

Double-barrier potentials for matter-wave gap solitons

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We investigate collisions of solitons of the gap type, supported by a lattice potential in repulsive Bose-Einstein condensates, with an effective double-barrier potential that resembles a Fabry-Perot cavity. We identify conditions under which the trapping of the entire incident soliton in the cavity is possible. Collisions of the incident soliton with an earlier trapped one are considered too. In the latter case, many outcomes of the collisions are identified, including merging, release of the trapped soliton with or without being replaced by the incoming one, and trapping of both solitons.

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I. INTRODUCTION

Bright matter-wave solitons [1,2] provide an exceptional test bed for studying quantum mechanics above the single-atom level. For the observation of quantum phenomena, coherence is a crucial requirement, and matter-wave solitons, which propagate without dispersion, may render this observation more feasible.

The generation of solitary waves in quasi-one-dimensional (Q1D) attractive Bose-Einstein condensates (BECs) [1] may be regarded as a tuned equilibrium between the dispersive effects that tend to spread the atomic wave function, and the nonlinear attractive interactions which oppose the spreading by providing an effective self-focusing of the matter waves. The consequence of such an equilibrium is a stable mesoscopic atomic wave packet propagating without dispersion.

Repulsive condensates in Q1D geometries can support matter-wave solitons of the gap type [2] if they are loaded in an optical lattice (OL), which gives rise to the band gap structure, and are placed at the edge of the first Brillouin zone, where there is a gap between the first and second bands [3]. Under these conditions, the soliton exhibits an effective *negative mass*, which permits the dispersion and the nonlinear interactions, even if they are repulsive to be balanced. Various effects generated by the negative effective mass were considered in Refs. [2,4–6].

Stability conditions for gap solitons (GSs) impose severe restrictions on their interactions with a potential well or barrier corresponding to a local modification of the periodic structure. Specifically, the requirement of stability allows only for perfect transmission or perfect reflection, but not the partial transmission and reflection that plane waves display [7]. In other words, GSs cannot split through the interaction with linear defects, and behave like particles exhibiting mesoscopic quantum features [5], including the quantum reflection of the entire soliton—an effect that has also been reported recently for matter-wave solitons in the self-attractive BEC case [8]. Although quantum reflection of ultracold atoms from a solid surface has been reported too [9], the limit

of complete reflection, predicted for solitons, is not achievable in that case. Interactions of matter-wave solitons with nonlinear traps and barriers produced by spatial variations of the scattering length have also been recently addressed [10].

In this work we aim to explore such quantum features in the case of the interaction of a matter-wave soliton of the gap type with a double-barrier potential resembling a Fabry-Perot cavity. In particular, we demonstrate that it is possible to trap a soliton in the cavity formed by the two potential barriers (a similar effect for optical solitons in fiber Bragg gratings was predicted in Ref. [11]). In the context of BEC, the propagation through a double-barrier potential acting as a Fabry-Perot interferometer for matter waves leads to bistability of the transmitted flux and resonant transport [12].

The paper is organized as follows. In Sec. II, we formulate the physical model which includes the effective cavity for the GS. Section III is devoted to the study of the conditions for the trapping of the entire soliton in the cavity. In Sec. IV, the collision of a second GS with one trapped in the cavity is studied (this interaction also bears some similarity to collisions between free and defect-trapped optical solitons in fiber gratings [13]). The paper is concluded in Sec. V.

II. THE PHYSICAL MODEL

The dynamics of bright GSs created in a Q1D geometry at zero temperature may be accurately described by the one-dimensional (1D) Gross-Pitaevskii equation (GPE)

$$i\hbar \frac{d\psi}{dt} = \left(-\frac{\hbar^2}{2m} \Delta + V(x) + g|\psi|^2 \right) \psi, \quad (1)$$

where the effective nonlinearity is $g \equiv 2\hbar a_s \omega_t$, with a_s the s -wave scattering length and ω_t the transverse trapping frequency. The effective axial potential

$$V(x) = (1/2)m\omega_x^2 x^2 + V_0 \sin^2(\pi x/d) \quad (2)$$

includes the parabolic trap with corresponding frequency ω_x and the OL with spatial period d and depth V_0 .

First, we briefly summarize the numerical procedure used to generate GSs, as per Ref. [4]. The ground state is found

for a ^{87}Rb condensate ($a_s=5.8$ nm) formed by $N=500$ atoms trapped magnetically with transverse and axial frequencies $\omega_t=715 \times 2\pi$ Hz and $\omega_x=14 \times 2\pi$ Hz, respectively, in the presence of an OL, with potential depth equal to the OL recoil energy, $V_0=E_r \equiv \hbar^2 k^2 / 2m$, where $k=\pi/d$ is the recoil momentum and $d=397.5$ nm the period. Then, the axial magnetic trap is suddenly turned off and an appropriate phase imprinting leads to the inversion of the sign of the wave function at each second site. As a result, the system evolves toward a self-sustained staggered soliton with a negative effective mass. The soliton, which contains approximately 35% of the initial number of atoms and extends over ≈ 11 sites of the OL potential, is generated at rest with the center at $x=0$. In order to set it into motion, we instantaneously impart an appropriate momentum to the soliton, to which it responds by self-adaptation to the new conditions, i.e., by expunging atoms until a new equilibrium state is reached. With momentum $p=0.1k\hbar$ lent to the GS, it settles down into a state with 27% of the initial number of atoms ($N_{\text{final}}=135$), total energy $0.92E_r$, and the kinetic energy of its motion $E_k=0.01E_r$ [5].

Our aim is to study the interaction of the moving matter-wave soliton so generated with a Fabry-Perot type potential, formed by two potential barriers creating a cavity for the soliton; cf. the similar configuration proposed for optical solitons in fiber gratings [11]. After turning off the magnetic trap, $V(x)$ in Eq. (1) accounts only for the OL, which is locally modified around two sites x_{m1} and x_{m2} to generate the double-barrier structure as follows:

$$V(x) = V_0 \sin^2(\pi x/d) + V_{\text{mod}}(x), \quad (3)$$

where $V_{\text{mod}}(x)$ reads

$$\begin{aligned} & V_{m1} \left(1 - \frac{(x-x_{m1})^2}{2\sigma^2} \right) & \text{if } x_{m1} - l/2 \leq x \leq x_{m1} + l/2, \\ & V_{m2} \left(1 - \frac{(x-x_{m2})^2}{2\sigma^2} \right) & \text{if } x_{m2} - l/2 \leq x \leq x_{m2} + l/2, \\ & 0 & \text{otherwise,} \end{aligned} \quad (4)$$

with $\sigma=6d$. Below, we consider the symmetric cavity created by two identical barriers, with $V_{m1}=V_{m2}=V_m < 0$. Note that, due to the negative effective mass of the GS, the barriers actually correspond to a local *decrease* of the periodic potential. Points $x_{m1,2}$ are fixed at local minima of the OL potential, and the distance between the barrier centers, $A \equiv x_{m2} - x_{m1}$, is assumed large enough to have the actual size of the cavity, $B=A-l$, much larger than the axial size of the soliton. The results presented below have been obtained with the potential given by Eq. (4), which actually represents a set of two nearly rectangular barriers. Such a potential can be created by means of a pair of sharply focused optical beams, and it permits control of the number of sites where the lattice is modified. We have checked that the soliton dynamics remain nearly the same when the cavity is formed, instead, by a pair of smooth Gaussian potentials.

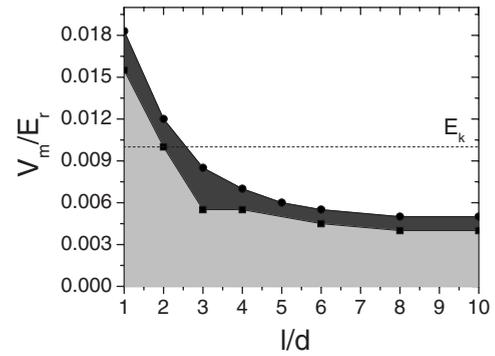


FIG. 1. Diagram of outcomes of the collisions of a moving lattice soliton with a cavity formed by two identical barriers: reflection, transmission, and trapping, in the white, light gray, and dark gray areas, respectively. The effective size of the cavity is $B=20d$. Parameters l (measured in units of the lattice period d) and V_m (measured in units of the recoil energy E_r) are the width and height of the two barriers, respectively. The horizontal dotted line denotes the kinetic energy of the moving soliton.

III. THE CAVITY

The interaction of a moving matter-wave GS with a single barrier in an OL was addressed recently in Ref. [5], where it was shown that the soliton does not split. For a fixed kinetic energy, there exists an abrupt transition from complete transmission to complete reflection, as the height of the barrier increases, for all considered values of the barrier widths. The border between these two outcomes of the collision of the soliton with the barrier is shown, in the plane of the barrier's parameters, height V_m and width l , by filled circles in Fig. 1. Complete reflection and transmission occur, respectively, above and below this border (i.e., the white area in the figure corresponds to the bounce of the entire soliton from the barrier). In the process of the collision, the soliton naturally decreases its velocity (even if does not bounce). This slowing down of the soliton in the region of the defect is more pronounced as we approach the border between the two behaviors.

Addressing the configuration with the second barrier placed at a certain distance from the first one, we notice that the behavior of the incident soliton that hits the first barrier remains as described before, i.e., a sudden transition from perfect transmission to perfect reflection occurs. Nevertheless, when the height of the barriers is close to the transition point, and the length of the cavity (B) is larger than the size of the soliton, the soliton that has passed the first barrier gets trapped in the cavity, in a state of oscillatory motion. Thus, three scenarios are identified in the interaction of the incident soliton with the double-barrier structure: complete reflection, complete transmission, and trapping into the oscillatory state. As said above, the reflection occurs in the same area of the parameter space (white region in Fig. 1) as for the single barrier. Complete transmission takes place for values of the barrier's height and width well inside the transmission region for the single barrier (the light gray area in Fig. 1), while trapping is observed close to the transmission-reflection border for the single barrier (the dark gray region in Fig. 1). The

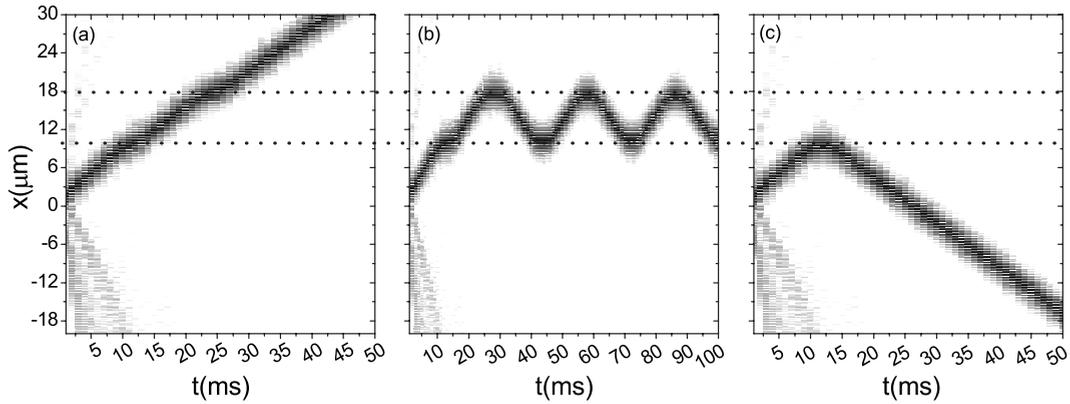


FIG. 2. Contour plots of the spatiotemporal evolution of the lattice soliton colliding with a set of two identical barriers centered at $x_{m1}=25d$ and $x_{m2}=45d$, each with width $l=2d$ and the following height: $|V_m| =$ (a) $0.008E_r$; (b) $0.011E_r$; (c) $0.014E_r$. Here as well as in the following figures the horizontal dotted lines show the positions of the centers of both barriers.

results shown in Fig. 1 correspond to a fixed cavity size of $B=20d$, and we have checked that the parameter space for trapping slightly increases with the size of the cavity. For a fixed width of the barriers, and a large enough distance between them, the three scenarios follow each other with increase of the barriers' height. These scenarios are illustrated in Fig. 2 by spatiotemporal trajectories of the soliton hitting the cavity formed by two identical barriers of width $l=2d$, which are separated by distance $A=20d$, giving the cavity enough room to trap the soliton, $B=18d$. Complete transmission is displayed in Fig. 2(a) for $|V_m|=0.008E_r$, Fig. 2(b) with $|V_m|=0.011E_r$ shows an example of trapping, and the

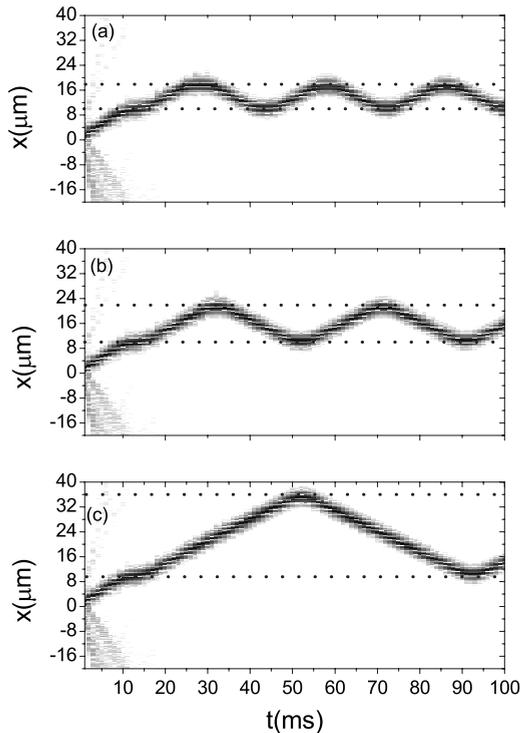


FIG. 3. Examples of the trapping of the lattice soliton by a pair of barriers of width $l=2d$ and height $|V_m|=0.011E_r$. The first barrier is centered at $x_{m1}=25d$, and the second at (a) $x_{m2}=45d$, (b) $55d$, and (c) $90d$.

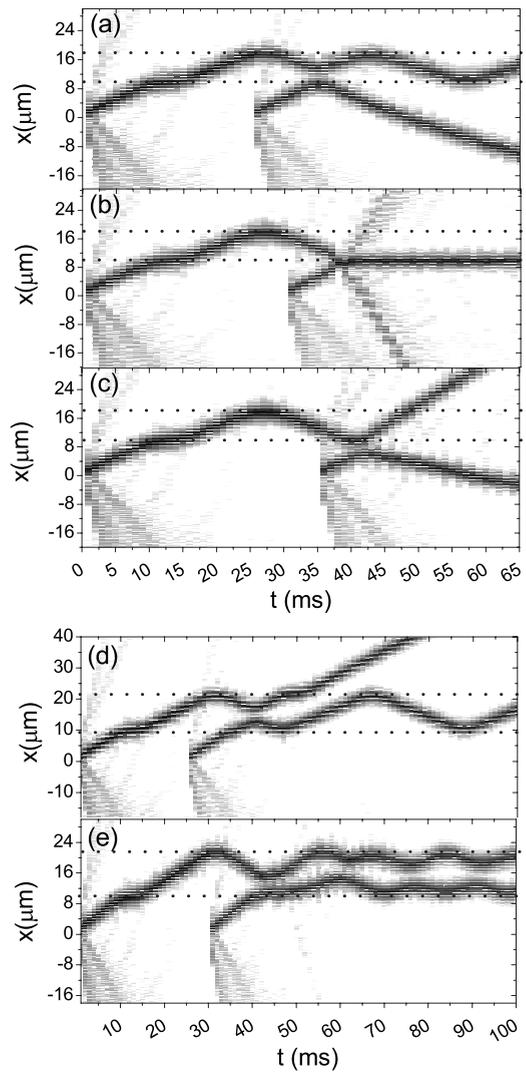


FIG. 4. Different outcomes of the collision between an incident GS with an identical one previously trapped. The barriers of width $l=2d$ and height $|V_m|=0.011E_r$ are separated by a distance $A=20d$ in (a)–(c), or $A=30d$ in (d) and (e). The second soliton is sent after the first one with time delay (a) $\Delta t=20$, (b) 30, (c) 35, (d) 25, and (e) 30 ms.

bounce (complete reflection) is observed in Fig. 2(c), for $|V_m|=0.014E_r$.

Figure 2(b) clearly demonstrates that the trapped soliton performs periodic oscillations in the cavity without any visible loss of atoms. The period of the oscillations, for given parameters of the barriers, can be modified by changing the size of the cavity, as shown in Fig. 3. Note that the smallest period, limited by the condition that the size of the cavity must be larger than the soliton's axial size, is ≈ 30 ms, in physical units. Faster oscillations were predicted for a lattice soliton trapped in a potential well [5]. It is relevant to note that, in the above-mentioned case of the trapping of optical solitons by a cavity in the model of a fiber grating [11], the maximum capture efficiency (in terms of the soliton's energy) is no more than 60%, contrary to what happens with the matter-wave GSs, which may be trapped *entirely*, without losses.

IV. COLLISIONS BETWEEN FREE AND TRAPPED SOLITONS

The next natural step in the analysis is to consider a collision between an incident soliton with the cavity already occupied by an (identical) earlier trapped GS. Different outcomes of the collisions are observed, depending on the time delay Δt between the two solitons. Figure 4 displays three cases for the cavity formed by two barriers of width $l=2d$ and height $|V_m|=0.011E_r$, separated by a distance of $A=20d$, and $\Delta t=$ (a) 20, (b) 30, and (c) 35 ms. In Fig. 4(a), the incident soliton bounces back, while the trapped one performs oscillations in the cavity; in Fig. 4(b), the two solitons *merge* into a single one, and in Fig. 4(c), the incident soliton bounces back, kicking out the trapped one in the forward direction. Two more cases are shown in Fig. 4 for the same parameters of the barriers, but for a different separation between them, $A=30d$, and time delays $\Delta t=25$ ms in Fig. 4(d), and $\Delta t=30$ ms in Fig. 4(e). In this case, the cavity has enough room to trap the two solitons, which gives rise to new collision scenarios: in Fig. 4(d) the incoming soliton gets trapped by kicking out the previously trapped one ("recharge"), while in Fig. 4(e) *both* solitons get trapped in the

cavity, oscillating in counterphase. These sundry dynamical behaviors suggest new experimental possibilities for the control and manipulation of matter-wave GSs.

V. CONCLUSION

In this work, we have studied the interactions of bright matter-wave solitons of the gap type, with negative effective mass, which are supported by the interplay of the OL and repulsive nonlinearity in a BEC, with a cavity formed by two far separated identical local potential barriers. We have shown that there exists a parameter region in which the incident soliton is trapped by the cavity into a shuttle state. This region can be found for all values of the barriers' width, provided that their height corresponds to the transmission of the soliton by a single barrier, but close to the reflection-transmission border.

The interaction of a second soliton which hits the cavity already occupied by a trapped oscillating soliton has been considered too. In that case, a number of different collision scenarios can be identified, depending on the time delay between the launch of the two solitons. Particularly interesting outcomes are the merging of the two solitons at the position of the first barrier, bounce of the second soliton kicking out the trapped one, the recharge, i.e., release of the originally trapped soliton which is replaced by the incident one, and trapping of both solitons into the state of shuttle oscillations in the cavity, with a phase shift of π between them.

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