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Collective Feshbach scattering of a superfluid droplet from a mesoscopic two-component Bose-Einstein condensate

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We examine the collective scattering of a superfluid droplet impinging on a mesoscopic Bose-Einstein condensate (BEC) as a target. The BEC consists of an atomic gas with two internal electronic states, each of which is trapped by a finite-depth external potential. An off-resonant optical laser field provides a localized coupling between the BEC components in the trapping region. This mesoscopic scenario matches the microscopic setup for Feshbach scattering of two particles, when a bound state of one submanifold is embedded in the scattering continuum of the other submanifold. Within the mean-field picture, we obtain resonant scattering phase shifts from a linear response theory in agreement with an exact numerical solution of the real-time scattering process and simple analytical approximations thereof. We find an energy-dependent transmission coefficient that is controllable via the optical field between 0 and 100%.

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The natural way to investigate quantum objects is scattering. By selecting a convenient physical "stencil" with a large interaction cross section, one can probe the structure as well as the excitation properties of a target. It mostly happens that quantum objects are either microscopically small, like atoms or nuclei, or they are embedded inside a solid-state system like electrons, electronic holes, or Cooper pairs. Thus, they have an elusive character, which usually shuns direct observation.

The generic response of a compound quantum object to bombardment with projectiles is either individual, i.e., by instantaneously ejecting another single particle of the compound, or it is collective, when after some transient period the target responds as a whole. Probably, the most drastic instance of this collective behavior is nuclear fission, when a heavy nucleus breaks up into large fragments—unleashing large amounts of kinetic energy. Modeling the dynamics of the atomic nucleus as a classical liquid drop [1,2] gave a very intuitive interpretation of the observed phenomenon.

Superconductivity in metals is another prominent collective effect. Within Ginzburg-Landau theory [3], one associates a collective wave function with Bose-condensed electronic Cooper pairs, and a hydrodynamic description is again successful. The quantum-mechanical nature of the fluidlike order parameter is usually discussed with the Josephson effect, but Andreev-Saint-James reflection [4] is an equally interesting phenomenon and much more in line with collective scattering theory, which will be presented in the following. This effect explains the unusual electrical transport properties through a normal-metal-superconductor (N-S) junction. The occurrence of collective quantum-mechanical resonances can be explained by a conversion of normal conductor electrons into holelike excitations at the interface [5].

The discovery of superfluidity in bosonic [6,7] and, most recently, fermionic atomic gases [8–10] was an amazing achievement and is another manifestation of collective many-particle physics. Probably, the use of interatomic Feshbach scattering resonances has been the most fruitful novel concept of the past few years. Today, they are a universal tool to manipulate the binary interaction in an atomic gas [11–13] in real time and they have paved the way to fermionic superfluidity [8–10,14].

In the present paper, we will demonstrate that the microscopic physics of binary Feshbach resonances can also be implemented at the mesoscopic level of an atomic BEC, giving rise to collective Feshbach resonances as shown in Fig. 4, below. In particular, we will study the scattering properties of a weak coherent perturbation on a two-component BEC confined in quasi-one-dimensional square-well potentials of finite depth. Based on a Gross-Pitaevskii (GP) mean-field picture, we derive a two-component linear response Bogoliubov theory [16] for the scattering phases of the continuum perturbations. This is compared with the numerical simulation of nonlinear wave-packet propagation in real time. Finally, we can explain the appearance of collective Feshbach resonances qualitatively from a simple Thomas-Fermi approximation of the Bogoliubov excitations.

The dynamical response of a BEC has been the subject of intensive investigations during the last decade. So far, laser light has been mostly the method of choice to impart momentum onto a BEC, to excite collective modes, and to measure the dynamic structure factor [17]. However, light has also been used indirectly to prepare colliding matter-wave packets that have exhibited stimulated amplification as a result of their nonclassical bosonic nature [18]. While the collision energy of these wave packets was rather low in this setup (a few times the speed of sound), more recently, high energy, but ultracold thermal [19] and condensed atomic clouds [20] of equal size have been used to measure the single-particle collision cross section. Theoretically, scattering of identical particles off a scalar BEC was already considered in the limits of low and high energies [21,22]. Scattering of vortices was investigated in [23] (already relating it to Andreev-Saint-James reflections [4]), and a generalization of Levinson's theorem was presented in [24].

Binary Feshbach scattering resonances cannot be described by a single-channel potential scattering but they require at least the interaction of two different state manifolds. This was discussed first in nuclear scattering theory by Feshbach [25–27] and in the context of optical spectra of two-electron atoms by Fano [28,29]. Thus, we will assume that the quasi-one-dimensional condensate consists of atoms with two distinct internal electronic states. In the condensed

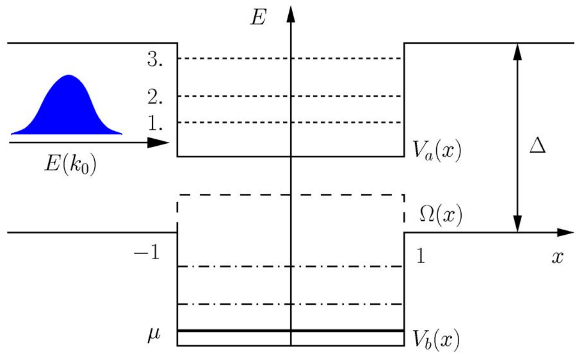

FIG. 1. (Color online) Schematic setup of a trapped two-component BEC with square-well potentials $V_a(x)$, $V_b(x)$ (solid line) and a coupling laser beam $\Omega(x)$ (long-dashed line) versus position x [15]. Scattering occurs in the two open (left and right) b channels, while the a channels are energetically closed with a threshold energy Δ. The chemical potential is μ (heavy solid line), the dash-dotted lines show the bound excitations, and the numbered quasibound energy levels (dashed line) are responsible for the collective Feshbach resonances.

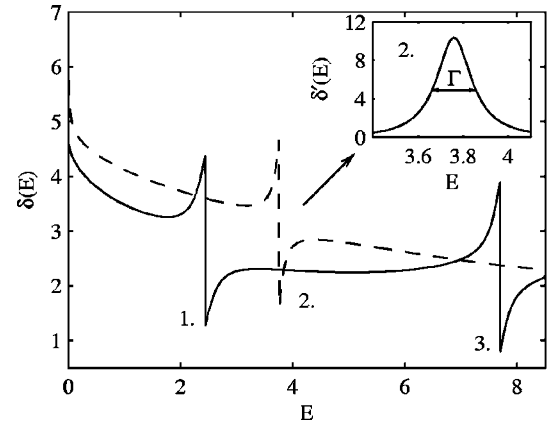

FIG. 2. Collective Feshbach resonances in the scattering phases δ_e (solid line) and δ_o (dashed line) of a weak perturbation of a two-component BEC versus energy E. Note that the π jumps of the collective Feshbach resonances occur at excitation energy $E_i=\mu+\epsilon_i$ of the even and odd quasibound Bogoliubov modes of Fig. 1. The inset shows the Lorentzian behavior of the phase derivative $\delta_o'(E)$ close to the resonance as in Eq. (4). For parameters see [15].

phase, the system is described by order parameters ψ_a and ψ_b, which are coupled by an optical laser beam $\Omega(x)$ with a large detuning Δ. Today, this can be achieved experimentally with very prolate traps, which freeze out the transverse degrees of motion effectively. The corresponding two-component GP equation is also known from the internal Josephson effect [30] and reads

$$\left[i\partial_t + \frac{1}{2}\partial_x^2 - \begin{pmatrix} V_a^{\mathrm{GP}}(x) & \Omega(x) \\ \Omega^*(x) & V_b^{\mathrm{GP}}(x) \end{pmatrix} \right] \begin{pmatrix} \psi_a \\ \psi_b \end{pmatrix} = 0,$$

$$V_a^{\mathrm{GP}}(x) = V_a(x) + g_{aa}|\psi_a(x)|^2 + g_{ab}|\psi_b(x)|^2,$$

$$V_b^{\mathrm{GP}}(x) = V_b(x) + g_{ab}|\psi_a(x)|^2 + g_{bb}|\psi_b(x)|^2, \qquad (1)$$

where V_a and V_b are external trapping potentials. For definiteness, we pick the square-well potentials as they lead to simple, analytically solvable approximations. This setup is depicted in Fig. 1. The coupling constants g_{aa}, $g_{ab}=g_{ba}$, and g_{bb} are proportional to the self- and cross-component s-wave scattering lengths. For typical experimental values of ^{87}Rb see [31], but from the theoretical point of view the choice of parameters is uncritical [15]. In order to describe the scattering of a superfluid droplet from the equilibrium BEC, we will determine the linear response modes of the two-component system [16]. The stationary Bogoliubov ansatz with particlelike $\boldsymbol{u}=(u_a,u_b)^\top$ and holelike $\mathbf{v}=(v_a,v_b)^\top$ excitations is

$$\boldsymbol{\psi}(x,t) = e^{-i\mu t}[\boldsymbol{\varphi}(x) + e^{-i\epsilon t}\mathbf{u}(\epsilon,x) + e^{i\epsilon t}\mathbf{v}^*(\epsilon,x)],$$

where μ is the chemical potential of the spinorial ground state $\varphi=(\varphi_a,\varphi_b)^\top$ and ϵ is the excitation energy. This perturbation to Eq. (1) yields the four-dimensional Bogoliubov equation

$$\left[\epsilon + \frac{\sigma_3}{2}\partial_x^2 - \begin{pmatrix} V^{\mathrm{B}} & M \\ -M^* & -V^{\mathrm{B}*} \end{pmatrix} \right] \begin{pmatrix} \mathbf{u} \\ \mathbf{v} \end{pmatrix} = 0,$$

$$M = \begin{pmatrix} g_{aa}\varphi_a{}^2 & g_{ab}\varphi_a\varphi_b \\ g_{ab}\varphi_a\varphi_b & g_{bb}\varphi_b{}^2 \end{pmatrix}, \qquad \sigma_3 = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix},$$

$$V^{\mathrm{B}} = \begin{pmatrix} V_a^{\mathrm{HF}}(x) & \Omega(x) + g_{ab}\varphi_a\varphi_b^* \\ \Omega^*(x) + g_{ab}\varphi_a^*\varphi_b & V_b^{\mathrm{HF}}(x) \end{pmatrix},$$

$$V_a^{\mathrm{HF}}(x) = V_a(x) + 2g_{aa}|\varphi_a|^2 + g_{ab}|\varphi_b|^2 - \mu,$$

$$V_b^{\mathrm{HF}}(x) = V_b(x) + g_{ab}|\varphi_a|^2 + 2g_{bb}|\varphi_b|^2 - \mu. \qquad (2)$$

The linear response energy matrix of Eq. (2) has the same structure as the well-known Bogoliubov equations in the case of a one-component condensate. It is symplectic and has a real-valued spectrum of pairwise positive and negative eigenvalues [32]. Due to the finite depth of the trapping potentials, the spectrum supports only a finite number of bound states and has a scattering continuum above a certain excitation energy. This is now the analogous situation as required for the two-particle Feshbach scattering resonances. If quasibound Bogoliubov modes coincide in energy with continuum modes, we will obtain a resonance behavior. These bound, positive-energy modes are depicted schematically in Fig. 1. For positive-energy solutions $\epsilon>0$, these resonances appear in the domain $0<\epsilon+\mu=k^2/2=E<\Delta$. In this regime, the mode components $u_a(\epsilon,x)$, $v_a(\epsilon,x)$, and $v_b(\epsilon,x)$ are localized on the condensate and vanish exponentially for $|x|\to\infty$. Only the particlelike component $u_b(\epsilon,x)$ can propagate outside:

$$\lim_{x\to\pm\infty} u_b(\epsilon,x) \propto \cos(kx \pm \delta_e), \text{ or } \sin(kx \pm \delta_o). \qquad (3)$$

Due to the reflection symmetry of the trapping potentials, the excitation can also be characterized by parity and we define even and odd phase shifts δ_e and δ_o, respectively. These scattering phases have been evaluated numerically from Eqs. (1)–(3) and are displayed in Fig. 2. The analogy to the phe-

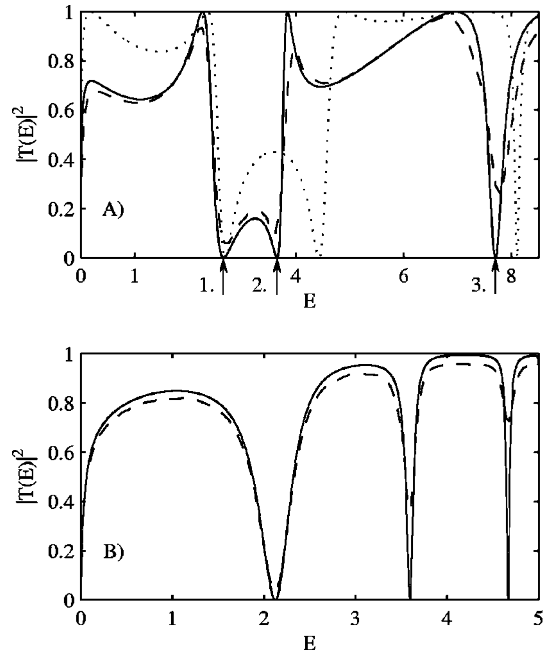

FIG. 3. Transmission coefficient of a weak coherent perturbation in the b component of a BEC in (A) square-well and (B) Pöschl-Teller traps versus energy E from the linear response Bogoliubov calculation (solid line). The dashed line is the transmission coefficient obtained from propagating a Gaussian wave packet in the nonlinear GP equation (1), in real time. The dotted line in (A) is the Thomas-Fermi approximation. Feshbach resonances are seen clearly in all curves in (A) and (B) superimposed on a background caused by potential scattering. For parameters see [15,35].

nomenon of Feshbach resonances stands out very clearly. According to the Breit-Wigner parametrization of an isolated resonance [27], one finds approximately

$$\delta^{\text{res}}(E) = \arctan\left(\frac{\Gamma/2}{E_R - E}\right), \quad \frac{2}{\Gamma} = \left.\frac{d\delta^{\text{res}}}{dE}\right|_{E=E_R}. \tag{4}$$

In turn, one can determine the resonance energy E_R and width Γ analytically from the poles of the S matrix in the vicinity of a Feshbach resonance [33,34].

From the phases shifts, one can obtain all scattering information, like reflection $R(E)$ or transmission amplitudes $T(E)$, by considering a causal wave $u_b^{(+)}(\epsilon,x)$, propagating to the right ($k>0$),

$$\begin{aligned}\lim_{x\to-\infty} u_b^{(+)}(\epsilon,x) &= e^{ikx} + Re^{-ikx} \\ &= e^{i\delta_e}\cos(kx-\delta_e) + ie^{i\delta_o}\sin(kx-\delta_o), \\ \lim_{x\to\infty} u_b^{(+)}(\epsilon,x) &= Te^{ikx} = e^{i\delta_e}\cos(kx+\delta_e) + ie^{i\delta_o}\sin(kx+\delta_o).\end{aligned} \tag{5}$$

The real valuedness of the scattering phases of Fig. 2 implies a current conservation $|R(E)|^2+|T(E)|^2=1$, and we can write the transmission coefficient in terms of the phases

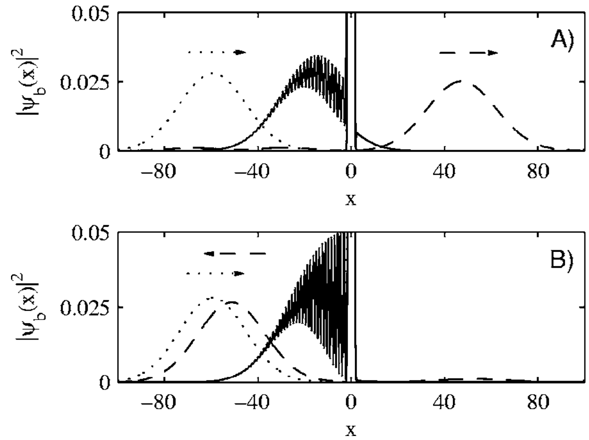

FIG. 4. Collective Feshbach resonance: complete transmission (A) or total refection (B) of a small coherent atomic wave packet $\psi_b(x,t)$ with incident momentum $k_0=2.05$ (A) or $k_0=2.35$ (B), when scattering off a stationary, two-component BEC, trapped around $-1<x<1$, with a maximal density $n_b(x)\approx 500$, which is off scale. The dimensionless density $n_b(x,t)$ is depicted versus position x (in natural units of the trap [15]) for three instants t: initially (t_i, dotted line), on impact (t_0, solid line), and finally (t_f, dashed line).

$$|T(E)|^2 = \cos^2[\delta_e(E) - \delta_o(E)]. \tag{6}$$

This transmission coefficient is shown in Fig. 3. In the vicinity of the resonance energies $E\approx\mu+\epsilon_i$, it changes rapidly between 0 and 100%.

In order to isolate the essential physical mechanism responsible for the collective resonance behavior, we approximate the GP equation (1) in the Thomas-Fermi limit. Then $\varphi_{\text{TF}}(x)$ is constant within the square-wells and vanishes exactly elsewhere. In an additional approximation, we disregard the matrix M in the Bogoliubov self-energy, Eq. (2). In this limit, the particle and hole excitations are decoupled and u satisfies a two-component Schrödinger equation

$$\left(\epsilon + \frac{\partial_x^2}{2} - V^{\text{B}}\right)\mathbf{u} = 0. \tag{7}$$

Due to the TF approximation, this equation has again a square-well character and the solution can be found analytically [26]. Sparing the details of the calculation of the scattering phases, we simply present the results in the dotted line in Fig. 3(A). A good qualitative agreement needs to be acknowledged, while there are obviously shifts in the resonance energies that are not accounted for in this simple approximation scheme. Refining this calculation iteratively would obviously improve the perturbative calculation.

The occurrence of collective Feshbach resonances is not related to the square-well shape of the traps, as we show in Fig. 3(B). Here, we have computed the transmission spectrum with Pöschl-Teller-like traps and laser coupling [35,36]

$$V_{PT}(x,\alpha,\lambda) = -\alpha^2\frac{\lambda(\lambda-1)}{\cosh^2\alpha x}, \tag{8}$$

$$V_a(x) = V_{PT}(x,\alpha_a,\lambda_a) + \Delta,$$

$$V_b(x) = V_{PT}(x,\alpha_b,\lambda_b), \quad \Omega(x) = V_{PT}(x,\alpha_\Omega,\lambda_\Omega). \tag{9}$$

We have chosen a laser beam diameter α_Ω^{-1} wider than those for the traps to describe a more realistic situation.

In a final step of the analysis of the collective Feshbach resonance, we have also performed a numerical simulation of the nonlinear, time-dependent GP equation (1). We propagated an incident traveling Gaussian wave packet on top of the stationary BEC solution depicted in Fig. 4,

$$\psi_b(x,t=0) = \varphi_b(x) + \sqrt[4]{\frac{\delta N^2}{\pi\sigma_x^2}}e^{-(x-x_0)^2/(2\sigma_x^2)+ik_0x}. \tag{10}$$

It can be seen in Fig. 4 that the b component of the ground-state solution φ_b is well localized in the trap center and has initially no overlap with the weak Gaussian perturbation if $\delta N \ll N$ and $x_0 \ll -1$. The initial momentum $k_0 > 0$ of the wave was varied to cover the energy range in Fig. 3. In order to resolve the resonance structure, one needs a small momentum spread ($\sigma_x=20$, $\sigma_k=\sigma_x^{-1}=0.05$) and we find that the wave-packet transmission spectrum matches the linear response approach very well. We show two instances of the propagation of an initial wave packet. In Fig. 4(A), the momentum $k_0 \approx 2.05$ corresponds to an energy below the resonance energy, marked as E_1 in Fig. 3, which leads to full transmission. In contrast, Fig. 4(B) shows the total reflection of the wave packet if the incident momentum $k_0 \approx 2.35$ corresponds exactly to the resonance energy E_1.

In conclusion, we have identified collective Feshbach resonances in a trapped two-component BEC. These resonances do not require a binary Feshbach resonance to modify the binary interaction, but quasibound Bogoliubov modes in the linear response spectrum. We have shown that this can be achieved easily with help of an optical laser beam. In contrast to the microscopic binary Feshbach resonance, this quantum-mechanical phenomenon exists on a mesoscopic, possibly macroscopic scale and can be used to control the transmission of matter waves between 0 and 100%. For conceptual simplicity, we have investigated a quasi-one-dimensional geometry with square-well potentials. None of this is important for the effect, and it can be implemented in various experimental configurations. For example, smooth, finite-range Pöschl-Teller potentials were examined numerically and gave the same result. But a large detuning is required in order to have a energetically closed upper collision channel. However, studying this resonance in the mean-field picture can be only a first step, and a proper inclusion of the thermal cloud is still lacking [37,38].

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