F > 0.999$), two more purification rounds have to be performed at the end points of the repeater chain. In numbers, an average of 41 pairs have to be postprocessed. Therefore, we introduce the average number of pairs $\bar{N} = 2^j / P_{\text{pur}}$ [see Eq. (19) for j purification rounds] involved in the final purification procedure, where j depends on $F^{(k)}$ [see Eq. (45)] with $k = \lceil \log_2 n \rceil$ and n is the number of elementary links.

Finally, the average rate is given by

$$R = \frac{1}{\bar{N}(T_{\text{link}}A_n + T_{\text{swap}})}. \quad (46)$$

We have already mentioned that the number of qubits available at the nodes is limited due to the current stage technology of conveyor belts. As this number is 19 (see Ref. [41]), we will consider no more than four iterations of the entanglement purification procedure. Although this number is

small, Fig. 5 shows that four iterations decrease extremely the average rate R of generated near-maximally-entangled pairs. This results clearly reflects the very expensive nature of purification protocols and shows that increasing the number of purification rounds leads to unrealistic demands of quantum memory. In the case of $N = 4$, this means that we require a quantum memory which is capable of protecting the coherency of the states for 10^8 s, i.e., more than 3 years. In the $N = 2$ case, we have much higher average rates; however, the fidelity of the states (still larger than 0.999) obtained after two purification rounds is affected by the swapping procedures and a few more purification rounds have to be carried out at the endpoints of the repeater chain. Here, we have considered an ideal scenario where $\eta = 1$ [see Eq. (8)] and $x = -0.5$, which according to Fig. 2 yields an elementary link length $L_0 \approx 3.5$ km.

As we increase the number of elementary links in order to obtain larger distances L , we are facing a situation where the fidelities of the pairs are more reduced by the swapping procedures. Thus, extra purification rounds are required at the end points of the repeater chain and this postprocess results in the decrease of the average rates. For example, let us consider that across all links we have purified pairs with fidelity $F = 1 - \epsilon$, where ϵ is a threshold number defining what we

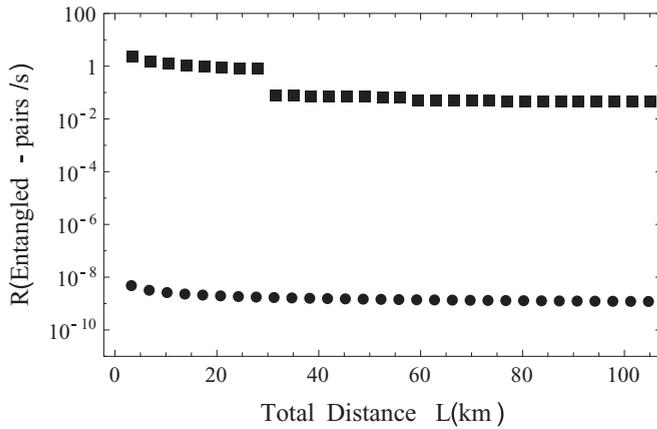


FIG. 5. Semilogarithmic plot of average rates of near-maximally-entangled ($F > 0.999$) pairs between two end points separated by the total distance L . The elementary link between two nodes is 3.5 km. Two scenarios are presented for different number of iterations of the purification protocol: $N = 2$ (points displayed as squares) and $N = 4$ (circles). Drops in the average rate are due to the destructive effects of the swapping procedure on the fidelities, according to Eq. (45), and therefore extra entanglement purification is required at the end points of the repeater chain. $1 - \eta = 0$; i.e., there are no photons reflected back from the surface of the second cavity (see Sec. II A), and the elementary link length determines accordingly the fidelity of the repeater protocol's input pairs in Eq. (14).

call a near-maximally-entangled pair ($\epsilon < 0.001$ in this paper). If we have n elementary links, then the fidelity after the swapping procedures is approximately equal to $1 - 2^{\lceil \log_2 n \rceil} \epsilon$. Therefore, strategies for large distances and with lower number of purification rounds may not generate pairs with sufficiently high fidelity, such that these fidelities do not drop way below

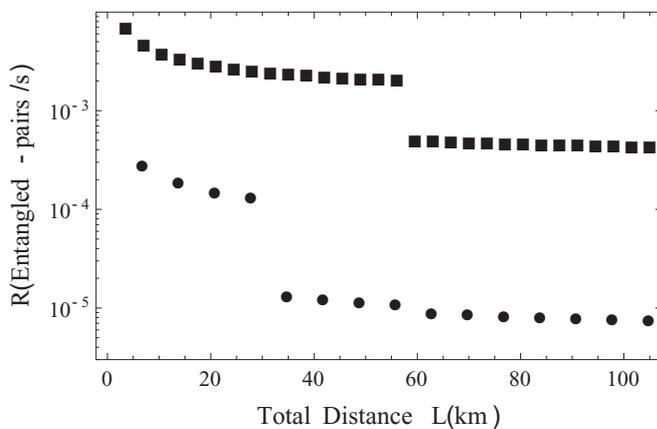


FIG. 6. Semilogarithmic plot of average rates of near-maximally-entangled ($F > 0.999$) pairs between two end points separated by the total distance L . Two cases are presented for different elementary link lengths: $L_0 = 3.5$ km (points displayed as squares), and $L_0 = 7$ km (circles). Both cases are depicted for $N = 3$ purification rounds. For a brief explanation for the drops in the average rates, see Fig. 5. $1 - \eta = 0$; i.e., there are no photons reflected back from the surface of the second cavity (see Sec. II A), and the elementary link lengths determine accordingly the fidelities of the repeater protocol's input pairs in Eq. (14).

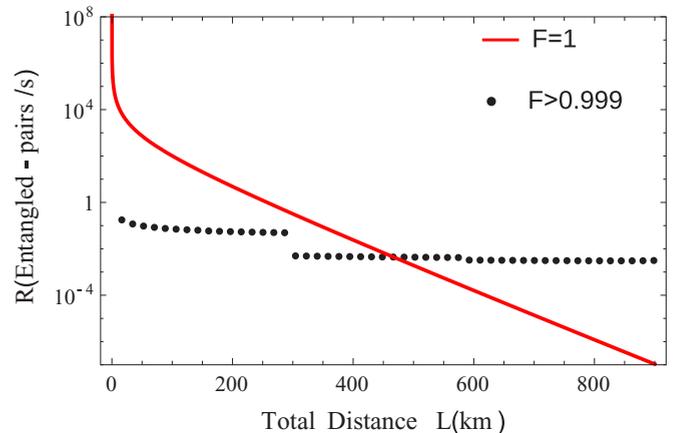
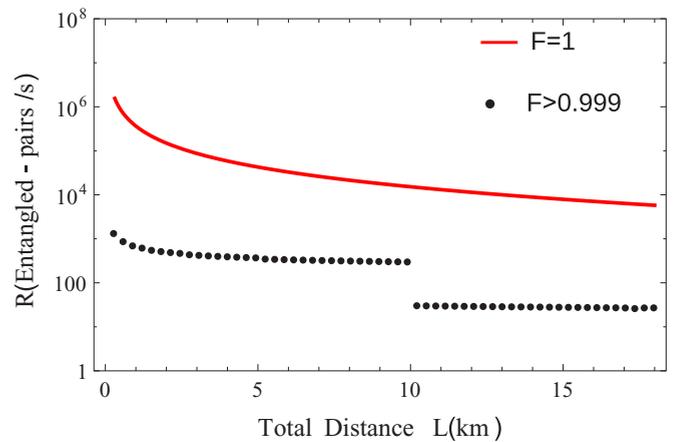


FIG. 7. Semilogarithmic plot of average rates of entangled pairs between two end points separated by the total distance L . Top panel: The scenario (black dots) with $L_0 = 0.3$ km long elementary links and $N = 1$ purification round is depicted up to 18 km, where near-maximally-entangled ($F > 0.999$) pairs are generated. Bottom panel: The 18-km-long repeater chain of the top figure is considered now as an elementary link and with $N = 1$ purification round is depicted up to 900 km. $1 - \eta = 0$; i.e., there are no photons reflected back from the surface of the second cavity (see Sec. II A), and the elementary link length determines accordingly the fidelity of the repeater protocol's input pairs in Eq. (14). The solid lines with $\eta = 1$ are the ultimate rates of pure fiber loss in repeaterless quantum communication [49].

the threshold fidelity $1 - \epsilon$ after the swapping procedures are applied. Depending how much they have dropped, more purification rounds have to be carried out at the end points of the repeater chain, which yield significantly reduced average rates. These reduced rates may be comparable with other strategies with high number of purification rounds.

In Fig. 6, we compare two quantum repeaters with the only difference being the elementary link length. We have considered a scenario where $\eta = 1$ and we have taken for $x = 0.5$, i.e., $L_0 \approx 3.5$ km, and 0.3 , i.e., $L_0 \approx 7$ km (see Fig. 2). Despite the longer distance of the elementary link length, low probabilities of purifying the state characterized by $x = 0.3$ reduce the average rate R of generated near-maximally-entangled pairs at the two end points separated by the total distance L . Furthermore, the destructive effects of the swapping procedures affect both strategies. The scenario with shorter elementary links is less affected, because here we

generate higher fidelity pairs between the repeater nodes than in the scenario with 7-km-long elementary links.

In the last case, we set $\eta = 1$ and $L_0 = 0.3$ km, which yield $x = 0.913$. This means that the fidelity of the generated pairs [see Eq. (14)] is high enough to obtain a near-maximally-entangled pair after only one round of the purification protocol. In the top panel of Fig. 7, we compare this scenario with the ultimate rate of Bell pairs per second in repeaterless quantum communication [49] and we see that the average rate of the quantum repeater protocol starts with lower values, but it seems that scales with L better. In this scenario, we are bound in our numerics to 18 km, because we have 60 elementary links and the binomial $\binom{n}{i}$ in (43) may take extremely large values which are multiplied with very small numbers. Above 18 km or 60 elementary links, the numerical instabilities are increasing and they lead to meaningless average rate numbers. It is also worth mentioning that in this case with 60 elementary links one must build 59 intermediate repeater nodes, which is an expensive procedure in regards to physical resources of cavity QED. In order to show that this repeater scenario exceeds the ultimate limit of Ref. [49], we embed the above discussed of an 18-km-long repeater chain into a longer repeater chain as an elementary link. Therefore, we determine the probability of generating an entangled pair with $x = 0.8769$ over 18 km, which yields 0.026. This approach circumvents the use of large valued binomials $\binom{n}{i}$. In the bottom panel of Fig. 7, we see that this longer repeater chain crosses the ultimate repeaterless rate around 500 km and for a total distance of $L = 900$ km the rate is found to be 3.6×10^{-3} . This achievement is contrasted with the required number of repeater nodes, which turns out to be 2999.

We therefore conclude that for a realistic implementation of our proposal, the generation of high-fidelity entangled states between the nodes is the most crucial ingredient. Entanglement purification is a very expensive procedure which cannot be properly compensated for by high repetition rates and the best strategy would be to generate entangled states which can be purified in one step [51]. Our numerical investigations shows that the parameter η [defined in Eq. (8)] quantifying the fraction of photons entering from the optical fiber into the cavity is the most crucial hurdle for obtaining high average rates, because the photon loss rate γ can be recompensated by choosing shorter elementary links. There exists experiments, which are able to obtain $\eta \approx 1$; however, they operate with single photons on short distances [34]. In scenarios where $\eta = 1$, we obtained an average rate $R = 23$ with 60 elementary links for $L = 18$ km and $R = 3.6 \times 10^{-3}$ with 3000 elementary links for $L = 900$ km. Furthermore, this scenario from a theoretical point of view outperforms the ultimate rate of repeaterless quantum communication [49]. A much lesser average rate $R = 5 \times 10^{-4}$ is found for 30 elementary links and a total distance $L = 105$ km. In summary, large distances and high repetition rates require a lot of repeater nodes at the expense of physical resources.

IV. CONCLUSIONS

We have presented a hybrid quantum repeater based on resonant qubit-field interactions. In our scheme, all two-qubit operations required for the building blocks are generated via

qubit-field interactions and postselective field measurements, thus making our proposal a good candidate for experimental implementation.

In the context of entanglement generation between the repeater nodes, we have investigated a system of two spatially separated material qubits coupled to single-mode cavities. In addition, these cavities are connected by an optical fiber. For the description of qubit-field interactions, we have used the resonant Jaynes-Cummings-Paul model and entanglement is generated between the distant qubits by a postselective balanced homodyne photodetection. Our model is subject to two type of decoherence, namely the photon loss in the optical fiber and the photon reflection from the surface of the cavity, an effect of the fiber-cavity coupling inefficiencies. These considerations extend former studies on hybrid quantum repeaters. Within this model, we have found that the quality of entangled qubit states, quantified via the concurrence, is very sensitive to the the photon reflection, which has a strong impact on the orthogonality of field states involved in postselective field measurements. In the case of small reflectivity and several-km-long optical fibers, we have shown that high-fidelity entangled states can be created with 50% success probability, which is an improvement of our former result in Ref. [17].

In the next step, entangled state obtained in the first building block of the quantum repeater have been considered as input states for an entanglement purification protocol, a recurrence protocol, introduced by us [19,51]. The theoretical model consists of two qubits, which sequentially interact with a single-mode cavity, and postselective field measurements. We have used the Jaynes-Cummings-Paul model and its solutions for the description of the interactions. These interactions and the field measurements generate a probabilistic two-qubit quantum operation, which takes over the role of the controlled-NOT gate used in standard purification protocols. We have found that the overall success probability of purifying near-maximally-entangled pairs is very low, and the results for four steps of iterations are already unrealistic. Therefore, it is more beneficial to use as few purification rounds as possible; otherwise the entanglement protocol becomes very expensive in regard to physical resources.

For the final building block of the quantum repeater, the entanglement swapping, we have considered two qubits, which interact simultaneously with single-mode cavities. We have employed the Tavis-Cummings model and its solutions. The Bell measurement are generated by postselecting the emerged cavity fields. This study, which is based on our former results [18,20], has been extended by a new set of field measurements, which are able to project on field states lying on the opposite side of phase space's axes. These postselective field measurements allow for deterministic realization of unambiguous Bell measurements, an improvement of our result in Ref. [20] and an important ingredient in the effective actuation of quantum repeaters.

As all three building blocks consists of the same cavity QED elements, we have collected some recent experimental developments with respect to these components. We have presented the parameters of these experiments and discussed the pros and cons of an implementation. A comprehensive analysis of the quantum repeater's rate of generating

near-maximally-entangled pairs per second has been given. In particular, we have found that moderately low rates can be achieved in the context of current experimental technologies for distances up to 100 km. This result is mainly due to the request that we purify near-maximally-entangled pairs ($0.999 < F < 1$) between the neighboring nodes. In addition, these purified pairs cannot reach in principle $F = 1$ and therefore the swapping procedures have destructive effects on the fidelities. This may result in extra purification rounds for the entangled pairs between the end points of the repeater chain. If the purification protocol is required at the end points, then the average rates are further reduced. We have also compared our results with the ultimate rate of repeaterless quantum communication [49] and we have shown that the average rates of our proposal with very high number of nodes exceeds this benchmark value around 500 km. This occurs in cases where the elementary links are a few hundred meters long and we use only one purification round.

In summary, the strength of our proposal is in the compatible and cavity-QED-based building blocks, which can easily augment each other. The main idealistic assumptions throughout this paper are the following: nondecaying qubits, i.e., perfect quantum memories, and unit efficiency detectors. In future work, we aim to relax one or both of these

assumptions. In view of these considerations, our proposal gives a better understanding of the influence of the building blocks on each other and shows its own limitations on the achievable repeater rates on moderate distances. These limitations may be surpassed only if in the first building block we generate such type of entangled states, which can be purified into a Bell state ($F = 1$) in one purification round [51]. Hence, this scenario is able to avoid the low success probabilities of several purification rounds and the destructive effects of swapping procedures.

We hope that our work is a step forward to an experimental realization of the first hybrid quantum repeater. In addition, the proposed scheme mainly relies on current technology and thus offers a clear perspective on a future experimental demonstration.

ACKNOWLEDGMENTS

This work is supported by the Bundesministerium für Bildung und Forschung project Q.com and the European Unions Horizon 2020 (Belgium) research and innovation programme under Grant Agreement No. 732894 (FET Proactive HOT). In addition, the author would like to thank the anonymous reviewer for helpful and constructive comments that greatly contributed to improving the paper.

-
- [1] M. A. Nielsen and A. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, UK, 2000).
 - [2] H. J. Kimble, *Nature (London)* **453**, 1023 (2008).
 - [3] B. Korzh, C. C. W. Lim, R. Houlmann, N. Gisin, M. J. Li, D. Nolan, B. Sanguinetti, R. Thew, and H. Zbinden, *Nat. Photon.* **9**, 163 (2015).
 - [4] T. Inagaki, N. Matsuda, O. Tadanaga, M. Asobe, and H. Takesue, *Opt. Express* **21**, 23241 (2013).
 - [5] J. Yin, Y. Cao, Y.-H. Li, S.-K. Liao, L. Zhang, J.-H. Ren, W.-Q. Cai, W.-Y. Liu, B. Li, H. Dai *et al.*, *Science* **356**, 1140 (2017).
 - [6] R. Ursin, F. Tiefenbacher, T. Schmitt-Manderbach, H. Weier, T. Scheidl, M. Lindenthal, B. Blauensteiner, T. Jennewein, J. Perdigues, P. Trojek *et al.*, *Nat. Phys.* **3**, 481 (2007).
 - [7] D. Dieks, *Phys. Lett. A* **92**, 271 (1982).
 - [8] W. K. Wootters and W. H. Zurek, *Nature (London)* **299**, 802 (1982).
 - [9] H.-J. Briegel, W. Dür, J. I. Cirac, and P. Zoller, *Phys. Rev. Lett.* **81**, 5932 (1998).
 - [10] W. Dür, H.-J. Briegel, J. I. Cirac, and P. Zoller, *Phys. Rev. A* **59**, 169 (1999).
 - [11] N. Sangouard, C. Simon, H. de Riedmatten, and N. Gisin, *Rev. Mod. Phys.* **83**, 33 (2011).
 - [12] P. van Loock, T. D. Ladd, K. Sanaka, F. Yamaguchi, K. Nemoto, W. J. Munro, and Y. Yamamoto, *Phys. Rev. Lett.* **96**, 240501 (2006).
 - [13] T. D. Ladd, P. van Loock, K. Nemoto, W. J. Munro, and Y. Yamamoto, *New J. Phys.* **8**, 184 (2006).
 - [14] P. van Loock, N. Lütkenhaus, W. J. Munro, and K. Nemoto, *Phys. Rev. A* **78**, 062319 (2008).
 - [15] D. Gonta and P. van Loock, *Phys. Rev. A* **88**, 052308 (2013).
 - [16] D. Gonta and P. van Loock, *Appl. Phys. B* **122**, 118 (2016).
 - [17] J. Z. Bernád and G. Alber, *Phys. Rev. A* **87**, 012311 (2013).
 - [18] J. M. Torres, J. Z. Bernád, and G. Alber, *Phys. Rev. A* **90**, 012304 (2014).
 - [19] J. Z. Bernád, J. M. Torres, L. Kunz, and G. Alber, *Phys. Rev. A* **93**, 032317 (2016).
 - [20] J. M. Torres, J. Z. Bernád, and G. Alber, *Appl. Phys. B* **122**, 117 (2016).
 - [21] S. Muralidharan, J. Kim, N. Lütkenhaus, M. D. Lukin, and L. Jiang, *Phys. Rev. Lett.* **112**, 250501 (2014).
 - [22] W. P. Schleich, *Quantum Optics in Phase Space* (Wiley-VCH, Weinheim, Germany, 2001).
 - [23] E. T. Jaynes and F. W. Cummings, *Proc. IEEE* **51**, 89 (1963).
 - [24] H. Paul, *Ann. Phys.* **466**, 411 (1963).
 - [25] A. W. Snyder and J. D. Love, *Optical Waveguide Theory* (Chapman and Hall, London, 1991).
 - [26] F. W. Cummings, *Phys. Rev.* **140**, A1051 (1965).
 - [27] A. I. Lvovsky and M. G. Raymer, *Rev. Mod. Phys.* **81**, 299 (2009).
 - [28] W. K. Wootters, *Phys. Rev. Lett.* **80**, 2245 (1998).
 - [29] C. H. Bennett, G. Brassard, S. Popescu, B. Schumacher, J. A. Smolin, and W. K. Wootters, *Phys. Rev. Lett.* **76**, 722 (1996); **78**, 2031 (1997).
 - [30] D. Deutsch, A. Ekert, R. Jozsa, C. Macchiavello, S. Popescu, and A. Sanpera, *Phys. Rev. Lett.* **77**, 2818 (1996); **80**, 2022 (1998).
 - [31] S. Gleyzes, S. Kuhr, C. Guerlin, J. Bernu, S. Deléglise, U. B. Hoff, M. Brune, J. M. Raimond, and S. Haroche, *Nature (London)* **446**, 297 (2007).
 - [32] Exponentially low probabilities are due to the constraint that at every iteration of the purification protocol the two-qubit operations on all the qubit pairs have to be successful. In Fig. 5

- of our former work [19], the overall success probability was erroneously overestimated and therefore the average number of qubits was underestimated.
- [33] M. Tavis and F. W. Cummings, *Phys. Rev.* **170**, 379 (1968).
- [34] S. Ritter, C. Nölleke, C. Hahn, A. Reiserer, A. Neuzner, M. Uphoff, M. Mücke, E. Figueroa, J. Bochmann, and G. Rempe, *Nature (London)* **484**, 195 (2012).
- [35] J. M. Raimond, M. Brune, and S. Haroche, *Rev. Mod. Phys.* **73**, 565 (2001).
- [36] R. Reimann, W. Alt, T. Kampschulte, T. Macha, L. Ratschbacher, N. Thau, S. Yoon, and D. Meschede, *Phys. Rev. Lett.* **114**, 023601 (2015).
- [37] B. Casabone, K. Friebe, B. Brandstätter, K. Schüppert, R. Blatt, and T. E. Northup, *Phys. Rev. Lett.* **114**, 023602 (2015).
- [38] A. Neuzner, M. Körber, O. Morin, S. Ritter, and G. Rempe, *Nat. Photon.* **10**, 303 (2016).
- [39] Z. Hu and H. J. Kimble, *Opt. Lett.* **19**, 1888 (1994).
- [40] M. Khudaverdyan, W. Alt, I. Dotsenko, T. Kampschulte, K. Lenhard, A. Rauschenbeutel, S. Reick, K. Schörner, A. Widera, and D. Meschede, *New J. Phys.* **10**, 073023 (2008).
- [41] L. Förster, W. Alt, I. Dotsenko, M. Khudaverdyan, D. Meschede, Y. Miroshnychenko, S. Reick, and A. Rauschenbeutel, *New J. Phys.* **8**, 259 (2006).
- [42] S. Kuhr, W. Alt, D. Schrader, I. Dotsenko, Y. Miroshnychenko, W. Rosenfeld, M. Khudaverdyan, V. Gomer, A. Rauschenbeutel, and D. Meschede, *Phys. Rev. Lett.* **91**, 213002 (2003).
- [43] C. Wittmann, M. Takeoka, K. N. Cassemiro, M. Sasaki, G. Leuchs, and U. L. Andersen, *Phys. Rev. Lett.* **101**, 210501 (2008).
- [44] H. Hansen, T. Aichele, C. Hettich, P. Lodahl, A. I. Lvovsky, J. Mlynek, and S. Schiller, *Opt. Lett.* **26**, 1714 (2001).
- [45] M. Cooper, C. Söller, and B. J. Smith, *J. Mod. Opt.* **60**, 611 (2013).
- [46] T. Chanelière, D. N. Matsukevich, S. D. Jenkins, T. A. B. Kennedy, M. S. Chapman, and A. Kuzmich, *Phys. Rev. Lett.* **96**, 093604 (2006).
- [47] M. Uphoff, M. Brekenfeld, G. Rempe, and S. Ritter, *Appl. Phys. B* **122**, 46 (2016).
- [48] A. Lenhard, M. Bock, C. Becher, S. Kucera, J. Brito, P. Eich, P. Müller, and J. Eschner, *Phys. Rev. A* **92**, 063827 (2015).
- [49] S. Pirandola, R. Laurenza, C. Ottaviani, and L. Banchi, *Nat. Commun.* **8**, 15043 (2017).
- [50] N. K. Bernardes, L. Praxmeyer, and P. van Loock, *Phys. Rev. A* **83**, 012323 (2011).
- [51] J. M. Torres and J. Z. Bernád, *Phys. Rev. A* **94**, 052329 (2016).