Quantum Information Theory

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Problem Set #2 **05 March 2013**

Problem 2.1 Indirect Measurement

Suppose a quantum system is prepared in one of two nonorthogonal states $|\varphi_1\rangle$ or $|\varphi_2\rangle$. We would like to make a measurement to determine which state was prepared, but do so without disturbing the state. To this end, we could consider making an indirect measurement in which we also prepare an auxiliary state $|\text{blank}\rangle$, apply a unitary U_{AB} which has the action

$$|\varphi_j\rangle_A|\text{blank}\rangle_B \to \mathrm{U_{AB}}|\varphi_\mathrm{j}\rangle_\mathrm{A}|\text{blank}\rangle_\mathrm{B} = |\varphi_\mathrm{j}\rangle_\mathrm{A}|\beta_\mathrm{j}\rangle_\mathrm{B},$$

and then measure system B in some way. This scheme evidently does not disturb the state of system A. What is the most we can learn about which state was prepared? What if the two states $|\varphi_j\rangle$ are orthogonal?

Problem 2.2 Teleportation Redux

a) Show that for the entangled state $|\Phi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ and any unitary operator U,

$$\left(U_A \otimes U_B^*\right)|\Phi\rangle_{AB} = |\Phi\rangle_{AB},$$

where $*$ denotes complex conjugation in the $|0\rangle, |1\rangle$ basis.

b) Show that for any state $|\psi\rangle$

$${}_A\langle\psi|\Phi\rangle_{AB} = \tfrac{1}{\sqrt{2}}|\psi^*\rangle_B.$$

c) Use the results of (a) and (b) to give a derivation of the teleportation protocol without resorting to components.

d) What happens if Alice and Bob use the state $(\mathbb{1}_A \otimes U_B)|\Phi\rangle_{AB}$ for teleportation? Or if Alice measures in the basis $U_{A'}^\dagger|\Phi_j\rangle_{A'A}$?

e) Instead of a single system state $|\psi\rangle_{A'}$, Alice has a bipartite state $|\psi\rangle_{A_1A_2}$. What happens if she performs the teleportation protocol on system A_2?

Problem 2.3 Remote Copy

Alice and Bob would like to create the state $|\Psi\rangle_{AB} = a|00\rangle_{AB} + b|11\rangle_{AB}$ from Alice's state $|\psi\rangle_A = a|0\rangle_A + b|1\rangle_A$, a "copy" in the quantum-mechanical sense. Additionally, they share the canonical entangled state $|\Phi\rangle$. Can they create the desired state by performing only local operations (measurements and unitary operators), provided Alice can only send <u>one</u> bit of classical information to Bob?