Storage and retrieval of coherent optical information in atomic populations

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Abstract

We propose a simple and powerful protocol to map an arbitrary atomic coherence between two quantum states into a population distribution of three metastable states, and later to retrieve the atomic coherence from the population distribution. The protocol applies simple sequences of radiation pulses with arbitrary temporal profile, either as coincident or as consecutive pulses. Mapping of rather short-lived atomic coherences into very long-lived atomic populations permits the prolongation of storage times (e.g. of optical information encoded in atomic coherences) by many orders of magnitude — without the need for complicated techniques to reduce homogeneous broadenings.

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1. Introduction

Storage of optical information in quantum systems typically relies on atomic coherences, i.e. coherent superpositions of states in a quantized memory. During storage and retrieval of such optical information, atomic coherences are subject to dephasing and decoherence. We use the terms “dephasing” and “decoherence” to distinguish between two different effects of phase changes in atomic coherences: (1) Inhomogeneous broadenings, which lead to steady, smooth evolution of the phases of two emitters. We will call such reversible processes “dephasing”, (2) Homogeneous broadenings, which lead to stochastic, irregular variation of the phases of two emitters. We will call such irreversible processes “decoherence”. Both dephasing and decoherence destroy the encoded information. Thus, we require protocols to reduce the perturbing effects of inhomogeneous and homogeneous broadenings. In this paper, we develop rather simple protocols to achieve this goal. In particular we deal with storage and retrieval of optical information by “stopped light” (often also termed “stored light”). The technique is based on electromagnetically-induced transparency (EIT), i.e. adiabatic transfer of the quantum state of photons to a collective atomic coherence in the medium [1]. The coupling scheme for light storage in a three-level A-type level scheme is depicted in Fig. 1.

A probe laser Pr drives the transition between a ground state |1〉 and an excited state |E〉. The coupling laser C drives the transition between the excited state |E〉 and a metastable state |2〉. When we choose appropriate temporal laser pulse profiles (i.e. coincident pulses, with a constant ratio of intensities in the falling edges of the laser pulses), and provided the pulses are sufficiently strong, we drive the medium to a coherent superposition of the long-lived states |1〉 and |2〉. The probability amplitudes in the two states are equal, i.e. we get a maximal atomic coherence. During the process, photons from the probe laser are converted into the atomic coherence. Thus, we store the probe pulse in the atomic coherence. This corresponds to the “write” process of a memory. To read out the quantized memory/atomic coherence, we apply the coupling pulse again. In a simplified explanation, this second “read” coupling pulse at frequency ω2 mixes with the atomic coherence at frequency ω12 and generates a radiation pulse at the beat frequency ω2−ω12=ωpr, i.e. the probe frequency. Thus, in the mixing process we retrieve the probe pulse from the atomic coherence as a directed radiation beam, coincident with the “read” coupling pulse. This is the basic concept of light storage in atomic coherences.

After basic demonstrations of stored light in atomic gases [2] the technique attracted considerable attention in the last decade, e.g. to store light pulses as carriers of optical data bits [1], or even to store extended images in atomic coherences [3–5]. A still small, but in the last years continuously growing number of experiments on light storage and EIT were performed in specific solid media, i.e. rare-earth ion doped solids [6–17]. These media combine the advantages of atoms in the gas phase (i.e. spectrally narrow transitions and moderate decoherence rates) with the advantages of solids (i.e. large density and scalability, as well as rather simple production and handling). In rare-earth doped media the atomic coherences are usually driven between hyperfine levels of a ground state. The population lifetimes of these hyperfine levels are typically very long.
types of media. Thus, we search for a simple and general alternative to
However, the additional setup with magnetic
control [19], involving sequences of radio-frequency (RF) pulses and
called stimulated photon echoes, see e.g. [20] and refs. therein). In the
technique a maximal coherence is converted onto populations in an
inhomogeneously broadened medium of two- or three-level atoms. As
an important feature of stimulated photon echoes, the initial informa-
tion is distributed over an ensemble of atoms. As we will discuss below,
our alternative mapping protocol is based on a three-level excitation
scheme. This enables us to store the full information of the initial
coherence (i.e. amplitude and phase) in each single atom of the
interaction volume. Thus, each single atom in the interaction volume
carries the full information of the initial coherence.

1.1. Level scheme and initial conditions

Atomic coherences in a medium are mathematically defined by the
off-diagonal elements of the corresponding density matrix. Thus, in a
two-level system of states [1] and [2], the density matrix element $\rho_{21} = C_1 C_2 \neq 0$
with the probability amplitudes $C_i$ indicates coherence. We note, that $\rho_{21}$
is a complex number, i.e. it includes amplitude and phase (or real and
imaginary part). Thus, if we want to map the coherence into populations $P_i$
of a closed level scheme, we require a minimum of three atomic states (as
the population of a third state is determined by the populations of the two
other states, i.e. $P_3 = 1 - P_1 - P_2$). In our above example of Pr:YSO, we can
provide such a three-state system by the three metastable hyperfine
ground states (compare Fig. 2).

We will consider now a loop interaction scheme, i.e. we assume
allowed single-photon transitions between all three states (see Fig. 3).

We note, that similar loop schemes were already investigated to
improve the efficiency of population transfer in stimulated Raman
adiabatic passage (STIRAP) [21]. The closed double-$A$ scheme was
discussed in Ref. [22]. We also note, that a three-level loop scheme is
not possible for optically-driven, single-photon electric dipole
transitions. An example of an optically-driven, three-level loop
system involving a two-photon transition is discussed in Ref. [23].
Moreover and in particular, the three-level loop scheme is possible for
magnetic dipole transitions, driven by RF fields. The latter is of
particular interest to application of light storage in doped solids, as

![excited state](image)

**Fig. 1.** Coupling scheme for light storage by EIT.

as there is no spontaneous decay via dipole allowed transitions
between these states.

As an example, Praseodymium Pr$^{3+}$ ions, doped in an Y$_2$SiO$_5$
lattice (in the following termed with the acronym Pr:YSO) exhibit a
level splitting in three hyperfine states (see Fig. 2), with level spacings
in the range of about 10MHz, and population lifetimes of about 100s
[18] in the ground states. In a light storage experiment, the atomic
coherence is driven between two of the long-lived hyperfine ground
states, while the third state is not used in the experiments. However,
while the population lifetime in such hyperfine states may be large,
the possible storage time (i.e. the lifetime of the atomic coherences) is
limited by dephasing and decoherence. These processes are typically
much faster than population decay. As an example, in Pr:YSO the
population lifetimes are in the range of 100s. Dephasing due to
inhomogeneous broadening occurs on the timescale of 10μs. Deco-
herence due to homogeneous broadening (mediated by interactions
with the host lattice) occurs on a time scale of 500μs [10]. Thus, the
storage times in atomic coherences are much smaller than the long
population lifetimes.

To overcome the problems of homogeneous broadenings, Longdell
et al. combined photon echo techniques and dynamic decoherence
control [19], involving sequences of radio-frequency (RF) pulses and
static magnetic fields [24,9].

This enables reduction of homogeneous broadenings approaching
the limit of the natural linewidth and storage times up to seconds.
However, the additional setup with magnetic fields and RF pulse
sequences is quite complex — and may also be not applicable to other
types of media. Thus, we search for a simple and general alternative to
preserve atomic coherences. The aim is to prolong storage times
beyond the limits set by homogeneous broadenings, up to the regime
of population lifetimes.

The key idea of our approach is as follows: Consider information,
initially stored in an atomic coherence in a quantum system (e.g. by
applying the “write” process in the concept of stopped light, see above).
We search now for a specific sequence of additional radiation pulses,
which converts the short-lived atomic coherence into long-lived atomic
populations, e.g. in a system of three metastable states. Depending on
the specific excitation scheme, the radiation pulses may be light pulses
or RF pulses — or also a combination of both. Prior to read-out of our
atomic memory, a similar sequence of radiation pulses shall serve to
convert the atomic populations back into the initial atomic coherence.
Now the memory is ready for read-out (e.g. by applying the “read”
process in the concept of stopped light, see above). In summary,
between writing and reading of our memory/atomic coherence, we map
the coherence onto populations and back again.

In the following we will term the required, specific sequence of
additional radiation pulses a “mapping sequence”. Provided a
mapping sequence exists and we are able to determine it, the concept
enables prolongation of storage times by many orders of magnitude.
The maximal storage time of all information in an arbitrary coherence
is then given by the long lifetime of the atomic populations. In the
above example of light storage in Pr:YSO, we could gain 6–7 orders of
magnitude compared to the coherence time. In the following we will
discuss in detail our theoretical concept of coherence mapping and
demonstrate several solutions for mapping sequences — which offer a
variety of possibilities for successful experimental implementation.

We note, that our approach of converting coherences to populations
is related to the technique of three pulse photon echoes (often also
called stimulated photon echoes, see e.g. [20] and refs. therein). In the
technique a maximal coherence is converted onto populations in an
inhomogeneously broadened medium of two- or three-level atoms. As
an important feature of stimulated photon echoes, the initial informa-
tion is distributed over an ensemble of atoms. As we will discuss below,
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magnetic dipole transitions, driven by RF fields. The latter is of
particular interest to application of light storage in doped solids, as

![metastable states](image)

**Fig. 2.** Level scheme of Pr:YSO.
briefly discussed above. In our example of Pr:YSGO, the level splittings in the range of 10MHz are very well suited for RF excitations.

We assume, that initially the medium is prepared in a coherent superposition of states $|1\rangle$ and $|2\rangle$, e.g. by EIT light storage. Thus, $\rho_{12}=c_1c_2 \neq 0$. Further we assume, that state $|3\rangle$ is initially empty. We note, that the condition of an empty state $|3\rangle$ is not necessary for the mathematical formalism discussed below. However, the assumption simplifies our calculations.

In this situation, the initial conditions for the atoms are given by $C_i(t_0)=[c_{i0}, c_{i20}, 0]^T$ with the column-vector $C_i(t)=[c_{i1}(t), c_{i2}(t), c_{i3}(t)]^T$ of the probability amplitudes for the three-state system. In the density matrix approach the initial conditions are:

$$\rho = \begin{pmatrix} c_{11} & c_{12} & 0 \\ c_{21} & c_{22} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

where $\rho_{0}=C_{0}C^{\dagger}$.

Our aim is now to determine sequences of radiation pulses, which permit mapping of an atomic coherence $\rho_{12}$ into populations $\rho_i$ of the three metastable states — and back again. In particular we will consider excitation by three pulses with center frequencies tuned to the three resonances at $\omega_{21}$, $\omega_{31}$, and $\omega_{23}$ in the loop scheme. Moreover, we will investigate the cases either of simultaneous excitation by our three radiation pulses, or alternatively excitation by three consecutive pulses. Such rather simple configurations for the mapping sequences are very appropriate for a realistic experiment.

We term the three radiation fields with indexes P, S and Q, which may e.g. denote pump pulse, Stokes pulse and an additional control pulse Q. This nomenclature is similar to related level schemes in the literature, e.g. tripod-STIRAP [25]. Field Q drives the transition between states $|1\rangle$ and $|2\rangle$, which are involved in the atomic coherence. Fields P and S drive transitions from states $|1\rangle$ or $|2\rangle$, towards the third state $|3\rangle$. We describe the corresponding fields by

$$B_p(t) = B_p(t) \cos(\omega_{21}t + \phi_p),$$

$$B_s(t) = B_s(t) \cos(\omega_{32}t + \phi_s),$$

$$B_Q(t) = B_Q(t) \cos(\omega_{23}t + \phi_Q),$$

with the transition frequencies $\omega_i = (\Omega_i + \Omega_j)/h$, the energies $\Omega_i$ of the three states and the pulse envelopes $B_i(t)$.

Our straightforward mathematical approach to search for mapping sequences is as follows: (Step 1) We describe the interaction of the loop quantum system with three radiation pulses by setting up the Hamiltonian for the interaction, including the above general expressions for the radiation fields. (Step 2) We use this Hamiltonian in the Schrödinger equation to derive general equations, which describe the state of the quantum system after excitation with the pulse sequence. (Step 3) We let the system evolve for a time interval, which is long compared to the coherence time in the medium, but still short compared to the population lifetime. After this evolution, there are no more coherences, but only populations in the medium. Thus, we mapped the initial atomic coherence into a population distribution. In the following, we will call this process “coherence-population mapping”, driven by a “write” mapping pulse sequence. (Step 4) We apply the Hamiltonian with a similar pulse sequence again in the Schrödinger equation to derive equations, which describe the final state of the quantum system after the second excitation with the pulse sequence. (Step 5) We consider the obtained, general equations for the final state of the quantum system. From the equations we determine specific conditions (e.g. for the intensities or pulse areas of the driving radiation pulses), which permit reconstruction of the original atomic coherence. This corresponds to mapping of the population distribution back to the initial atomic coherence. In the following, we will call this process “population-coherence mapping”, driven by a “read” mapping pulse sequence.

We assume, that the pulses are much shorter than the relaxation times in the system. We note, that this is no strong restriction for appropriate quantum systems, and also in general exhibits no principal limitation of the concept. In this case, we can apply the Schrödinger equation to obtain the state vector $C(t)$ immediately after interaction of the three-state system with the radiation pulses (i.e. after mapping of the atomic coherence onto populations). In this approach we mathematically describe decoherence by introducing independent random phases to each component of the state vector $C(t)$ before application of the “read” mapping sequence. We note, that in our treatment we may also apply a density matrix approach rather than the Schrödinger equation.

1.2. Dynamics in the loop scheme

Before we start our calculation through steps 1–5 as described above, we introduce a new basis, which facilitates our calculations:

$$|\psi_1\rangle = |1\rangle,$$

$$|\psi_2\rangle = e^{-i\Omega_2 t - i\phi_2} |2\rangle,$$

$$|\psi_3\rangle = e^{-i\Omega_3 t - i\phi_3} |3\rangle.$$  

The components of the state vector $C(t)$ in the new basis are connected with the components of the state vector $C(t)$ in the basis of the bare states by the relations:

$$C_1(t) = c_{11}(t),$$

$$C_2(t) = e^{-i\Omega_2 t - i\phi_2} c_{22}(t),$$

$$C_3(t) = e^{-i\Omega_3 t - i\phi_3} c_{33}(t).$$  

The state vector $C(t)$ satisfies the time-dependent Schrödinger equation:

$$ih \frac{d}{dt} C(t) = H(t) C(t),$$

with the Hamiltonian in rotating wave approximation:

$$H(t) = \frac{h}{2} \begin{bmatrix} 0 & \Omega_Q(t)e^{-i\phi} & \Omega_P(t) \\ \Omega_Q(t)e^{i\phi} & 0 & \Omega_S(t) \\ \Omega_P(t) & \Omega_S(t) & 0 \end{bmatrix}.$$  

The Hamiltonian includes the Rabi frequencies:

$$\Omega_P(t) = -\frac{\mu_3 B_p(t)}{h},$$

$$\Omega_S(t) = -\frac{\mu_2 B_s(t)}{h},$$

$$\Omega_Q(t) = -\frac{\mu_3 B_Q(t)}{h}.$$  

Fig. 3. Loop interaction scheme.
and the generalized phase $\Phi = \psi_P - \psi_S - \psi_Q$. Without loss of generality we assume real values for the matrix elements $\mu_{ij}$ of all transition moments.

We obtain the following relation between the elements of the density matrix $\rho_{nm}$ ($n,m=1,2,3$) in the basis of the bare states and the elements $\sigma_{nm}$ in the basis (3):

$$
\rho_{12} = e^{i\omega_{12}t + k}\sigma_{12}, \\
\rho_{23} = e^{i\omega_{23}t + k}\sigma_{23}, \\
\rho_{13} = e^{i\omega_{13}t + k}\sigma_{13}, \\
\rho_{nn} = \sigma_{nn}.
$$

(8)

We note, that all other non-diagonal elements of the density matrix are determined by $\rho_{00}=\rho_{0}$. The density matrix $\sigma$ in rotating wave approximation satisfies the equation

$$
\frac{d}{dt} \sigma(t) = \left[ H(t), \sigma \right],
$$

(9)

with the Hamiltonian $H(t)$ given by Eq. (6). In explicit form we get the following equations for the elements of the density matrix:

$$
\frac{\partial}{\partial t} \sigma_{11} = \frac{1}{2} \left[ \Omega \left( \sigma_{12} - \sigma_{33} \right) + \Omega_0 \left( \sigma_{12} e^{i\phi} - \sigma_{21} e^{-i\phi} \right) \right],
$$

$$
\frac{\partial}{\partial t} \sigma_{13} = -i \left[ \Omega \left( \sigma_{13} - \sigma_{21} \right) + \Omega_0 \left( \sigma_{13} e^{i\phi} - \sigma_{21} e^{-i\phi} \right) \right],
$$

$$
\frac{\partial}{\partial t} \sigma_{22} = \frac{1}{2} \left[ \Omega \left( \sigma_{21} - \sigma_{12} \right) - \Omega_0 \left( \sigma_{12} e^{i\phi} - \sigma_{21} e^{-i\phi} \right) \right],
$$

$$
\frac{\partial}{\partial t} \sigma_{23} = \frac{1}{2} \left[ \Omega \left( \sigma_{23} - \sigma_{31} \right) - \Omega_0 \left( \sigma_{23} e^{i\phi} - \sigma_{31} e^{-i\phi} \right) + \Omega_0 \sigma_{13} \right],
$$

$$
\frac{\partial}{\partial t} \sigma_{33} = \frac{1}{2} \left[ \Omega \left( \sigma_{31} - \sigma_{13} \right) - \Omega_0 \left( \sigma_{31} e^{i\phi} - \sigma_{13} e^{-i\phi} \right) - \Omega_0 \sigma_{23} \right].
$$

(10)

To write Eq. (10) in a more compact form, we omitted the argument $t$. We also note, that the diagonal elements of the density matrix obey the normalization condition $\sigma_{11} + \sigma_{22} + \sigma_{33} = 1$. For the following calculation it is convenient to define new variables with real values, i.e. $S_{ij} = \langle \psi_i | \psi_j \rangle$, $V_{ij} = \psi_i + \psi_j$ (i.e. $\sigma_{11}$). The above equations for the new variables read

$$
\frac{\partial}{\partial t} S_{11} = \frac{\Omega_P}{2} S_{31} + \frac{\Omega_Q}{2} S_{21} \cos \Phi - V_{21} \sin \Phi, \\
\frac{\partial}{\partial t} S_{13} = -\frac{\Omega_P}{2} S_{31} - \frac{\Omega_Q}{2} S_{33} \sin \Phi, \\
\frac{\partial}{\partial t} S_{22} = \frac{\Omega_P}{2} S_{32} + \frac{\Omega_Q}{2} S_{21} \cos \Phi - V_{21} \sin \Phi, \\
\frac{\partial}{\partial t} S_{31} = -\Omega_Q (S_{12} - S_{21}) + \frac{\Omega_Q}{2} V_{21} \cos \Phi - \frac{\Omega_Q}{2} V_{21} \sin \Phi, \\
\frac{\partial}{\partial t} V_{21} = -\frac{\Omega_Q}{2} S_{32} \cos \Phi - \frac{\Omega_Q}{2} V_{21} \sin \Phi + \frac{\Omega_P}{2} S_{21},
$$

(11)

We see, that for two pulses in A-type configuration the population dynamics include only the real part of the density matrix element $\sigma_{12}$ through $V_{21} = \sigma_{12} + \sigma_{21}$, analogous arguments hold true, if we apply two pulses $P$ and $P$ (but no pulse $S$), or $Q$ and $Q$ (but no pulse $P$). The only difference with regard to the A-type configuration is, that the populations depend on the imaginary part of $\sigma_{12} e^{i\phi}$. Hence, it is impossible to map the full information about the initial coherence $\sigma_{12}^{0}$ onto populations $-P$ or $S$, i.e. the full loop scheme.

2. Coincident mapping pulses

In the following section we will consider interaction of the medium with three coincident pulses $P$, $S$, and $Q$. We assume fixed temporal pulse shapes (e.g. rectangular or Gaussian temporal profiles) for all pulses. However, we allow for different peak pulse intensities (rp. different pulse areas) in the coherence-passage mapping process and population-coherence-"read" mapping sequence. We note, that these conditions (i.e. similar temporal profiles, but the possibility to vary peak intensities) are easily implemented in experiments, involving optical or RF excitations.

Our goal is to determine pulse areas, which permit mapping of an initial coherence $\sigma_{12}$ onto populations $S_{11}$, $S_{22}$ and $S_{33}$, and back again. Thus, a successful "write" and "read" mapping sequence will enable us to retrieve the initial coherence again. To have some more freedom in the optimization, we also consider retrieval of a fixed fraction of the initial coherence as a successful mapping process. Thus, we search for mapping sequences, which permit $\sigma_{12}^{0} \leftrightarrow \sigma_{12}^{\text{final}}$. To start the theoretical treatment (step 1), we describe the three coincident pulses by the Rabi frequencies $\Omega_Q(t) = \Omega_S(t) = \Omega_P(t)$, with the generalized phase $\Phi = 0$. We note, that coherent interaction requires fixed relative phase between the radiation pulses. This is fulfilled, in particular, by our definition of the Rabi frequencies with phase $\Phi = 0$. In this case the Hamiltonian in rotating wave approximation yields:

$$
H(t) = \frac{\hbar}{2} \begin{bmatrix}
0 & \Omega(t) & \Omega(t) \\
\Omega(t) & 0 & \Omega(t) \\
\Omega(t) & \Omega(t) & 0
\end{bmatrix}.
$$

(13)
For coincident pulses we can analytically solve the time-dependent Schrödinger equation

\[ i\hbar \frac{d}{dt}c(t) = H(t)c(t). \tag{14} \]

We get

\[ c(t_1) = M c(t_0) \tag{15} \]

with the evolution matrix:

\[ M = \begin{bmatrix} a & b & b \\ b & a & b \\ b & b & a \end{bmatrix}. \]

with \( a = \frac{1}{3} \left( 2e^{i\phi_2/2} + e^{-i\phi_2} \right) \), \( b = \frac{1}{3} \left( -e^{i\phi_2/2} + e^{-i\phi_2} \right) \) and the pulse area \( A = \int t \Omega(t) dt \). We consider now the situation at a certain time \( t_0 \) in the interval \([t_0, t_1]\), i.e. between the start time \( t_0 \) with the initial coherence \( c_{\text{coh}}(t_0) = c_1(t_0) c_2(t_0) \), and the end of the coherence-population ("write") mapping sequence at time \( t_1 \). We use solution (15), introducing the pulse area of the coherence-population ("write") sequence \( A = A_w \) and applying the initial condition \( c(t_0) = [c_1(t_0), c_2(t_0), 0]^T \). We obtain (step 2):

\[
\begin{align*}
    c_1(t_1) &= a_1 c_1(t_0) + b_1 c_2(t_0), \\
    c_2(t_1) &= b_2 c_1(t_0) + a_2 c_2(t_0), \\
    c_3(t_1) &= b_3 c_1(t_0) + c_2(t_0). 
\end{align*}
\tag{17}
\]

After interaction with the radiation pulses, i.e. during the interval \([t_1, t_2]\) with the end of the coherence-population ("write") mapping sequence at time \( t_1 \) and the beginning of the population-coherence ("read") mapping sequence at time \( t_2 \), the system evolves freely in time. The time difference \( t_2 - t_1 \) (i.e. the storage time of the coherence) shall be still short to the very long population lifetimes of the metastable states. However, decoherence occurs. We mathematically describe decoherence by the transformation \( c(t_2) = A_{\text{decoh}} c(t_1) \) with the evolution matrix

\[ A_{\text{decoh}} = \begin{bmatrix} e^{i\phi_1} & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & e^{i\phi_3} \end{bmatrix}. \tag{18} \]

The phases \( \phi_1, \phi_2, \phi_3 \) are totally random. Thus, the ensemble averaging is

\[ \langle \exp[-i(\phi_2 - \phi_3)] \rangle = \delta_{ij}. \tag{19} \]

After free evolution involving decoherence (step 3), introducing the pulse area of the population-coherence ("read") sequence \( A = A_r \), we get from the above solution (15):

\[
\begin{align*}
    c_1(t_2) &= a_w c_1(t_0) + b_w c_2(t_0) e^{i\phi_1}, \\
    c_2(t_2) &= b_w c_1(t_0) + a_w c_2(t_0) e^{i\phi_2}, \\
    c_3(t_2) &= b_w c_1(t_0) + c_2(t_0) e^{i\phi_3}. 
\end{align*}
\tag{20}
\]

Finally, at the end \( t_3 \) of the population-coherence mapping sequence we obtain (step 4):

\[
\begin{align*}
    c_1(t_3) &= a_{1w} c_1(t_0) + a_{2w} c_2(t_0) e^{i\phi_1} + b_{1w} b_w c_1(t_0) + a_{2w} c_2(t_0) e^{i\phi_2} + b_{1w} b_w c_1(t_0) + c_2(t_0) e^{i\phi_3}, \\
    c_2(t_3) &= b_{1w} a_w c_1(t_0) + b_{2w} c_2(t_0) e^{i\phi_1} + a_{1w} b_w c_1(t_0) + a_{2w} c_2(t_0) e^{i\phi_2} + b_{1w} b_w c_1(t_0) + c_2(t_0) e^{i\phi_3}, \\
    c_3(t_3) &= b_{1w} a_w c_1(t_0) + b_{2w} c_2(t_0) e^{i\phi_1} + b_{1w} b_w c_1(t_0) + a_{2w} c_2(t_0) e^{i\phi_2} + a_{1w} b_w c_1(t_0) + c_2(t_0) e^{i\phi_3}. 
\end{align*}
\tag{21}
\]

We can write the density matrix element \( \alpha_{12} \) at the end of the population-coherence mapping sequence as

\[ \alpha_{12}(t_3) = \alpha_{11}(t_0) + \alpha_{22}(t_0) + \alpha_{11}(t_0) + \alpha_{22}(t_0) + \alpha_{22}(t_0). \tag{22} \]

Mapping of the coherence to populations and back will be successful, if we fulfill the requirements \( \alpha_{11} = \alpha_{22} = \alpha_{12} = 0 \) and \( \alpha_{12} \neq 0 \). Using solution (21) and ensemble averaging (19), we obtain the conditions (step 5):

\[
\begin{align*}
    \alpha_{11} &= \frac{1}{27} \left( 1 + 2 \cos \frac{3}{2} A_w \right) \left( \cos \frac{3}{2} A_r - 1 + 3i \sin \frac{3}{2} A_r \right), \\
    \alpha_{12} &= \frac{2}{27} \left( 1 - \cos \frac{3}{2} A_r \right) \left( \cos \frac{3}{2} A_w - \cos \frac{3}{2} A_r \cos \frac{3}{2} A_w - 3 \sin \frac{3}{2} A_r \sin \frac{3}{2} A_w \right), \\
    \alpha_{13} &= \frac{1}{27} \left( 1 - \cos \frac{3}{2} A_r \right) \left( \cos \frac{3}{2} A_w - \cos \frac{3}{2} A_r \cos \frac{3}{2} A_w + 3 \sin \frac{3}{2} A_r \sin \frac{3}{2} A_w \right), \\
    \alpha_{23} &= -\frac{1}{27} \left( 1 + 2 \cos \frac{3}{2} A_w \right) \left( 1 - \cos \frac{3}{2} A_r \right) \left( 1 + 3i \sin \frac{3}{2} A_r \right). \tag{23} \end{align*}
\]

From these equations, we see that \( \alpha_0 = 0 \), if

\[ \cos \frac{3}{2} A_w = -\frac{1}{2}. \tag{24} \]

This yields the required pulse area in the coherence-population ("write") mapping sequence

\[ A_w = \frac{4}{3} n + \frac{4}{3} m, \tag{25} \]

with an integer number \( n \).

The condition for the pulse area of the population-coherence "read" mapping sequence follows from \( \alpha_{12} = 0 \) and \( \alpha_{13} \neq 0 \). This yields:

\[ A_r = \frac{4}{3} n + \frac{4}{3} m, \tag{26} \]

and

\[ A_r = \frac{4}{3} n + \frac{4}{3} m. \tag{27} \]

We consider now the obtained final coherence after the mapping sequences. In case (26), the final density matrix reads:

\[ \alpha_{\text{fin}} = \frac{1}{3} \begin{bmatrix} 1 & \alpha_{12}^{\text{fin}} & \alpha_{22}^{\text{fin}} \alpha_{21}^{\text{fin}} \alpha_{22}^{\text{fin}} \\ \alpha_{12}^{\text{fin}} & 1 & \alpha_{22}^{\text{fin}} \alpha_{21}^{\text{fin}} \alpha_{22}^{\text{fin}} \\ \alpha_{22}^{\text{fin}} & \alpha_{21}^{\text{fin}} & 1 \end{bmatrix}. \tag{28} \]

We note, that also case (27) yields a similar matrix. Thus, after the mapping sequences we retrieve the coherence between states \( |1\rangle \) and \( |2\rangle \) with a factor of \( 2 \) compared to the initial coherence. However, this does not matter for an experiment on data storage. As long as there is coherence, we can generate a signal from the medium (e.g. by EIT-driven light storage and retrieval). We also note, that the initial coherence \( \alpha_{\text{fin}}^{\text{int}} \) is mapped onto \( \alpha_{12}^{\text{fin}} \) and \( \alpha_{22}^{\text{fin}} \) with additional phase shifts \( \pm \frac{\pi}{4} \). However, also this does not matter for data storage.
We note, that the quite simple conditions (26) and (27) fully define the required pulse areas for coincident pulses in the mapping sequence. Via the definition of the pulse area, the conditions implicitly include the specific temporal intensity pulse shapes (which for experimental simplicity we initially assumed similar for all pulses). We also stress the important point, that the deduced mapping sequences permit writing and reading of an arbitrary coherent superposition. The pulse sequences are not restricted, e.g. to mapping of a maximal coherence (as applied in EIT-driven light storage), but work for any coherent superposition.

2.1. Numerical simulations (coincident pulses)

To depict the dynamics of the quantum system in the case of coincident mapping pulses, we will show and discuss now numerical simulations of the process. Initially the system is in a coherent superposition of wave functions \( \psi_1 \) and \( \psi_2 \). We express the superposition as \( \Psi = \phi_1 \cos \chi + \phi_2 e^{-i \delta} \sin \chi \) with the amplitudes \( \cos \chi \) and \( \sin \chi \) and the phase \( \phi \). Thus, the only non-zero initial density matrix elements are \( \sigma_{11}(t_0) = \cos^2 \chi, \sigma_{22}(t_0) = \sin^2 \chi, \sigma_{12}(t_0) = e^{i \delta} \sin \chi \cos \chi \). As an example for our numerical simulation we choose the parameters of the coherent superposition \( \psi = \frac{\pi}{2} \) and \( \chi = \frac{\pi}{3} \). According to our above calculation we choose mapping pulses:

\[
\Omega(t) = \begin{cases} 
\frac{4}{\pi} A_{w2} \cos^4 \left( \pi - \frac{t-t_\text{write}}{2\tau} \right), & |t-t_\text{write}| \leq \frac{\tau}{2}, \\
0, & |t-t_\text{write}| > \tau,
\end{cases}
\]  

where \( t_\text{write} \) and \( t_\text{read} \) define the temporal position of peak Rabi frequencies in the “write” and “read” pulse sequence. The difference \( t_\text{read} - t_\text{write} \) gives the desired storage time. \( \tau \) is the pulse duration, which we can choose arbitrarily (provided the pulse duration is small compared to the decoherence time in the system). Fig. 4 shows the evolution of populations and coherence \( \sigma_{12} \) in the quantum system, driven by these mapping sequences with coincident pulses.

In our example, the initial absolute value of the coherence is \( \cos \chi \sin \chi = 0.43 \) and the initial phase is \( \phi = 0.2 \pi \) (see frame (c) for early times). After the coherence-population “write” mapping sequence, the system is driven to some specific distribution of populations in states \( |1 \rangle, |2 \rangle, |3 \rangle \) (see frame (b) for times immediately after the “write” mapping sequence). We note, that there is also some coherence in the medium (see frame (c) for times immediately after the “write” mapping sequence). However, for long storage times \( t_\text{read} - t_\text{write} \) the coherence fully decays (compare, step (3) of the analytic treatment).

Thus, the system evolves towards a purely incoherent superposition, fully described by the atomic populations. As the population lifetime is long compared to the decoherence time, the population distribution remains unchanged (compare frame (b) for times immediately after the “write” sequence and just before the beginning of the “read” sequence). After some storage time, the population-coherence “read” mapping sequence is applied onto the populations and generates a coherence from the population distribution (see frame (c) for times immediately after the “read” mapping sequence). Comparison of the initial values of \( \phi_{12} \) in frame (c) and the final value shows, that the phase of the coherence is fully restored after the mapping sequences. Note that the coherence amplitude before the beginning of the “read” mapping sequence is equal to zero. Thus, the coherence phase is undefined. When the “read” sequence starts, the phase becomes definite and equals to \( -\phi \). The final absolute value of the coherence reaches \( \frac{1}{2} \) of the initial magnitude — as expected from Eq. (28). Thus, the obtained pulse sequences successfully mapped the initial coherence onto long-lived populations and back again.

3. Consecutive mapping pulses

In the previous sections we discussed mapping sequences with coincident radiation pulses. In the following section we will now consider interaction of the medium with three consecutive pulses \( P, Q \) and \( Q \). Such a configuration might be appropriate for specific experimental situations, e.g. when only a single source is available to generate the three pulses. We assume fixed temporal pulse shapes (e.g. rectangular or Gaussian temporal profiles) for all pulses. The pulse overlap should be negligible, i.e. we assume well delayed pulses. We allow for different peak pulse intensities (rp. different pulse areas) in all pulses. However, coherent interaction requires fixed relative phases between the radiation fields. We denote pulse areas (step 1) for the radiation fields \( P, Q \) and \( Q \) in the coherence-population “write” sequence as \( A_{wP}, A_{wQ}, \) and \( A_{Qw} \) with phases \( \varphi_P, \varphi_Q, \varphi_Q \). The pulse areas for the radiation fields \( P, Q \) and \( Q \) in the population-coherence “read” sequence are \( A_{rP}, A_{rQ} \) and \( A_{Qr} \) with phases \( \varphi_P, \varphi_Q, \varphi_Q \). As we deal with separated pulses, we have to agree on the order of \( P, Q \) and \( Q \) pulses in the sequences. In the following, we will investigate a pulse order of \( P, Q, S \) in the coherence-population “write” stage and a reversed pulse order \( S, Q, P \) in the population-coherence “read” stage. We also assume \( \varphi_Q = 0 \). As in the case of coincident pulses, our goal is to determine pulse areas, which permit mapping of an initial coherence \( \sigma_{12} \) onto populations \( \sigma_{11}, \sigma_{22} \) and back again. A successful “write” and “read” mapping sequence will enable us to retrieve a fixed fraction of the initial coherence. Thus, we search for mapping sequences, which permit \( \sigma_{12}^{\text{final}} = \sigma_{12}^{\text{initial}} / 2 \).

The theoretical treatment for the interaction of the three-state quantum system with three separate pulses is less complicated compared to the case of coincident pulses. As each pulse couples only two states, we can solve the Schrödinger equation for three consecutive excitations in two-level systems, both for the “write” and the “read” sequence. The general procedure of the theoretical treatment is the same as already discussed in the previous sections: We calculate the state of the system after interaction with the coherence-population “write” sequence (step 2). We let the system freely evolve during the storage time. We assume, that only fast decoherence occurs, but slow population decay is negligible on our timescales (step 3). We apply the population-coherence “read” sequence to obtain the final state of the system (step 4). We consider the obtained, general expressions for the final state to determine the required parameters for successful mapping sequences (step 5). The calculation is cumbersome, but straightforward, and in general similar to the case of coincident pulses. Thus, we will not go through details of the calculation now, but discuss the final result only.

From the calculation we obtain the relevant density matrix element for the coherence at the end of the interaction with the “write” and “read” sequence:

\[
\sigma_{12}^{\text{final}} = \alpha_{11}\sigma_{11}^{\text{initial}} + \alpha_{22}\sigma_{22}^{\text{initial}} + 2\alpha_{12}\sigma_{12}^{\text{initial}}.
\]  

The coefficients \( \alpha_{11}, \alpha_{22} \) and \( \alpha_{12} \) vanish, if the pulse areas for \( P \) and \( S \) pulses in the “write” and “read” sequence are \( A = \pi/2 \), but one of these pulses must exhibit negative pulse area, i.e. \( A = -\pi/2 \) or \( A = -3\pi/2 \).

The pulse area of the \( Q \) pulse yields for both the “write” and “read” sequence \( A_{Qw} = A_{Qr} = \arccos \left( \frac{2}{3} \right) \). Table 1 shows examples of appropriate pulse areas, which permit storage and retrieval of the coherence.

The numerical coefficient \( \alpha_{12} \) yields \( \pm 1/3 \). The sign depends on the choice, whether either the \( P \) or the \( S \) pulse exhibits negative pulse area \( A = -\pi/2 \) or \( A = 3\pi/2 \). We note, that the other density matrix elements of the final state (which are not of interest here) yield expressions similar to Eq. (28). Thus, similar to the case of
coincident pulses, we also find for consecutive pulses, that the absolute value of the retrieved coherence is 1/3 of the initial value.

3.1. Numerical simulations (consecutive pulses)

To depict the dynamics of the quantum system in the case of consecutive mapping pulses, we will show and discuss now numerical simulations. Fig. 5 shows the evolution of populations and coherence $\sigma_{12}$ in the quantum system, driven by mapping sequences with coincident pulses. The initial conditions (e.g. for absolute value and the phase of the coherence $\sigma_{12}$) are the same as in the case of coincident pulses (compare Fig. 4). Also the temporal pulse shapes of the driving radiation fields are similar to the case of coincident pulses. However, the pulses are well delayed now, i.e. the pulse delays are larger than the pulse duration (see frame (a)).

The discussion of the population and coherence dynamics is similar to the case of mapping sequences with coincident pulses (see above, compare Fig. 4). Comparison of the initial and the final values of $\sigma_{12}$ in frame (c) shows, that the phase of the coherence is fully restored after the mapping sequences. The final absolute value of the coherence reaches $\frac{1}{3}$ of the initial magnitude. Thus, also the sequences of consecutive pulses successfully mapped the initial coherence onto long-lived populations and back again.

We note, that in the above treatment we ignored decoherence processes during the single pulses in the “write” and “read” mapping sequences. This is very well justified, if we assume sufficiently short pulse durations, i.e. below the decoherence time. We confirmed this by numerical simulations, yielding only a very small sensitivity of the restored phase with regard to coherence decay. If we assume, e.g. a decay rate $\gamma=0.01/\tau$ for all coherences, the restored phase differs only by 3% from the ideal case without any decoherence. For larger decay rate $\gamma=0.1/\tau$ the restored phase is 35% different from the ideal case. From these numerical simulations we conclude, that pulse durations should be roughly two orders of magnitude shorter than the coherence decay time in order to permit successful implementation of the protocol.

We also considered the effect of fluctuations in the pulse areas during the mapping sequences. As an example, we performed numerical simulations of the mapping process depicted in Fig. 5, when we change the optimal pulse areas by 1%. In this case, the restored phase differs only by 3% from the optimal case. A more detailed analysis shows, that for small fluctuations of the pulse areas the restored phase depends practically linear on the deviations from the optimal pulse area. Thus, the required stability in the pulse area is still in a tolerable range.

4. Discussion and notes

We note, that the population and coherence dynamics (e.g. as derived in the equations of the previous sections, or depicted by the numerical simulations in Figs. 4 and 5) look quite complicated — at least on a first glance. However, the obtained conditions for the pulse areas are indeed rather simple. The solution for consecutive pulses yields mapping sequences of simple $n/2$ pulses. Such pulses are well known from the field of coherent light–matter interactions. The conditions for the pulse areas in the case of coincident pulses look a bit more complicated (see Eqs. (26) and (27)). This mirrors the more complicated dynamics for three coincident excitations in a three-level system. However, also here we get solutions for the pulse areas which resemble common features of fractions of $n$ pulses in coherent light–matter interaction. Thus, on a second glance the dynamics and solutions of our mapping sequences appear rather familiar.

We also like to stress the point, that our concept to determine mapping sequences exhibits a very general approach. Above we discussed stopped light in doped solids as one possibility for coherent information storage. However, our mapping strategy will work also for other types of quantized memories — no matter which type of medium we use (e.g. an ensemble of ultracold atoms in a trap, spin states in a color center, ...) and no matter which type of technique is applied to initially store information in an atomic coherence. Moreover, mapping sequences may be implemented with RF excitations between hyperfine states or magnetic sub-levels (e.g. as discussed for the example of a doped solid above). However, also this is only one of several possibilities. Due to selection rules, it is not possible to implement the loop scheme with optically-driven, single-photon electric dipole transitions alone. However, we may replace a single-photon RF transition between two specific states by a two-photon optical transition between the same states. Thus, the general mathematical approach also permits determination of optical mapping sequences involving multi-photon excitations. This may be of advantage for specific experimental configurations.

5. Conclusions

We developed protocols to map an atomic coherence between two quantum states $|1\rangle$ and $|2\rangle$ into population distributions in a three-level system of states $|1\rangle$, $|2\rangle$ and $|3\rangle$, and later retrieve the coherence back from the populations. The process is driven by a coherence-population “write” pulse sequence (which converts the initial
coherence into populations) and a population-coherence “read” pulse sequence (which converts the population distribution back to a coherence). Both mapping sequences consist of three radiation pulses P, S and Q, which couple the three quantum states in a loop scheme. Field Q drives the transition between states |1\rangle and |2\rangle, which are involved in the atomic coherence. Fields P and S drive transitions from states |1\rangle or |2\rangle, towards the third state |3\rangle.

The radiation pulses in the mapping sequences may be applied as coincident or consecutive (i.e. separated) pulses of arbitrary temporal profile. As control parameters we choose the pulse areas (i.e. the pulse intensities and relative phases). We show, that rather simple pulse sequences permit the coherent information transfer between the population distribution and the atomic coherence. The protocol is expected to permit significantly enhanced storage times of atomic coherences. The mapping protocol exploits the fact, that in most atomic memories the lifetime of atomic populations is much longer compared to the atomic coherence decay times. Thus, the protocol permits conversion of a short-lived atomic coherence into long-lived atomic populations — and back again. Afterwards we may apply the retrieved coherence, e.g. to read out an optical data bit.

Field strength (arb. units)

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<th>A_S</th>
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<td>$\pi$</td>
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<tr>
<td>Read</td>
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Fig. 5. Numerical simulation of populations and coherence $c_{12}$, driven by consecutive, i.e. temporally separated mapping pulses P, Q and S. Frame (a) shows the time dependence of Rabi frequencies of “write” and “read” pulses. Frames (b) depicts the temporal evolution of the populations of states $|\psi_1\rangle$, $|\psi_2\rangle$ and $|\psi_3\rangle$. Frame (c) shows the temporal evolution of absolute value (blue, solid line, right y-axis) and phase (red, dashed line, left y-axis) of the coherence $c_{12}$. The phase is given in units of $\pi$. Pulse areas according to example no. 1 in Table 1.

Table 1

<table>
<thead>
<tr>
<th>No.</th>
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