# Quantum Information Theory

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**Problem Set #4** **07 March 2013**

### Problem 4.1 Some common quantum channels

Find Kraus representations for the following qubit channels

- The dephasing channel: $\rho \to \rho' = \$(\rho) = (1-p)\rho + p\,\mathrm{diag}(\rho_{00}, \rho_{11})$ (the off-diagonal elements are annihiliated with probability $p$).
- The depolarizing channel: $\rho \to \rho' = \$(\rho) = (1-p)\rho + p\mathbb{1}/2$.
- The amplitude damping (damplitude) channel, defined by the action $|00\rangle \to |00\rangle$, $|10\rangle \to \sqrt{1-p}|10\rangle + \sqrt{p}|01\rangle$

### Problem 4.2 Unital Channels

A superoperator \$ is unital if $\$(\mathbb{1}) = \mathbb{1}$, or in terms of Kraus operators, $\sum_k A_k A_k^\dagger = \mathbb{1}$. Show that the eigenvalues of the output of a unital superoperator majorize the eigenvalues of the input. (Recall that a vector $p$ majorizes $q$ if there exists a doubly stochastic matrix $D$ such that $q = Dp$.) *Hint:* Express the input (output) as $U\Lambda U^\dagger$ ($V\Lambda' V^\dagger$) for $U$, $V$ unitary and $\Lambda$, $\Lambda'$ diagonal.

### Problem 4.3 "All-or-Nothing" Violation of Local Realism

Consider the three qubit state $|\mathrm{GHZ}\rangle = \frac{1}{\sqrt{2}}(|000\rangle - |111\rangle)_{123}$, the Greenberger-Horne-Zeilinger state.

a) Show that $|\mathrm{GHZ}\rangle$ is a simultaneous eigenstate of $X_1Y_2Y_3$, $Y_1X_2Y_3$, and $Y_1Y_2X_3$ with eigenvalue $+1$, where $X$ and $Y$ are the corresponding Pauli operators.

b) Use the results of part (a) to argue by Einstein locality that each qubit has well-defined values of $X$ and $Y$. For qubit $j$, denote these values by $x_j$ and $y_j$. We say that these values are *elements of reality*. What would local realism, i.e. the assumption of realistic values that are undisturbed by measurements on other qubits, predict for the product of the outcomes of measurements of $X$ on each qubit?

c) What does quantum mechanics predict for the product of the outcomes of measurements of $X$ on each qubit?